A Compositional Semantics for Verified Separate Compilation & Linking

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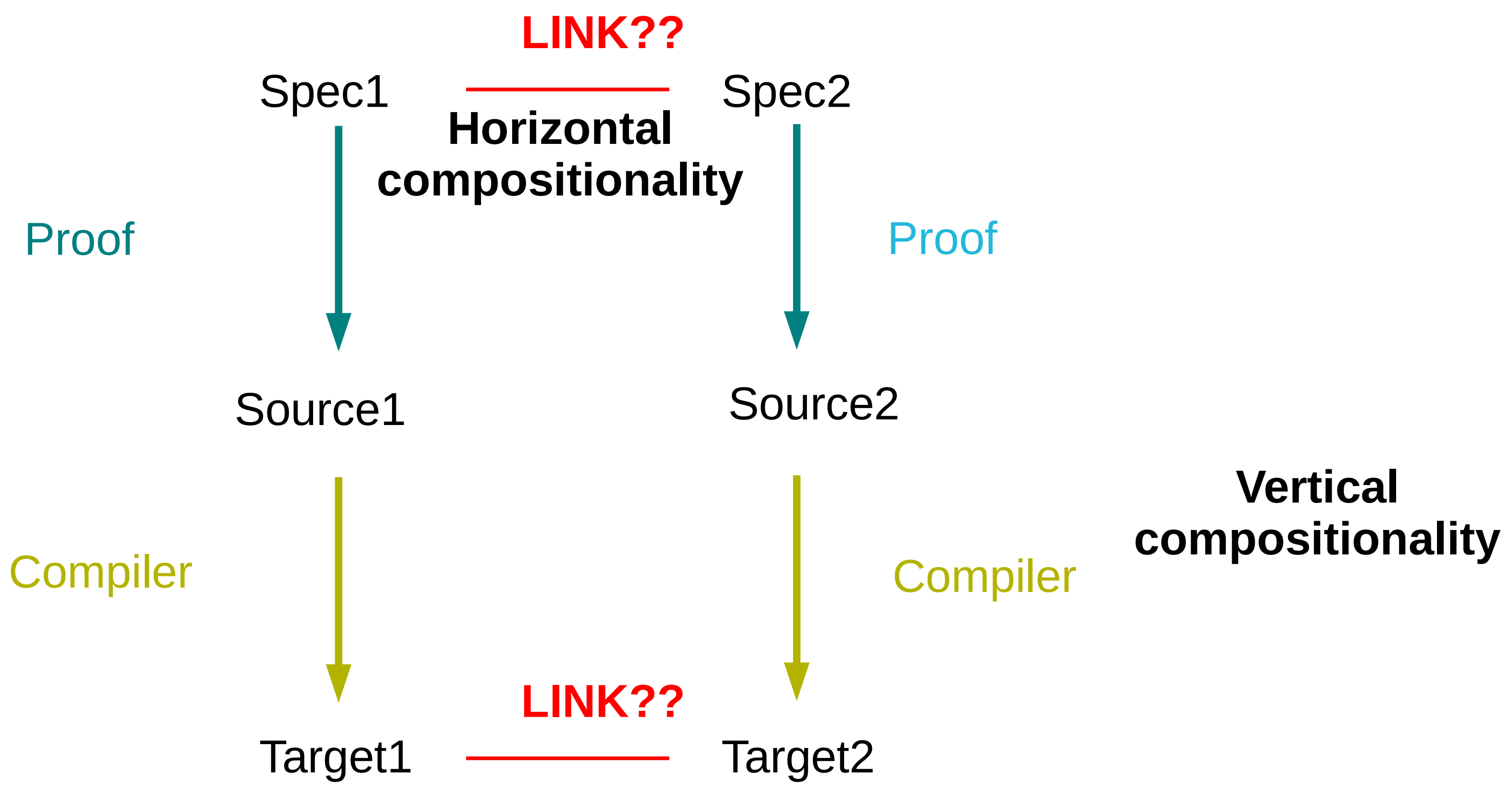
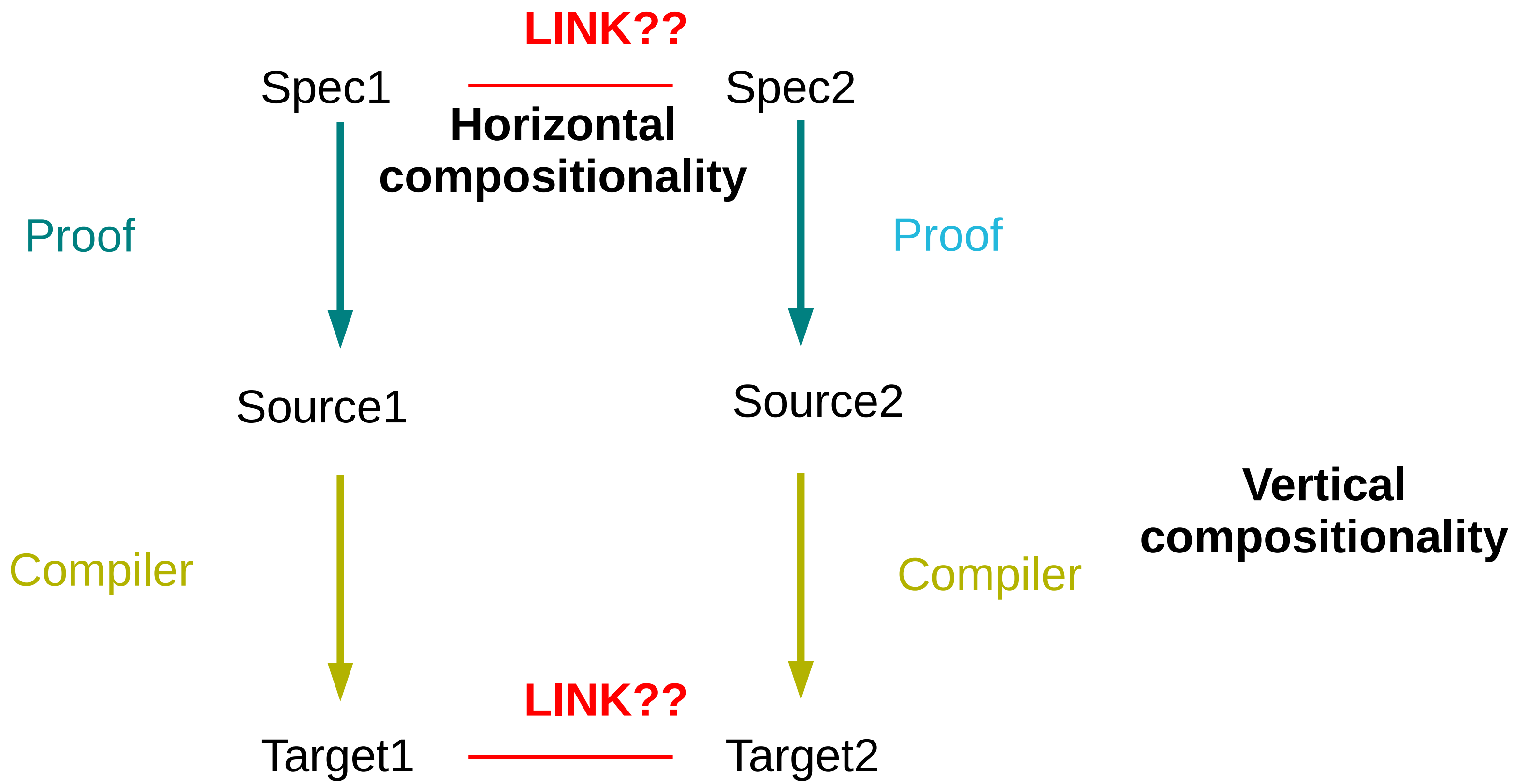
Vertical vs. horizontal composition (Hur et al. POPL 2012)



