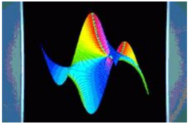
The Dynamic Dictionary of Mathematical Functions

Special Functions, Computer Algebra and High-Precision Arithmetic

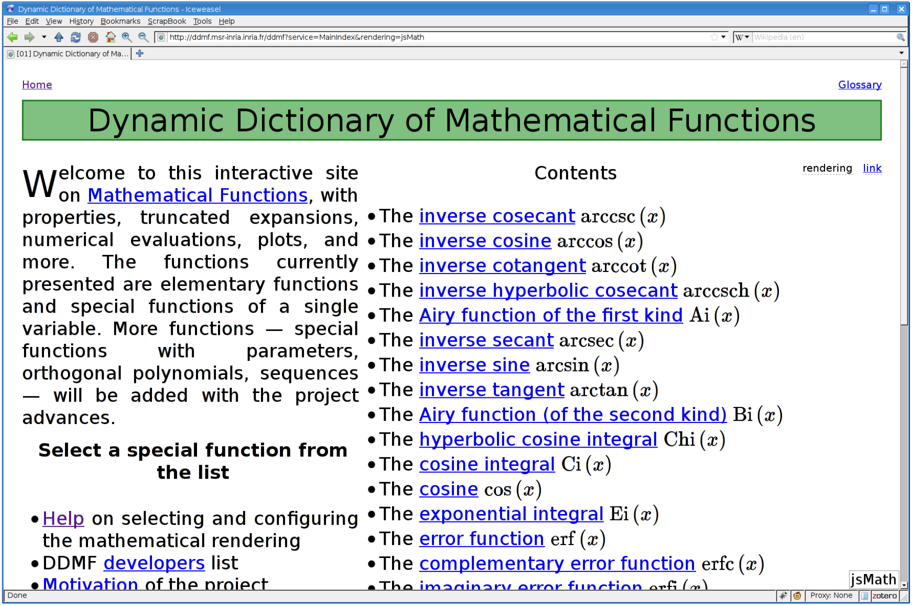
Marc MEZZAROBBA



Projet ALGORITHMS

Séminaire des doctorants de Rocquencourt
2011-09-20

The DDMF


Dynamic Dictionary of Mathematical Functions - Iceweasel

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[01] Dynamic Dictionary of Ma...

Home

Glossary

Dynamic Dictionary of Mathematical Functions

Welcome to this interactive site on Mathematical Functions, with properties, truncated expansions, numerical evaluations, plots, and more. The functions currently presented are elementary functions and special functions of a single variable. More functions — special functions with parameters, orthogonal polynomials, sequences — will be added with the project advances.

Select a special function from the list

- Help on selecting and configuring the mathematical rendering
- DDMF developers list
- Motivation of the project

rendering link

Contents

- The inverse cosecant $\operatorname{arccsc}(x)$
- The inverse cosine $\arccos(x)$
- The inverse cotangent $\operatorname{arccot}(x)$
- The inverse hyperbolic cosecant $\operatorname{arccsch}(x)$
- The Airy function of the first kind $\operatorname{Ai}(x)$
- The inverse secant $\operatorname{arcsec}(x)$
- The inverse sine $\arcsin(x)$
- The inverse tangent $\arctan(x)$
- The Airy function (of the second kind) $\operatorname{Bi}(x)$
- The hyperbolic cosine integral $\operatorname{Chi}(x)$
- The cosine integral $\operatorname{Ci}(x)$
- The cosine $\cos(x)$
- The exponential integral $\operatorname{Ei}(x)$
- The error function $\operatorname{erf}(x)$
- The complementary error function $\operatorname{erfc}(x)$
- The imaginary error function $\operatorname{erfi}(x)$

