## Manipulation des Interactions dans les Gaz Quantiques

#### **David Papoular**

Laboratoire de Physique Théorique et Modèles Statistiques

11 juillet 2011

#### LPTMS, Université Paris-Sud









Introduction: interactions in ultracold gases

A two-dimensional crystal of composite bosons

Microwave-induced Feshbach resonances

Summary

#### Introduction: Ultracold collision between two atoms

 Interaction modelled by the rotationally invariant potential V(r) whose range is b.





- For very low energies E = ħ<sup>2</sup>k<sup>2</sup>/m such that λ = 2π/k ≫ b: isotropic scattering
   Ψ<sub>s-wave</sub>(r) ∝ sin[k(r - a)]/kr
  - where *a* is the scattering length.

• Effective interaction:  $V_{\rm eff} = \frac{4\pi\hbar^2}{m} a \,\delta(r)$ 

Ultracold interaction properties are encoded in the single real number a.

### Manipulation of interactions: Why?

 Bose–Einstein condensates
 Vary magnitude and sign of *a* → explore various
 behaviours of quantum gas.



#### 2. Metrology

Cold collisions cause a change in the transition frequency:  $\delta \nu \propto n(a_{\beta\beta} - a_{\alpha\alpha})$  (*n* = atomic density) Tune  $(a_{\beta\beta} - a_{\alpha\alpha}) \rightarrow$  control clock shift.

3. Explore novel quantum phases (crystal of composite bosons)

# Manipulation of interactions: Why?





3. Explore novel quantum phases (crystal of composite bosons)

# Manipulation of interactions: Why?





3. Explore novel quantum phases (crystal of composite bosons)

Manipulation of interactions: How?

Feshbach resonances in a static magnetic field

Using a static B, tune the energy of a scattering state of the two atoms in one internal state to resonance with a bound state in a different internal state.



▶ Broad Feshbach resonances ( $\Delta B \gtrsim 10G$ ) = valuable tools Example: <sup>7</sup>Li:  $B_{res} = 700 G$ ,  $\Delta B = 200 G$ 



Introduction: interactions in ultracold gases

#### A two-dimensional crystal of composite bosons

Microwave-induced Feshbach resonances

Summary

#### Composite bosons in a heteronuclear Fermi mixture

Start from an ultracold gas of fermionic atoms containing two different species, e.g. <sup>6</sup>Li and <sup>40</sup>K.



- Weakly bound ( $|E_b| \lesssim k_{\rm B} \cdot 10 \,\mu{\rm K}$ ) and large: size  $\sim 1000 \,{\rm \AA}$
- Long lifetime: ~ 1 s at densities 10<sup>13</sup> atoms/cm<sup>3</sup> due to Pauli repulsion between identical fermions.

#### Effective interaction between composite bosons

- ► Heavy atoms: 2D motion. Light atoms: 3D (2 × 3) or 2D (2 × 2) motion.
- Born–Oppenheimer approach.

Zero–range interaction between heavy and light fermions. Dilute: mean distance  $\overline{R}$  between dimers > molecular size  $\kappa_0^{-1}$ .

- $U_{2\times3}$  and  $U_{2\times2}$  are repulsive.
- Functions of  $\kappa_0 R$ .

 $\kappa_0$  determined by scattering length a.

• Both potentials  $\propto |\varepsilon_0| = \frac{\hbar^2 \kappa_0^2}{2m}$ 

• Competition with zero–point vibrations  $\propto 1/M$ .

#### Effective interaction between composite bosons

- ► Heavy atoms: 2D motion. Light atoms: 3D (2 × 3) or 2D (2 × 2) motion.
- Born–Oppenheimer approach.

Zero-range interaction between heavy and light fermions.

Dilute: mean distance  $\overline{R}$  between dimers > molecular size  $\kappa_0^{-1}$ .

- $U_{2\times3}$  and  $U_{2\times2}$  are repulsive.
- Functions of κ<sub>0</sub>R.
   κ<sub>0</sub> determined by scattering length *a*.
- Both potentials  $\propto |\varepsilon_0| = \frac{\hbar^2 \kappa_0^2}{2m}$



• Competition with zero–point vibrations  $\propto 1/M$ .

#### Effective interaction between composite bosons

- ► Heavy atoms: 2D motion. Light atoms: 3D (2 × 3) or 2D (2 × 2) motion.
- Born–Oppenheimer approach.

Zero-range interaction between heavy and light fermions.

Dilute: mean distance  $\overline{R}$  between dimers > molecular size  $\kappa_0^{-1}$ .