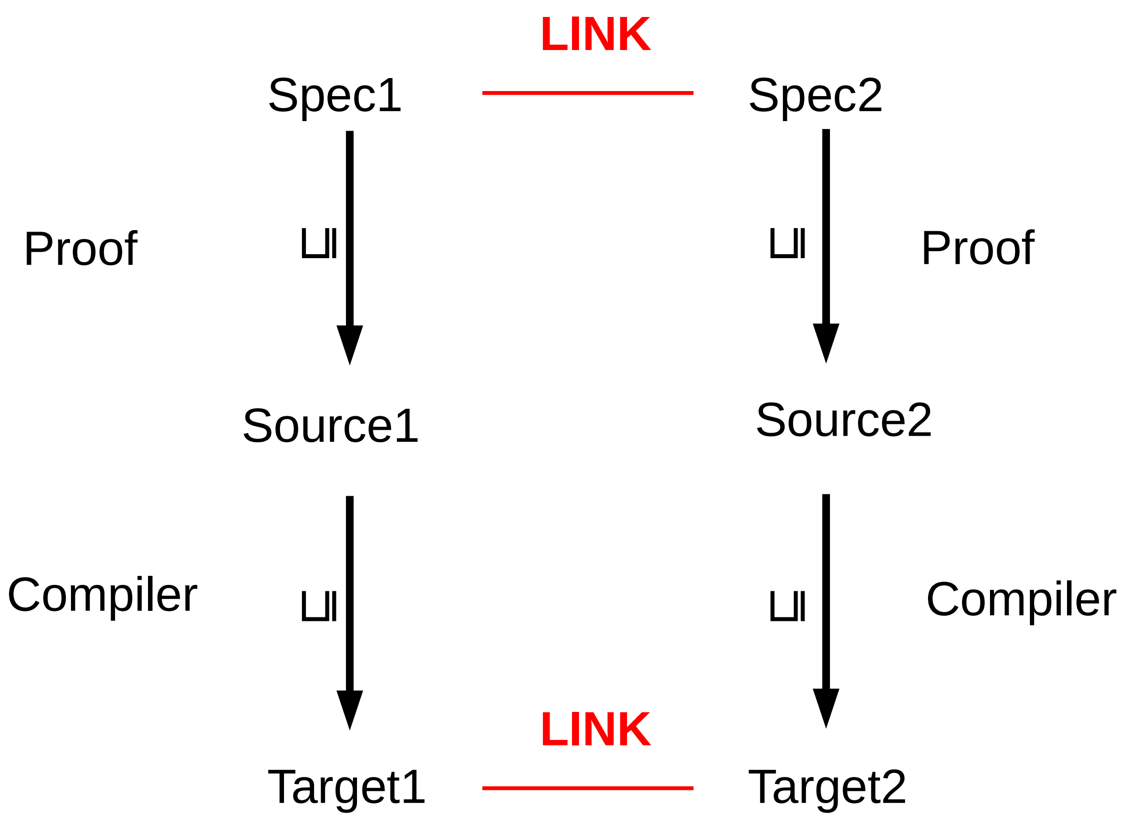
How to preserve component proofs by compilation and linking?

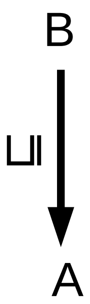


CompCert (Leroy POPL 2006) cannot preserve proofs by linking

- Works only for whole programs
- No correctness statement for open modules

Our unified approach: compositional semantics +refinement





A ⊑ B, reads “A refines B”
Based on a compositional semantics

<u>Research challenges</u>

- What is the semantics of an open module?
- How to generalize compiler correctness to open modules?
- How to connect to compositional program logics?

Our contributions

1. Semantics of open modules
2. Semantic linking operator
3. Linking theorem
4. Compositional refinement
5. Refinement for memory-changing passes

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Reminder: Operational small-step semantics

- Usual way to describe the machine semantics of the executable

$$s \xrightarrow{\text{eventList}} s'$$

- Not suitable to describe compiler correctness at the higher level
  - Too fine-grained
  - Optimizations can change intermediate states
- Not compositional for linking purposes
  - Only for whole programs

Observable program behaviors

- Big-step the small-step semantics
- **[**Prog**]** ⊆

  {Terminates(eventList), Stuck(eventList), Diverges(eventList), Reacts(eventStream)}
- Compiler correctness: program behavior refinement:

  [Compiler(Prog)**]** ⊑ **[**Prog**]**

Examples

The C program...	… has the behavior

`int main () {`
`   printf('a');`
`   return 2;`
`}`
	OUT (a) . Terminates(2)

`int main () {`
`   printf('a');`
`   3/0;`
`   return 4;`
`}`
	OUT (a) . Stuck

`int main () {`
`   printf('a');`
`   while (1) {};`
`   return 5;`
`}`
	OUT (a) . Diverges

`int main () {`
`   while (1) { printf('b'); };`
`   return 6;`
`}`
	OUT (b) :: … :: OUT (b) :: … (Reacts)