jsMath

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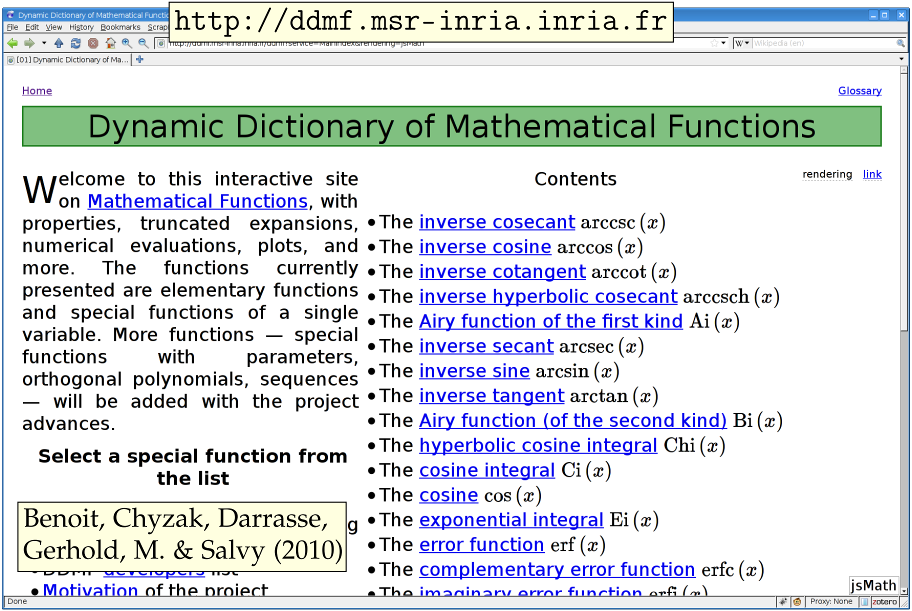

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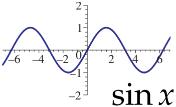
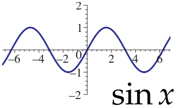
Benoit, Chyzak, Darrasse, Gerhold, M. & Salvy (2010)

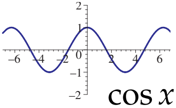
Mathematical Functions



$\sin x$

Mathematical Functions





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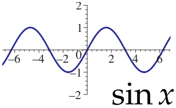

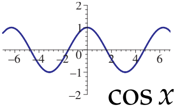

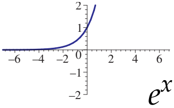

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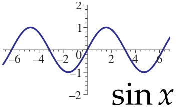
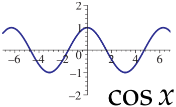
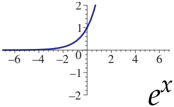
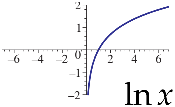
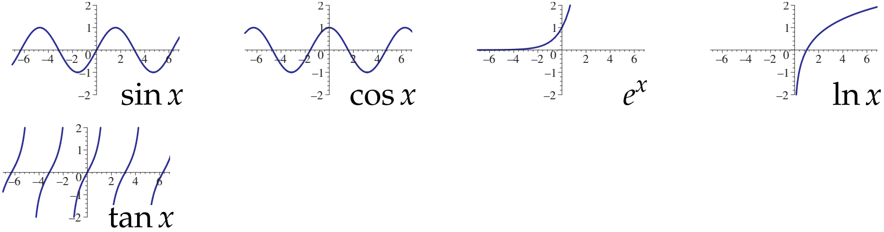

$\sin x$


$\cos x$


e^x


$\ln x$

Mathematical Functions



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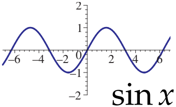
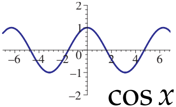
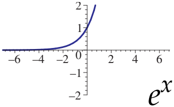