- $U_{2\times3}$  and  $U_{2\times2}$  are repulsive.
- Functions of κ<sub>0</sub>R.
   κ<sub>0</sub> determined by scattering length *a*.
- Both potentials  $\propto |\varepsilon_0| = \frac{\hbar^2 \kappa_0^2}{2m}$



• Competition with zero–point vibrations  $\propto 1/M$ .

#### Gas–Crystal phase diagram at T = 0

- Phase (gas or crystal) results from competition between repulsive interaction (∝ 1/m) and zero-point vibrations (∝ 1/M).
- Vary 2D density n and mass ratio M/m.
- Motion of heavy atoms is 2D.
   Motion of light atoms is 3D or 2D.
- Triangles and circles: Quantum Monte Carlo results.
- Dashed lines:
   harmonic/Lindemann
- Solid lines: low-n hard-disk limit.

Crystal expected for  $M/m \gtrsim 100$  and appropriate densities.

#### Gas–Crystal phase diagram at T = 0

- Phase (gas or crystal) results from competition between repulsive interaction (∝ 1/m) and zero-point vibrations (∝ 1/M).
- Vary 2D density n and mass ratio M/m.



Crystal expected for  $M/m \gtrsim 100$  and appropriate densities.

# Two simple approaches to the phase diagram Assume crystal has hexagonal lattice (confirmed by QMC)

1. Harmonic/Lindemann approach: valid for all 2D densities



Evaluate RMS displacement  $l_0$  of a dimer from lattice site. Lindemann criterion: crystal melts when  $l_0 > \gamma d$  ( $\gamma \approx 0.24$ )

**2**. For very low densities, model dimers by hard spheres conserve dimer–dimer scattering length  $a_{dd}(M/m)$  critical density:  $n^{crit}(M/m) = 0.33/a_{dd}^2$  (Xing 1990)

# Two simple approaches to the phase diagram Assume crystal has hexagonal lattice (confirmed by QMC)

1. Harmonic/Lindemann approach: valid for all 2D densities



Evaluate RMS displacement  $l_0$  of a dimer from lattice site. Lindemann criterion: crystal melts when  $l_0 > \gamma d$  ( $\gamma \approx 0.24$ )

**2**. For very low densities, model dimers by hard spheres conserve dimer–dimer scattering length  $a_{dd}(M/m)$  critical density:  $n^{crit}(M/m) = 0.33/a_{dd}^2$  (Xing 1990)

#### Gas–Crystal phase diagram at T = 0

- Phase (gas or crystal) results from competition between repulsive interaction (∝ 1/m) and zero-point vibrations (∝ 1/M).
- Vary 2D density n and mass ratio M/m.



Crystal expected for  $M/m \gtrsim 100$ 

and appropriate densities.

1. Suggestion for a new experiment:



- Vary n by changing number of atoms
- Vertical optical lattice confines heavy atoms to 2D.
- To achieve 2x2 regime, add vertical optical lattice acting on light atoms.
- Horizontal optical lattice conveys effective mass M\* to heavy atoms.

Use *e.g.*  $\Lambda_{horizontal} = 250 \text{ nm}, M^* = 20M, a = 500 \text{ nm}$  to observe crystal.

 (theory) Devise a method to distinguish crystal and gas phases (for example, compare low-energy excitation spectra).

1. Suggestion for a new experiment:



- Vary n by changing number of atoms
- Vertical optical lattice confines heavy atoms to 2D.
- To achieve 2x2 regime, add vertical optical lattice acting on light atoms.

 Horizontal optical lattice conveys effective mass M\* to heavy atoms.

Use *e.g.*  $\Lambda_{horizontal} = 250 \text{ nm}, M^* = 20M, a = 500 \text{ nm}$  to observe crystal.

 (theory) Devise a method to distinguish crystal and gas phases (for example, compare low-energy excitation spectra).

1. Suggestion for a new experiment:



- Vary n by changing number of atoms
- Vertical optical lattice confines heavy atoms to 2D.
- To achieve 2x2 regime, add vertical optical lattice acting on light atoms.
- Horizontal optical lattice conveys effective mass *M*\* to heavy atoms.

Use *e.g.*  $\Lambda_{horizontal} = 250 \text{ nm}, M^* = 20M, a = 500 \text{ nm}$  to observe crystal.

 (theory) Devise a method to distinguish crystal and gas phases (for example, compare low–energy excitation spectra).

1. Suggestion for a new experiment:



- Vary n by changing number of atoms
- Vertical optical lattice confines heavy atoms to 2D.
- To achieve 2x2 regime, add vertical optical lattice acting on light atoms.
- Horizontal optical lattice conveys effective mass *M*\* to heavy atoms.