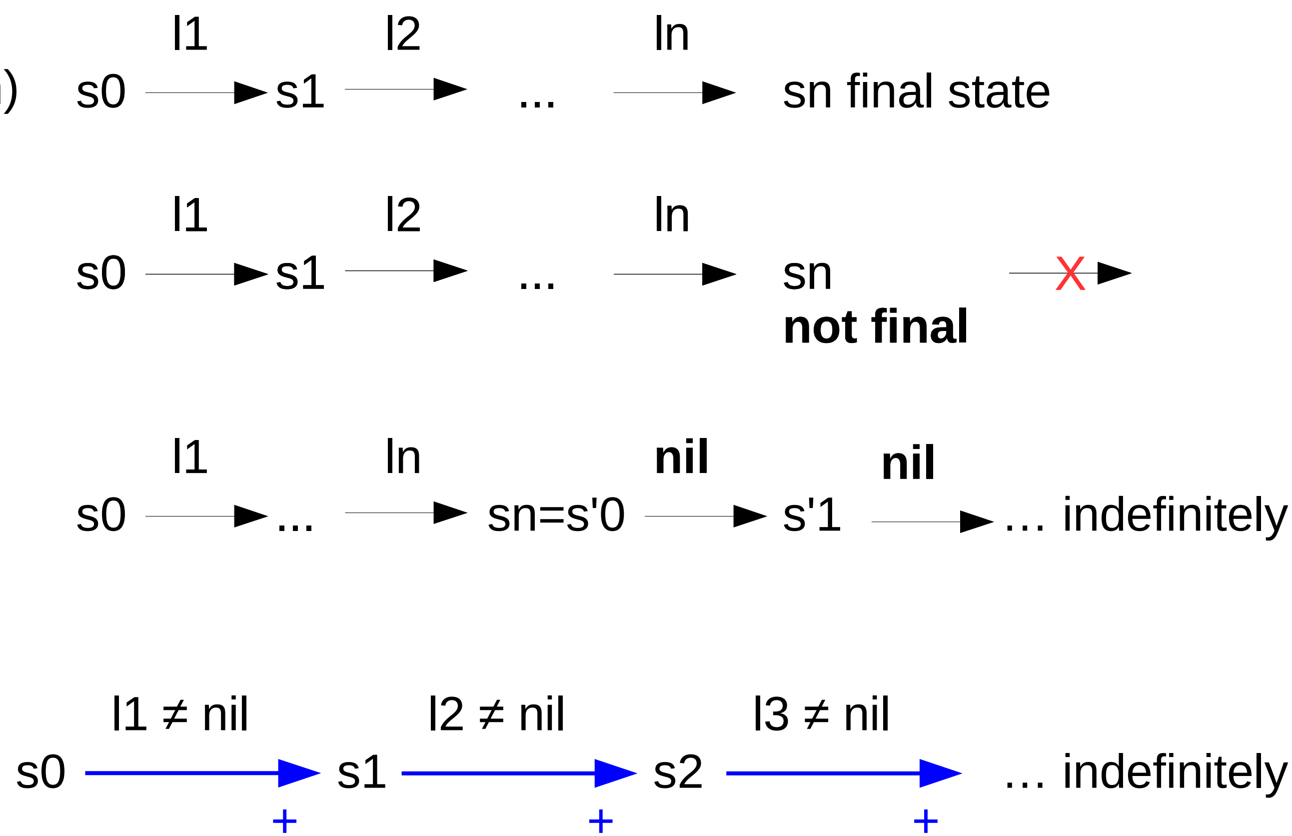
Big-stepping
the small-step semantics

Terminates(l1 ++ l2 ++ … ++ ln)

Stuck(l1 ++ l2 ++ … ++ ln)

Diverges(l1 ++ l2 ++ … ++ ln)

Reacts(l1 +++ l2 +++ ...)



How to deal with input?

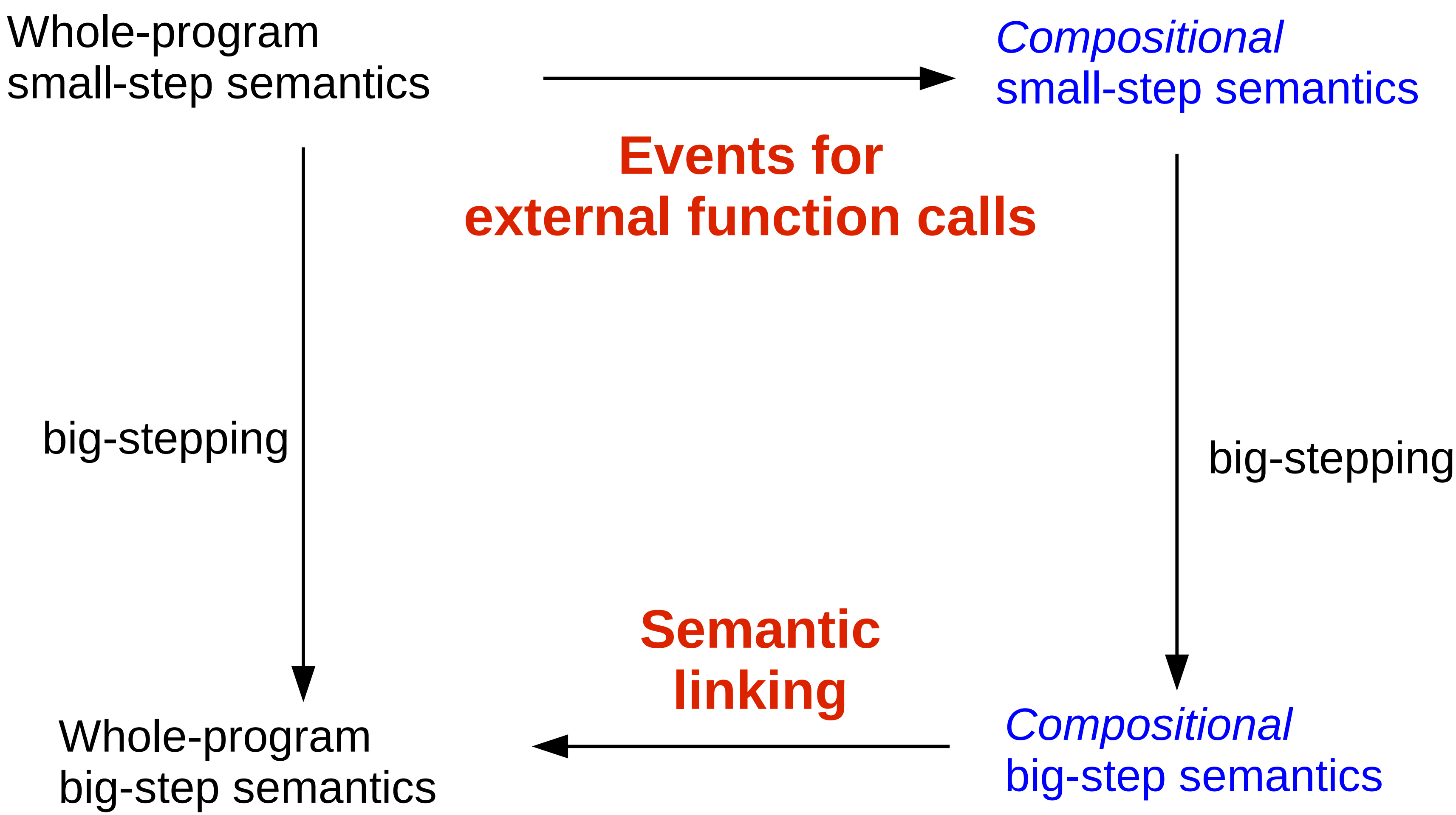
Provide a behavior for each possible input value.