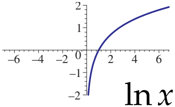
$\sin x$



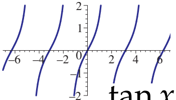
$\cos x$



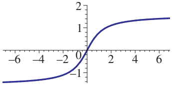
e^x



$\ln x$

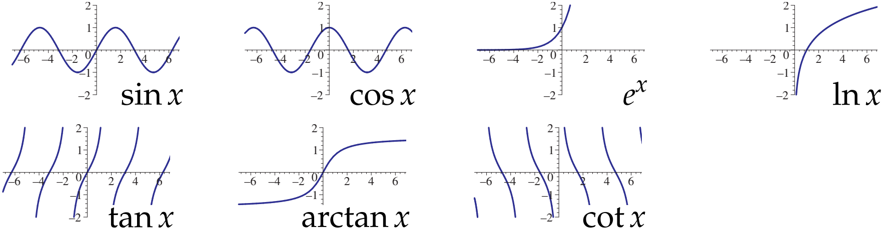


$\tan x$



$\arctan x$

Mathematical Functions



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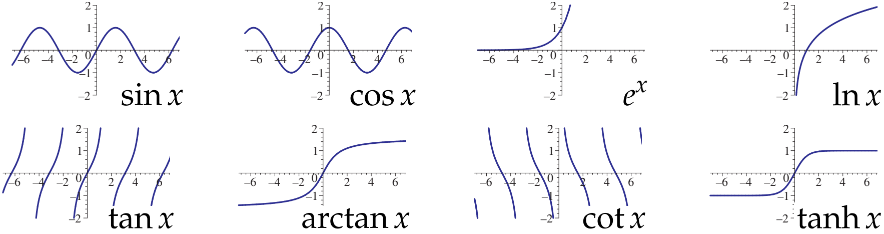

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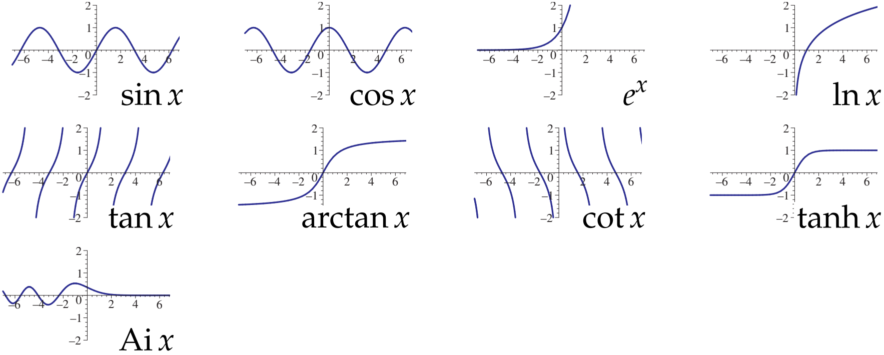

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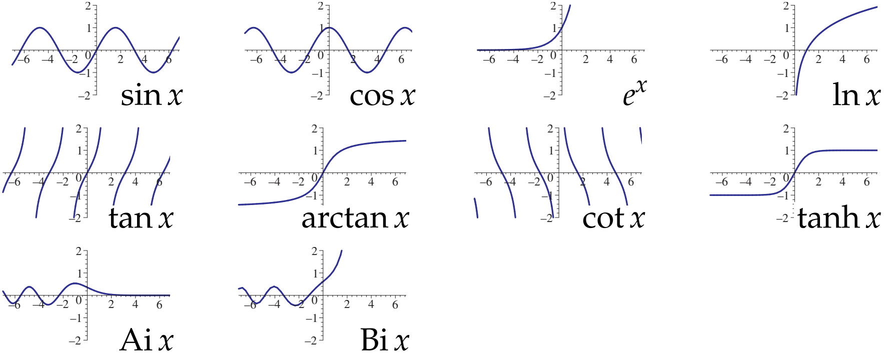

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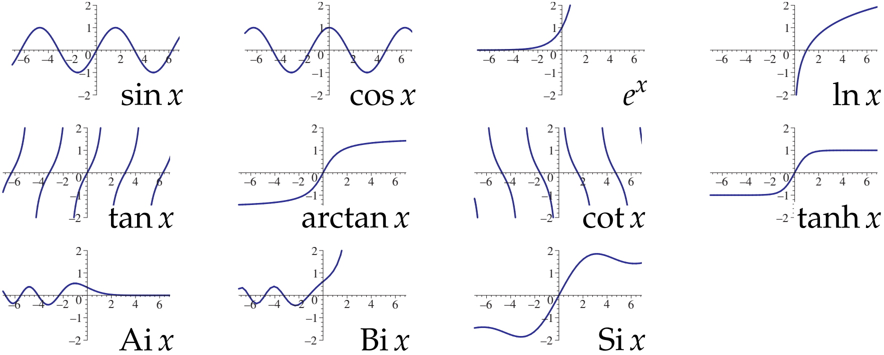