Use *e.g.*  $\Lambda_{horizontal} = 250 \text{ nm}$ ,  $M^* = 20M$ , a = 500 nm to observe crystal.

 (theory) Devise a method to distinguish crystal and gas phases (for example, compare low–energy excitation spectra).



Introduction: interactions in ultracold gases

A two-dimensional crystal of composite bosons

Microwave-induced Feshbach resonances

Summary

Manipulation of interactions: How?

Feshbach resonances in a static magnetic field

Using a static B, tune the energy of a scattering state of the two atoms in one internal state to resonance with a bound state in a different internal state.



▶ Broad Feshbach resonances ( $\Delta B \gtrsim 10G$ ) = valuable tools Example: <sup>7</sup>Li:  $B_{res} = 700 G$ ,  $\Delta B = 200 G$ 

#### Feshbach resonances in static *B* fields: Limitations

- 1. Resonances occur for fixed (often large) values of B.
- 2. Broad Feshbach resonances not available for all atoms.
- Examples: Feshbach resonances in

<sup>23</sup>Na  $\longrightarrow \Delta B = 1G$ ,  $B_{\rm res} = 1200 \,\rm G$ 

<sup>87</sup>Rb  $\longrightarrow \Delta B = 0.2 \text{ G}, \quad B_{\text{res}} = 1000 \text{ G}$ 

 $\rightarrow$  harder to use in experiments.

 $\implies$  Look for another way to tune the scattering length.

#### Alternatives to Feshbach resonances in static B fields

Resonances in a static magnetic field

proposed by Verhaar et al. (1992), first observed by Ketterle (1998).

- RF magnetic field (Moerdijk & Verhaar 1996)
- optical electric field (Fedichev et al. 1996, add experiment)
- Manipulate existing FFRs using MW or RF magnetic fields (Thompson et al 2005, Zhang et al 2009, Kaufman et al 2009)

#### Our suggestion:

#### Microwave–Induced Feshbach resonances

- 2 bosonic alkali atoms in a given internal state collide in the presence of a microwave oscillating magnetic field.
- If ω is tuned correctly, coupling to a bound state corresponding to different internal atomic states.



#### A simple approach: the Two–Channel Model Replace with



$$a(\omega) = a_{bg} \left(1 + \frac{\Delta \omega}{\omega - \omega_{res}}\right)$$

Res. frequency:  $\hbar\omega_{\rm res}^0 = \Delta E_{\rm hf} - |E_T|$  (energy of resonant bound state) Res. width:  $\Delta\omega \propto \frac{\mu}{a_{\rm bg}} B_0^2 |\langle \Psi_{\rm closed} | S^+ | \Psi_{\rm open} \rangle|^2$ 

#### A simple approach: the Two–Channel Model Replace with



$$a(\omega) = a_{\rm bg} \left(1 + \frac{\Delta \omega}{\omega - \omega_{\rm res}}\right)$$

Res. frequency:  $\hbar \omega_{\rm res}^0 = \Delta E_{\rm hf} - |E_T|$  (energy of resonant bound state)

Res. width: 
$$\Delta \omega \propto \frac{\mu}{a_{\rm bg}} B_0^2 |\langle \Psi_{\rm closed} | S^+ | \Psi_{\rm open} \rangle|^2$$

 $\Delta \omega$  scales with  $B_0^2$ . (So does a small shift on  $\omega_{\text{res}}$ :  $\omega_{\text{res}} = \omega_{\text{res}}^0 + \alpha B_0^2$ )

#### Internal states for alkali atoms with nuclear spin 3/2



-2

-1

 $m_{f}$ 

► Two–atom states,  $\sigma^+$ –polarised mw with  $\hbar \omega \approx \Delta E_{\rm hf}$  connecting  $| bb \rangle$  to  $| ab \rangle$ 



#### Beyond the two-channel model

Use realistic electronic potentials  $V_S(r)$  and  $V_T(r)$ , which depend on total electronic spin S ( $S = s_1 + s_2$ )  $\rightarrow$  spin recoupling:

ATOMS CLOSE TOGETHER:

The two electronic spins couple together.

ATOMS FAR APART:

The electronic spin of each atom couples with its nuclear spin.





 $|V_T(r) - V_S(r)| \gg V_{\text{hyperfine}}$ 

 $|V_T(r) - V_S(r)| \ll V_{\text{hyperfine}}$ 

Accounted for in the coupled-channel method.