The remaining behavior depends on that input value.

```
int main() {
  char x = getchar();
  printf("%c", x);
  return 0;
}
```

{
IN(a) :: OUT(a) . Terminates(0),
IN(b) :: OUT(b) . Terminates(0),
IN(c) :: OUT(c) . Terminates(0),
...}

From Operational to Compositional Semantics



Compositional semantics

- Events for external function calls
  - Provide behaviors for each possible return value and return memory state (like input)

m = {f ↦ `int x=18; int y=g(&x); printf("%d %d", y, x);` **}**

《 m 》 (f) = {
Extcall(g, [x->18], &x, 0, [x->0]) :: OUT 0 :: OUT 0 . Terminates,
Extcall(g, [x->18], &x, 0, [x->1]) :: OUT 0 :: OUT 1 . Terminates,
…
Extcall(g, [x->18], &x, 1, [x->0]) :: OUT 1 :: OUT 0 . Terminates,
Extcall(g, [x->18], &x, 1, [x->1]) :: OUT 1 :: OUT 1 . Terminates,
…}

Compositional semantics

- Events for external function calls
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m = {f ↦ `int x=18; int y=g(&x); printf("%d %d", y, x);` **}**

《 m 》 (f) = {

Extcall(g, [x->18], &x, 0, [x->0]) :: OUT 0 :: OUT 0 . Terminates,
Extcall(g, [x->18], &x, 0, [x->1]) :: OUT 0 :: OUT 1 . Terminates,
…
Extcall(g, [x->18], &x, 1, [x->0]) :: OUT 1 :: OUT 0 . Terminates,
Extcall(g, [x->18], &x, 1, [x->1]) :: OUT 1 :: OUT 1 . Terminates,
}

Callee

Memory state before call

Arguments

Compositional semantics

- Events for external function calls
  - Provide behaviors for each possible return value and return memory state (like input)

```
m = {f ↦ int x=18; int y=g(&x); printf("%d %d", y, x);}
```

《 m 》 (f) = {
Extcall(g, [x->18], &x, 0, [x->0]) :: OUT 0 :: OUT 0 . Terminates,
Extcall(g, [x->18], &x, 0, [x->1]) :: OUT 0 :: OUT 1 . Terminates,
…
Extcall(g, [x->18], &x, 1, [x->0]) :: OUT 1 :: OUT 0 . Terminates,
Extcall(g, [x->18], &x, 1, [x->1]) :: OUT 1 :: OUT 1 . Terminates,
}

Return value

Memory state after return

Compositional semantics

- Function semantics parameterized on arguments and memory state before call
- Terminating behaviors also bear return value and return memory state

m' = {g(int* x**) ↦** `int y=*x; printf("%d", y-1); *x=y+1; return y;` **}**

《 m' 》 (g)(p)[p->*n*] = {
OUT (*n*-1) . Terminates(*n*, [p->*n*+1])
}

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Semantic Linking

m1 = {f1 ↦ call f2}

《 m1 》 (f1) = Extcall(f2) Terminates

⋈

m2 = {f2 ↦ call f1}

《 m2 》 (f2) = Extcall(f1) Terminates

《 m1 》⋈《 m2 》 (f1) = {Diverges}
《 m1 》⋈《 m2 》 (f2) = {Diverges}

m1 ⊎ m2 = {f1 ↦ call f2, f2 ↦ call f1}

《 m1 ⊎ m2 》 = 《 m1 》⋈《 m2 》

- Defined at the semantic level of big-step behaviors
  - No need for the underlying small-step semantics
  - Can link semantics of modules of different languages
- Main technical challenge (mechanized Coq proof)

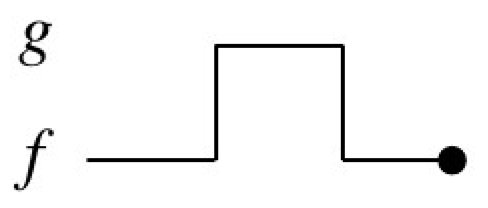
The linking operator

- “Replace” each external function call event with a behavior of the callee
- Linking based on a *resolution* operator R performing those replacements:

$$\psi 1 \bowtie \psi 2 = R(\psi 1 \uplus \psi 2)$$

The resolution operator
Example #1: Terminating case

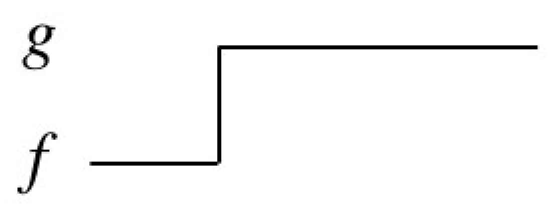
ψ = {g ↦ Terminates ;
f ↦ Extcall(g) . Terminates }



R(ψ) = {g ↦ Terminates ;
f ↦ Terminates }

The resolution operator
Example #2: Diverging case

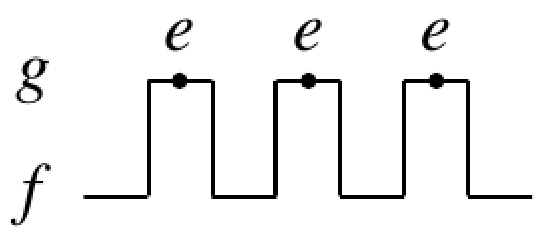
ψ = {g ↦ Diverges ;
f ↦ Extcall(g) :: Print 2 . Terminates }



R(ψ) = {g ↦ Diverges ;
f ↦ Diverges }

The resolution operator
Example #3: Infinitely many externals

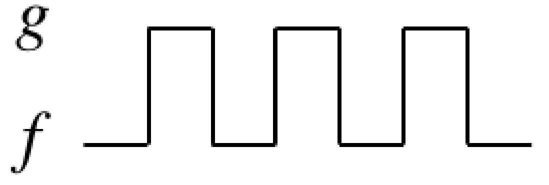
ψ = {g ↦ Print(e) . Terminates ;
f ↦ Extcall(g) :: Extcall(g) :: … :: Extcall(g) :: ... }



R(ψ) = {g ↦ Print(e) . Terminates ;
f ↦ Print(e) :: Print(e) :: … :: Print(e) :: ... }

The resolution operator
Example #4: Infinitely many externals

ψ = {g ↦ Terminates ;
f ↦ Extcall(g) :: Extcall(g) :: … :: Extcall(g) :: ... }

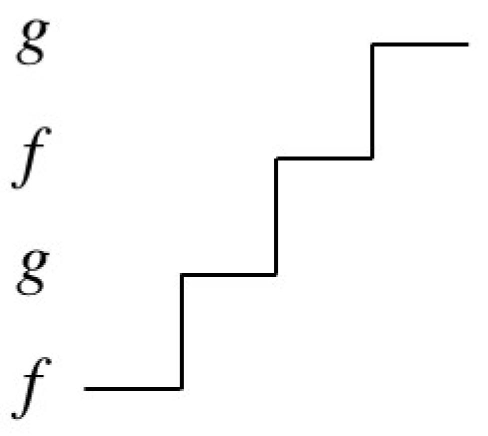


R(ψ) = {g ↦ Terminates ;
f ↦ Diverges }

Eager replacement will fail.

