A walk in the *universe* of *categories*
Part I : Notion of Bicategory

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Abstract

In this talk i will present the notion of bicategory which was introduced by Bénabou in []. The idea is to construct some kind of "categories" wich are not "really" categories in the usual sens. Roughly speaking a bicategory S consists of a given collection objects Ob(S) and for any two object A and B instead of having a hom-object Hom(A,B) we have a category S(A,B) and we would like this one to "behave as" a hom-object. If one's familiar with the notion of *enriched categories*, a bicategory looks like a category enriched in Cat (the category of all small categories) which is known in the higher-category jargon as *2-category*. But enriched categories are linked with monoidal categories and Bénabou remarked in his paper that monoidal categories are bicategories with one object, so the construction of bicategories avoid using monoidal categories and enrichment, leaving then possible ways of constructing higher (weak)-categories[1] . We will focus on local definition and local properties. Here by local we mean that the object of S form a mere set. The global definition uses *bigraph* with given operations which must satisfy some diagrammatical coherences in the same manner as we do when we construct category from graphs (see Mac Lane [])