# Modelling the interaction between two alkali atoms $H = \frac{p^2}{2\mu} + \underbrace{V_{el}(r)}_{V_S P_S + V_T P_T} + \underbrace{V_{hf}}_{a_{hf}(s_1 \cdot l_1 + s_2 \cdot l_2)} + \underbrace{\hbar \omega \ a^{\dagger} a}_{photon \ en^{sites}} + \underbrace{W^{\sigma^{\dagger}}}_{W_1 \ (S^+ a + S^- a^{\dagger})}$

8 coupled channels: *H* is an  $8 \times 8$  matrix, |

 $\mathbf{\Psi}$  ) is an 8–comp. wavefunction.

#### Modelling the interaction between two alkali atoms





8 coupled channels: *H* is an  $8 \times 8$  matrix,

 $| oldsymbol{\Psi} 
angle$  is an 8–comp. wavefunction.

# Modelling the interaction between two alkali atoms $H = \frac{p^2}{2\mu} + \underbrace{V_{el}(r)}_{V_S P_S + V_T P_T} + \underbrace{V_{hf}}_{a_{hf}(s_1 \cdot i_1 + s_2 \cdot i_2)} + \underbrace{\hbar \omega \ a^{\dagger} a}_{photon \ en^{sites}} + \underbrace{W^{\sigma^{\dagger}}}_{W_1 \ (S^+ a + S^- a^{\dagger})}$



8 coupled channels: *H* is an 8  $\times$  8 matrix,  $|\Psi\rangle$  is an 8–comp

# Modelling the interaction between two alkali atoms $H = \frac{p^2}{2\mu} + \underbrace{V_{el}(r)}_{V_S P_S + V_T P_T} + \underbrace{V_{hf}}_{a_{hf}(s_1 \cdot i_1 + s_2 \cdot i_2)} + \underbrace{\hbar \omega \, a^{\dagger} a}_{photon \, en^{gies}} + \underbrace{W_{\tau}^{\sigma^+}}_{W_1 \, (S^+ a + S^- a^{\dagger})}$

8 coupled channels: *H* is an 8  $\times$  8 matrix,  $|\Psi\rangle$  is an 8–comp. w

# Modelling the interaction between two alkali atoms $H = \frac{p^2}{2\mu} + \underbrace{V_{el}(r)}_{V_S P_S + V_T P_T} + \underbrace{V_{hf}}_{a_{hf}(s_1 \cdot i_1 + s_2 \cdot i_2)} + \underbrace{\hbar\omega a^{\dagger} a}_{photon en^{gies}} + \underbrace{W_{\tau}^{\sigma^+}}_{W_1(S^+a + S^-a^{\dagger})}$

e<sub>z</sub> B

8 coupled channels: *H* is an  $8 \times 8$  matrix,

 $|\Psi\rangle$  is an 8–comp. wavefunction.



8 coupled channels: *H* is an  $8 \times 8$  matrix,

 $|\Psi\rangle$  is an 8–comp. wavefunction.



8 coupled channels: *H* is an  $8 \times 8$  matrix,

 $|\Psi\rangle$  is an 8–comp. wavefunction.

#### How do we calculate multichannel wavefunctions? I use my own C++ implementation of the coupled-channel method.

- Inputs: electronic pots. V<sub>S</sub>, V<sub>T</sub> for r ≥ 20a<sub>0</sub> and scattering lengths a<sub>S</sub>, a<sub>T</sub>.
- Encode small—r Physics in "Accumulated Phase" boundary condition at finite r<sub>0</sub> [Verhaar *et al.*, PRA 2009].
  - Large-r boundary condition: atoms in relevant mw-dressed state.
  - Solve coupled Schrödinger equations numerically.
     One trick: take advantage of spin recoupling.





The coupled-channel approach:

#### Results



		<sup>7</sup> Li	<sup>23</sup> Na	<sup>41</sup> K	<sup>87</sup> Rb	<sup>133</sup> Cs
<i>E<sub>b</sub></i>  / <i>h</i>	(MHz)	12000	200	140	25	$5 \cdot 10^{-3}$
$\omega_{ m res}^0/2\pi$	(GHz)	12	1.6	0.12	6.8	9.2
$\Delta \omega_{\rm CC}/2\pi$	$(Hz/G^2)$	6	1400	350	60	-6 · 10 <sup>9</sup>

Narrow  $\Delta \omega$  for <sup>7</sup>Li due to no weakly–bound state in closed channel.

Narrow  $\Delta \omega$  for <sup>87</sup>Rb due to  $a_T = 99 a_0$  being very close to  $a_S = 90 a_0$ .