The resolution operator Example #5: (mutual) recursion

ψ = {g ↦ Extcall(f) :: Print(a) . Terminates ;
f ↦ Extcall(g) :: Print(b) . Terminates }



R(ψ) = {g ↦ Diverges ;
f ↦ Diverges }

How resolution works

- *Behavior simulation* small-step semantics
- Big-step this small-step semantics

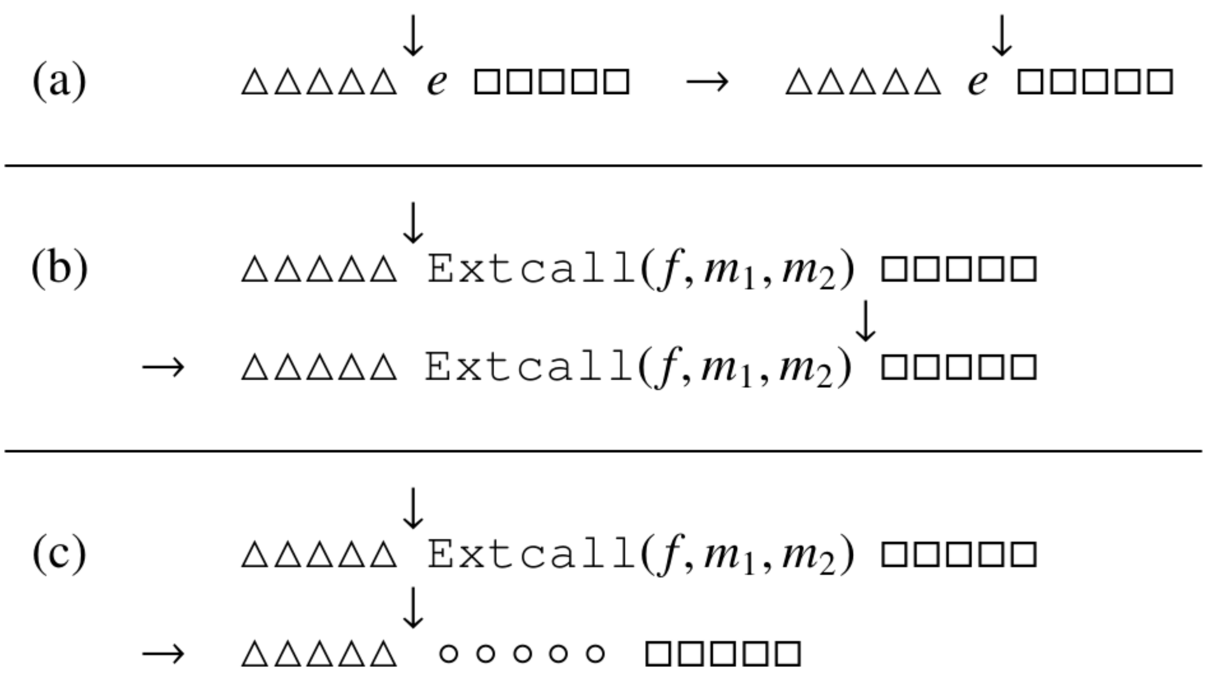


Fig. 1. Three cases in behavior simulation: (a) regular event; (b) $f \notin \mathrm{dom}(\psi)$; (c) $\circ\circ\circ\circ\circ \in \psi(f)$.

How resolution works

$\psi = \{g \mapsto \text{Print}(e) \,.\, \text{Terminates} \;;$
$f \mapsto \text{Extcall}(g) :: \text{Extcall}(g) \,.\, \text{Terminates}\;\}$

How to compute $R(\psi)(f)$?

How resolution works

ψ = {g ↦ Print(e) . Terminates ;
f ↦ Extcall(g) :: Extcall(g) . Terminates }

How to compute R(ψ)(f) ?

Extcall(g) :: Extcall(g) . Terminates , []

How resolution works

ψ = {g ↦ Print(e) . Terminates ;
f ↦ Extcall(g) :: Extcall(g) . Terminates }

How to compute R(ψ)(f) ?

Extcall(g) :: Extcall(g) . Terminates , []

How resolution works

ψ = {g ↦ Print(e) . Terminates ;
f ↦ Extcall(g) :: Extcall(g) . Terminates }

How to compute R(ψ)(f) ?

Print(e) . Terminates , [Extcall(g) . Terminates]

How resolution works

$\psi = \{g \mapsto \text{Print(e) . Terminates ;}$
$f \mapsto \text{Extcall(g) :: Extcall(g) . Terminates } \}$

How to compute $R(\psi)(f)$?

Print(e) . Terminates , [Extcall(g) . Terminates]

How resolution works

$\psi = \{g \mapsto \text{Print}(e) \,.\, \text{Terminates} \;;$
$f \mapsto \text{Extcall}(g) :: \text{Extcall}(g) \,.\, \text{Terminates}\, \}$

How to compute $R(\psi)(f)$?

Terminates , [Extcall(g) . Terminates]

Print(e) .

How resolution works

ψ = {g ↦ Print(e) . Terminates ;
f ↦ Extcall(g) :: Extcall(g) . Terminates }

How to compute R(ψ)(f) ?

Terminates , [Extcall(g) . Terminates]

Print(e) .

How resolution works

ψ = {g ↦ Print(e) . Terminates ;
f ↦ Extcall(g) :: Extcall(g) . Terminates }

How to compute R(ψ)(f) ?

Extcall(g) . Terminates , []

Print(e) .

How resolution works

ψ = {g ↦ Print(e) . Terminates ;
f ↦ Extcall(g) :: Extcall(g) . Terminates }

How to compute R(ψ)(f) ?

Extcall(g) . Terminates , []

Print(e) .

How resolution works

ψ = {g ↦ Print(e) . Terminates ;
f ↦ Extcall(g) :: Extcall(g) . Terminates }

How to compute R(ψ)(f) ?

Print(e) . Terminates , [Terminates]

Print(e) .

How resolution works

ψ = {g ↦ Print(e) . Terminates ;
f ↦ Extcall(g) :: Extcall(g) . Terminates }

How to compute R(ψ)(f) ?

Print(e) . Terminates , [Terminates]

Print(e) .

How resolution works

$\psi = \{g \mapsto \text{Print}(e)\ .\ \text{Terminates}\ ;$
$f \mapsto \text{Extcall}(g) :: \text{Extcall}(g)\ .\ \text{Terminates}\ \}$

How to compute $R(\psi)(f)$?

Terminates , [Terminates]

Print(e) :: Print(e) .

How resolution works

ψ = {g ↦ Print(e) . Terminates ;
f ↦ Extcall(g) :: Extcall(g) . Terminates }

How to compute R(ψ)(f) ?

Terminates , [Terminates]

Print(e) :: Print(e) .

How resolution works

ψ = {g ↦ Print(e) . Terminates ;
f ↦ Extcall(g) :: Extcall(g) . Terminates }

How to compute R(ψ)(f) ?

Terminates , []

Print(e) :: Print(e) .

How resolution works

$\psi = \{g \mapsto \text{Print}(e) \ . \ \text{Terminates} \ ;$
$f \mapsto \text{Extcall}(g) :: \text{Extcall}(g) \ . \ \text{Terminates} \ \}$

How to compute $R(\psi)(f)$?

$\text{Print}(e) :: \text{Print}(e) \ . \ \text{Terminates}$

Resolution and (mutual) recursion

$\psi = \{g \mapsto \text{Extcall}(f) :: \text{Print}(a) \,.\, \text{Terminates} \,;$
$f \mapsto \text{Extcall}(g) :: \text{Print}(b) \,.\, \text{Terminates} \,\}$

How to compute $R(\psi)(g)$?

Resolution and (mutual) recursion

$\psi = \{g \mapsto \text{Extcall}(f) :: \text{Print}(a) \,.\, \text{Terminates} \,;$
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How to compute $R(\psi)(g)$?

Extcall(f) :: Print(a) . Terminates ,
[]

Resolution and (mutual) recursion

ψ = {g ↦ Extcall(f) :: Print(a) . Terminates ;
f ↦ Extcall(g) :: Print(b) . Terminates }

How to compute R(ψ)(g) ?

Extcall(g) :: Print(b) . Terminates ,
[Print(a) . Terminates]

Resolution and (mutual) recursion

ψ = {g ↦ Extcall(f) :: Print(a) . Terminates ;
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How to compute R(ψ)(g) ?

Extcall(f) :: Print(a) . Terminates ,
[Print(b) . Terminates ;
Print(a) . Terminates]

Resolution and (mutual) recursion

ψ = {g ↦ Extcall(f) :: Print(a) . Terminates ;
f ↦ Extcall(g) :: Print(b) . Terminates }

How to compute R(ψ)(g) ?

Extcall(g) :: Print(b) . Terminates ,
[Print(a) . Terminates ;
Print(b) . Terminates ;
Print(a) . Terminates]

Resolution and (mutual) recursion

ψ = {g ↦ Extcall(f) :: Print(a) . Terminates ;
f ↦ Extcall(g) :: Print(b) . Terminates }

How to compute R(ψ)(g) ?

… and so on : it will diverge
[Print(b) . Terminates
Print(a) . Terminates ;
Print(b) . Terminates ;
Print(a) . Terminates]

Relation with denotational semantics

- Parameterization needs care (intension, or coinductive local vs. global knowledge) to make fixpoints work
- Our work needs no such “fixpoint” thing besides regular big-stepping of small-step semantics

Higher-order functions

- iter f (x1 :: … :: xn :: nil)
- Semantics depends on symbol resolution and module in which iter is defined:
  - If f is an external function, then: Extcall(f) :: … :: Extcall(f) . Terminates
  - If f is a function defined in the same module as iter, then no such events

Challenges

- How to cope with return values / return memory state?

Reminder: compositional semantics

- Provide behaviors for each possible return value and each possible return memory state

m1 = {f ↦ `int x=18; int y=g(&x); printf("%d %d", y, x);`**}**

《 m1 》 (f) = {
Extcall(g, [x↦18], &x, **0, [x↦0]**) :: print 0 :: print 0 . Terminates,
Extcall(g, [x↦18], &x, **0, [x↦1]**) :: print 0 :: print 1 . Terminates,
…
Extcall(g, [x↦18], &x, **1, [x↦0]**) :: print 1 :: print 0 . Terminates,
Extcall(g, [x↦18], &x, **1, [x↦1]**) :: print 1 :: print 1 . Terminates,
…}

Behavior simulation with return value and memory state

ψ1(f) = {
Extcall(g, [x↦18], &x, 0, [x↦0]) :: print 0 :: print 0 . Terminates(x↦0),
Extcall(g, [x↦18], &x, 0, [x↦1]) :: print 0 :: print 1 . Terminates(x↦1),
…
Extcall(g, [x↦18], &x, 1, [x↦0]) :: print 1 :: print 0 . Terminates(x↦0),
Extcall(g, [x↦18], &x, 1, [x↦1]) :: print 1 :: print 1 . Terminates(x↦1)
…}

$\bowtie$

m2 = {g ↦ `(int* px) *px = 1729; return 42;`**}**

《 m2 》 (g) = { Terminates(42, (x↦1729)) }

Behavior simulation with return value and memory state

Extcall(g, [x↦18], &x, 1, [x↦0]) :: print 1 :: print 0 . Terminates(x↦0), []

$\bowtie$

ψ2(g) = { Terminates(42, (x↦1729)) }

Behavior simulation with return value and memory state

Extcall(g, [x↦18], &x, 1, [x↦0]) :: print 1 :: print 0 . Terminates(x↦0), []

⋈

ψ2(g) = { Terminates(42, (x↦1729)) }

Behavior simulation with return value and memory state

Terminates(42, (x↦1729)) ,

[(1, [x↦0]) . print 1 :: print 0 . Terminates(x↦0)]

Behavior simulation with return value and memory state

Terminates(42, (x↦1729)) ,
[(1, [x↦0]) . print 1 :: print 0 . Terminates(x↦0)]

- The result expected by the caller is not the result returned by the callee.
- How to **choose in advance** a behavior of the **caller** in accordance with all its callees' behaviors?
  - What if the caller performs infinitely many external function calls?

Behavior simulation with return value and memory state

Terminates(42, (x↦1729)) ,
[(1, [x↦0]) . print 1 :: print 0 . Terminates(x↦0)]

SPURIOUS

This behavior will be removed from the behavior simulation big-step semantics

(consider it as a terminating behavior with result SPURIOUS).

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Does resolution make sense?

- How to relate behavior simulation with actual program linking?

Yes, resolution makes sense!

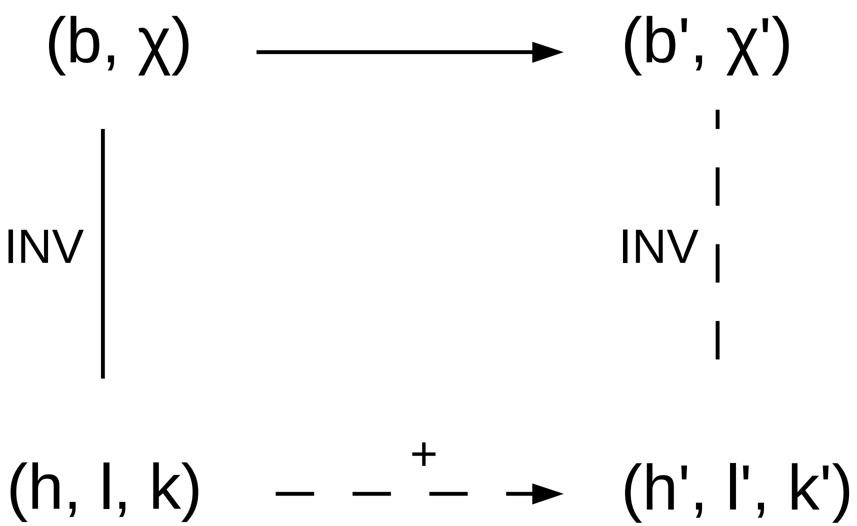
- How to relate behavior simulation with actual program linking?
  - Linking two modules written in the same language

$$\langle\langle\, \mathbf{m1} \uplus \mathbf{m0} \,\rangle\rangle = \langle\langle\, \mathbf{m1} \,\rangle\rangle \bowtie \langle\langle\, \mathbf{m0} \,\rangle\rangle$$

Mechanized Coq proof

《 m1 》 ⋈ 《 m0 》 ⊆ 《 m1 ⊎ m0 》

- Simulation diagram



Invariant INV

Stack: k = p ++ q

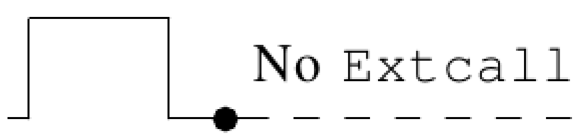
(h, l, p) behaves b in 《 m1 ⊎ m0 》

χ similarly matches q

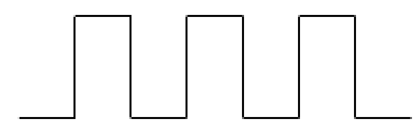
$\langle\!\langle\, m1 \uplus m0 \,\rangle\!\rangle \subseteq \langle\!\langle\, m1 \,\rangle\!\rangle \bowtie \langle\!\langle\, m0 \,\rangle\!\rangle$