<sup>133</sup>Cs: very weakly–bound state  $\rightarrow$  non–hyperbolic resonance for  $B_0 \gtrsim 4 \text{ mG}$ .

SYRTE's static-field resonances in Cesium 133

• Atoms in  $|f_1 = 4, m_1 = 0\rangle \& |f_2 = 3, M_F\rangle, M_F = 1, 2, \text{ or } 3.$ 

Measured at SYRTE

Calculated (coupled channels)



• Due to *s*-wave triplet bound state with  $|E_T| = h \cdot 5 \text{ kHz}$ .

▶ 3 triplet 2-atom states in  $(f_1 = 4, f_2 = 3, M_F = 2) \rightarrow 3$  peaks.

	$-11 \pm 5$					
	2.7		4.0			
	5.1		7.2		4.2	
$18 \pm 3$	16.5	25	22	$5\pm1$	5.5	

SYRTE's static-field resonances in Cesium 133

• Atoms in  $|f_1 = 4, m_1 = 0\rangle \& |f_2 = 3, M_F\rangle$ ,  $M_F = 1, 2, \text{ or } 3$ .

Measured at SYRTE Calculated (coupled channels) 0.4 - m. = 2 scattering length a [1000 a<sub>0</sub>] 0- -5 - -5 - -5 0.2 0.0 Re(a) -0.2 Im(a) -0.6 -0.8 -1.0- $M_{E}=2$ -15 -60 -40 -20 0 B (mG) 40 60 80 100 -20 0 20 40 Static magnetic field B [mG]

- Due to *s*-wave triplet bound state with  $|E_T| = h \cdot 5 \text{ kHz}$ .
- ▶ 3 triplet 2-atom states in  $(f_1 = 4, f_2 = 3, M_F = 2) \rightarrow 3$  peaks.

Resonance positions <i>B</i> <sub>res</sub> [mG]						
$M_F = 1$		M <sub>F</sub>	= 2	$M_F = 3$		
measured	calculated	measured	calculated	measured	calculated	
	$-11 \pm 5$					
	2.7		4.0		3.0	
	5.1	8	7.2		4.2	
$18\pm3$	16.5	25	22	$5\pm1$	5.5	

#### A giant MW-induced resonance in Cesium



#### ▶ $B_0 \lesssim 1 \text{ mG}$ : hyperbolic resonance with $\Delta \omega / 2\pi = 6 \text{ GHz} / G^2$

Larger values of B<sub>0</sub>: non–hyperbolic resonance.

Dressed-state effects important for  $B_0 \gtrsim |E_T|/\mu_{\rm B} = 4 \, {\rm mG}$ 

Lowest–energy dressed state is different for  $\delta < 0$  and  $\delta > 0$  $\rightarrow$  different scattering lengths.

#### A giant MW-induced resonance in Cesium



- ▶  $B_0 \lesssim 1 \text{ mG}$ : hyperbolic resonance with  $\Delta \omega / 2\pi = 6 \text{ GHz} / G^2$
- ► Larger values of *B*<sub>0</sub>: non–hyperbolic resonance.



Dressed-state effects important for  $B_0 \gtrsim |E_T|/\mu_B = 4 \text{ mG}$ 

Lowest–energy dressed state is different for  $\delta < 0$  and  $\delta > 0$  $\rightarrow$  different scattering lengths.

#### MW-induced resonances: Conclusion and Outlook

		<sup>7</sup> Li	<sup>23</sup> Na	<sup>41</sup> K	<sup>87</sup> Rb	<sup>133</sup> Cs
$\Delta \omega/2\pi$	$(Hz/G^2)$	6	1400	350	60	$-6 \cdot 10^{9}$

• Realistic mw amplitudes:  $B_0 \lesssim 10 \,\mathrm{G}$ 

Optimistic prospects for experiments with Na, K, Rb, Cs.

- Wide resonance in <sup>133</sup>Cs: width  $\sim 5 \text{ kHz}$  for  $B_0 = 1 \text{ mG}$ . Observable in a Cesium fountain clock.
- Our scheme can be transposed to fermionic atoms and heteronuclear mixtures.
- It can be used to tune the interaction in spinor gases.
- It can be used to improve control over clock shifts.

#### Manipulation of Interactions in Quantum Gases



A new way to manipulate interactions:

microwave-induced Feshbach resonances

[PRA 81, 041603(R) (2010)]

One novel quantum phase:
 2D crystal of composite bosons
 [PRL 99, 130407 (2007)]



Thanks to G. Shlyapnikov, J. Dalibard, C. Salomon, D. Petrov, S. Bize, P. Rosenbusch.