- Easy induction for finite (terminating/stuck) behaviors
- For infinite (diverging/reacting) behaviors, distinguish between 3 cases:

Some external function call does not return



Finitely many external function calls that all terminate



Infinitely many external function calls that all terminate

Summary

- Resolution and linking operator at the semantic level
- Independent of the underlying language
- Allows linking behaviors of modules written in different languages

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Compositional refinement

- Expressed at the semantic level
- CompCert behavior improvement (cf. Dockins' PhD thesis)
  - Beh1 improves Beh2 iff
    Beh1 = Beh2 or Beh2 is a stuck prefix of Beh1
- **Vertical composition:** Already known to be transitive
- **Horizontal composition:** Compatible with linking

$$\psi 1 \sqsubseteq \psi 2 \ \Rightarrow\ \psi \bowtie \psi 1 \sqsubseteq \psi \bowtie \psi 2$$

- **Mechanized (Coq) proof**

Compositional compiler correctness

$$\langle\!\langle \text{Compiler(Prog)} \rangle\!\rangle \sqsubseteq \langle\!\langle \text{Prog} \rangle\!\rangle$$

- Refinement with compositional semantics
- No deep change in CompCert proofs
  - External function call events treated like ordinary events
  - We have instantiated our framework with a CompCert optimization proof
    - common subexpression elimination with value numbering

Vertical and horizontal composition

User proves: $\langle\langle P \rangle\rangle \bowtie Ls \sqsubseteq S$ and $\langle\langle Li \rangle\rangle \sqsubseteq Ls$

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Horizontal composition: $\langle\langle P \rangle\rangle \bowtie \langle\langle Li \rangle\rangle \sqsubseteq \langle\langle P \rangle\rangle \bowtie Ls$

Vertical and horizontal composition

User proves: $\langle\langle P \rangle\rangle \bowtie Ls \sqsubseteq S$ and $\langle\langle Li \rangle\rangle \sqsubseteq Ls$

Horizontal composition: $\langle\langle P \rangle\rangle \bowtie \langle\langle Li \rangle\rangle \sqsubseteq \langle\langle P \rangle\rangle \bowtie Ls$

Vertical composition: $\langle\langle P \rangle\rangle \bowtie \langle\langle Li \rangle\rangle \sqsubseteq S$

Vertical and horizontal composition

User proves: $\langle\!\langle P \rangle\!\rangle \bowtie Ls \sqsubseteq S$ and $\langle\!\langle Li \rangle\!\rangle \sqsubseteq Ls$

Horizontal composition: $\langle\!\langle P \rangle\!\rangle \bowtie \langle\!\langle Li \rangle\!\rangle \sqsubseteq \langle\!\langle P \rangle\!\rangle \bowtie Ls$

Vertical composition: $\langle\!\langle P \rangle\!\rangle \bowtie \langle\!\langle Li \rangle\!\rangle \sqsubseteq S$

$\parallel$

Linking theorem: $\langle\!\langle P \uplus Li \rangle\!\rangle \sqsubseteq S$

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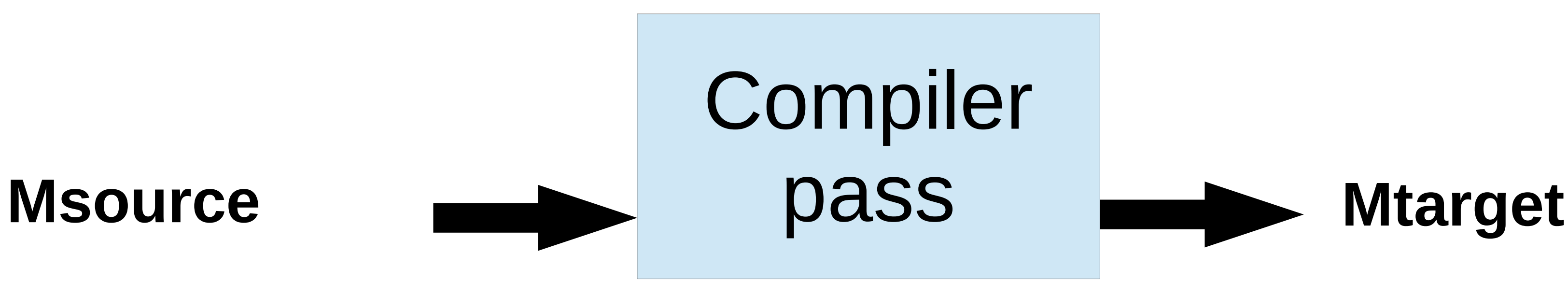
Memory-changing transformations: α-refinement

- CompCert refinement relation works for all passes that do not change memory
- For other passes (e.g. C#minor-to-Cminor), argue that compilation "preserves information" through some bijection α



Memory-changing transformations: α-refinement

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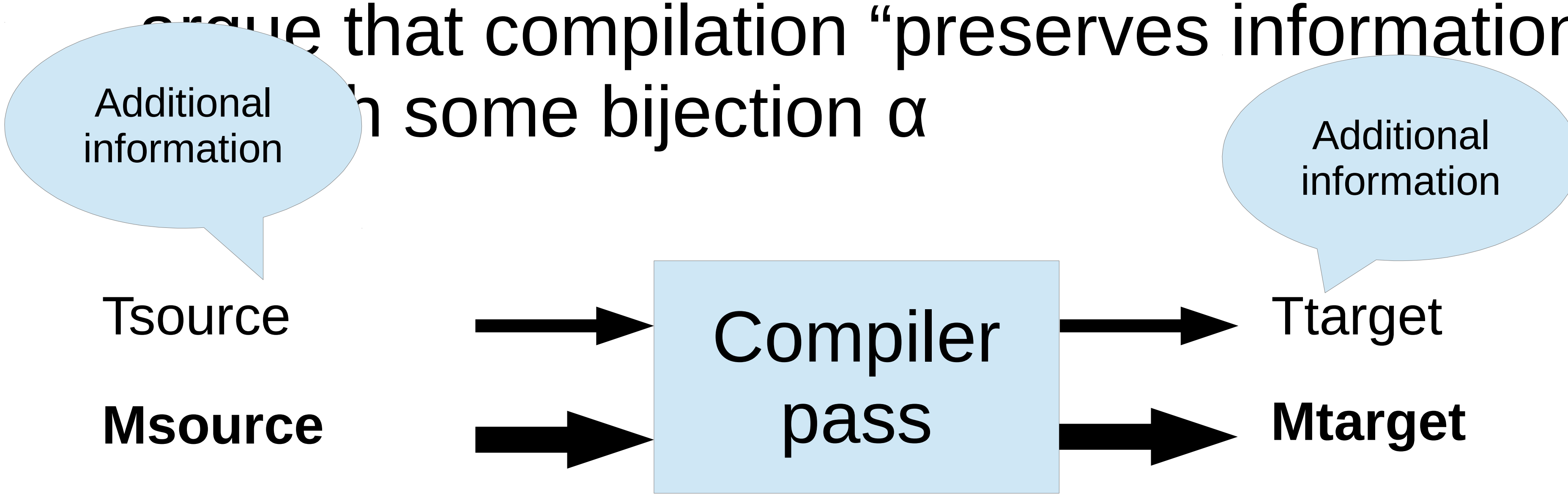

Memory-changing transformations: α-refinement

(Ttarget, Mtarget) = α(Tsource, Msource)

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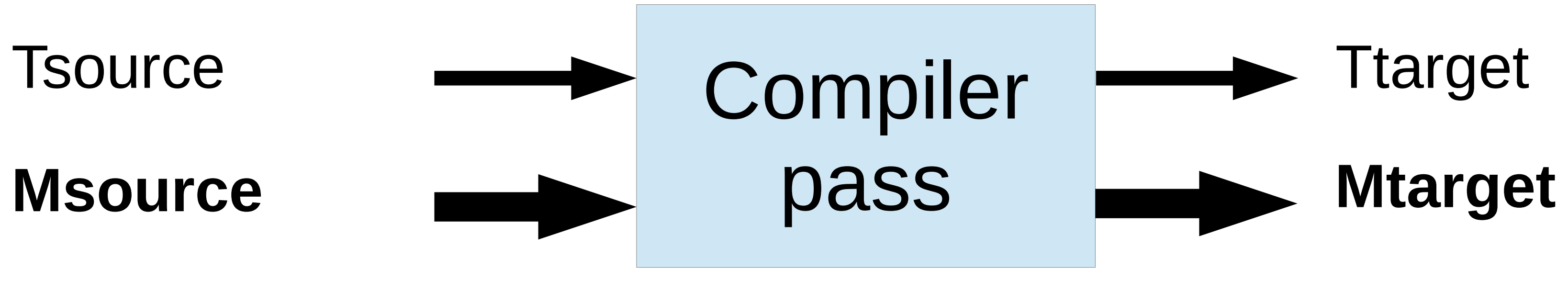
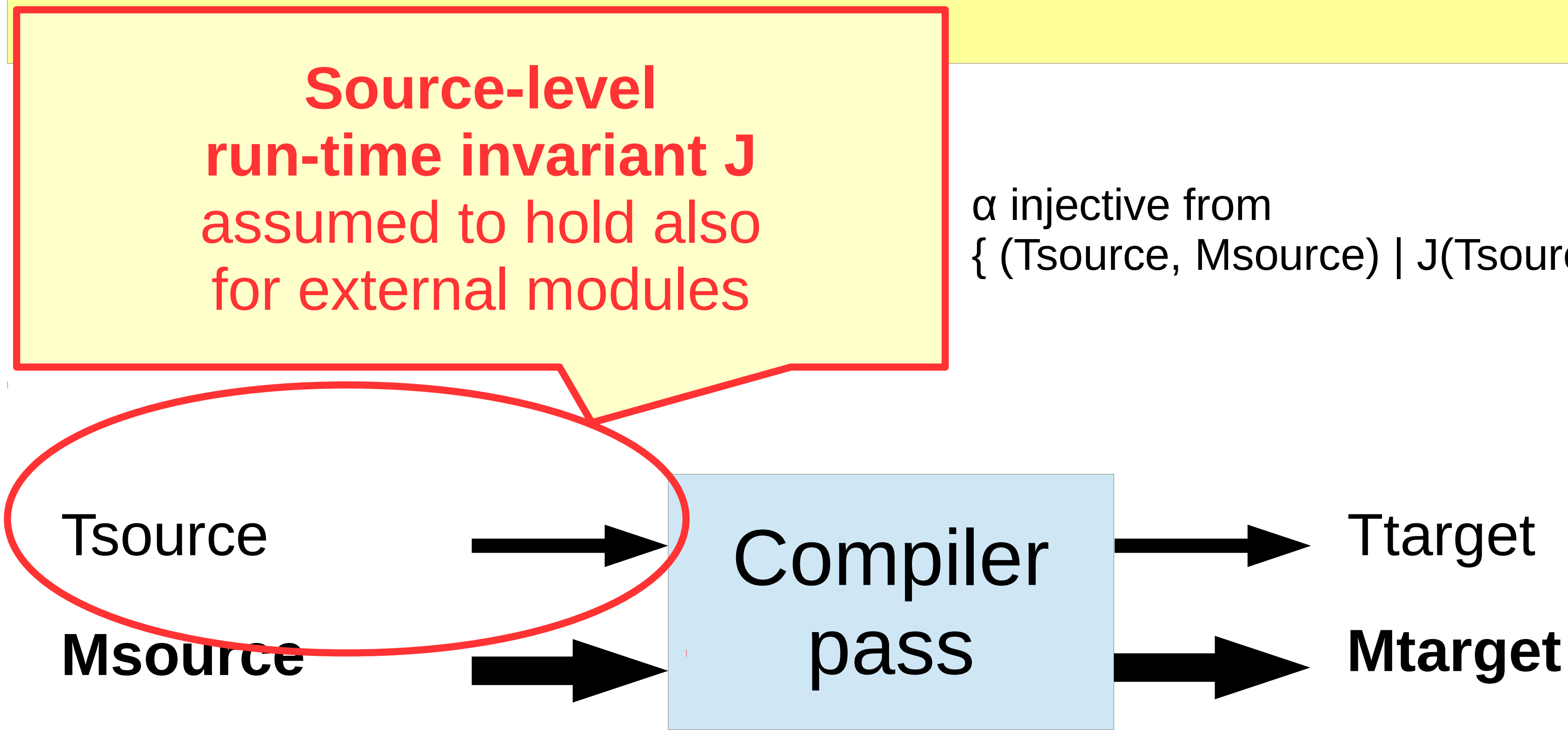


Memory-changing transformations: α-refinement

(Ttarget, Mtarget) = α(Tsource, Msource)

α injective from
{ (Tsource, Msource) | J(Tsource, Msource)}



Implementation

- Instantiated with CSE optimization and its CompCert proof
- Memory-changing transformation: C#minor to Cminor (local variable layout)
- Proofs available on the Internet
  - `http://flint.cs.yale.edu/publications/vscl.html`

Related work

- Hur et al. (POPL 2012)
  - coined “horizontal vs. vertical composition” problem
  - Based on parameterized operational semantics without intension
  - Local vs. global knowledge
- Stewart et al. (POPL 2015)
  - Based on operational small-step semantics: focus on simulation diagrams
  - Full-scale CompCert
  - We provide a linking theorem
  - Potential for simpler, less redundant proofs?
  - Go attend their talk on Friday at 10am, HBA!

Related work

- Perconti et al. (ESOP 2014)
  - Devices to unify different languages and compose their semantics
  - Can extend our linking theorem to cross-language linking
- Ghica et al. (MFPS 2012)
  - Based on game semantics
  - Opponent = unknown module to link with

Conclusion

- Language-independent compositional semantics and semantic linking operator
- Semantic vs. syntactic linking theorem
- Generalization of CompCert's event-based semantics
  - Point out minimal proof changes (at least in simpler settings)