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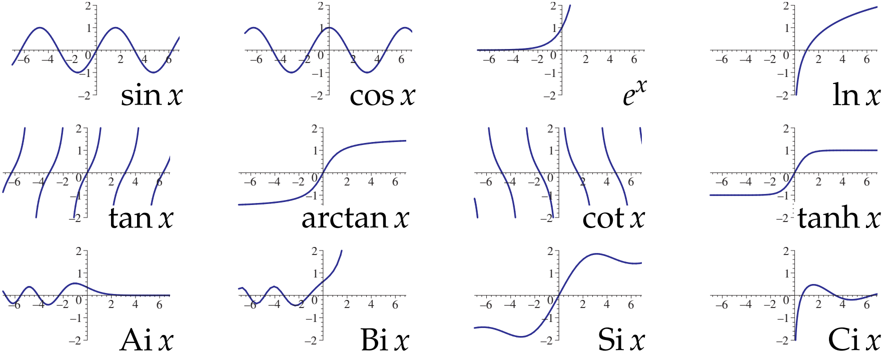

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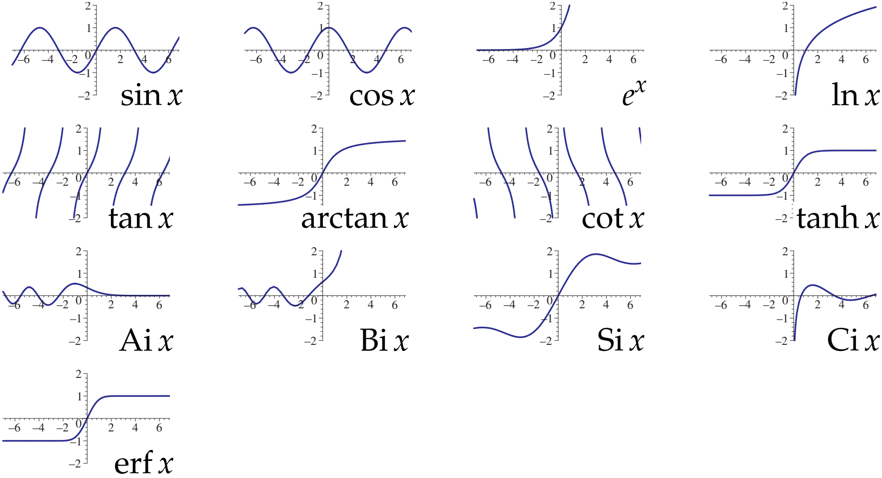

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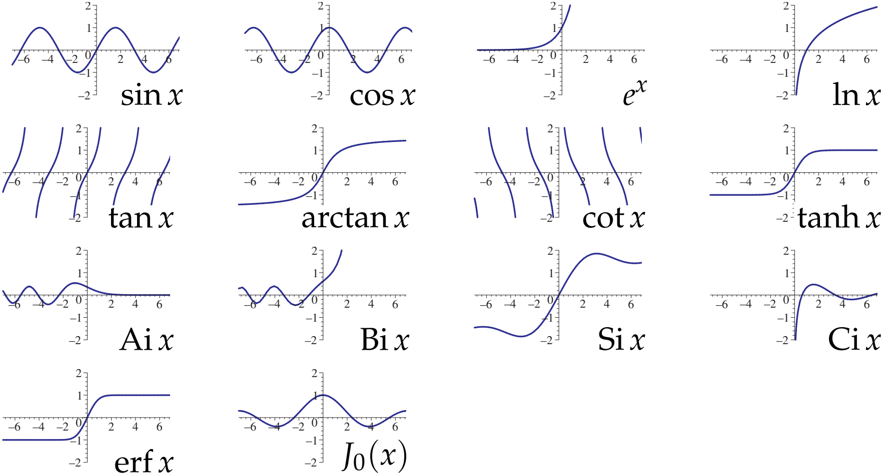

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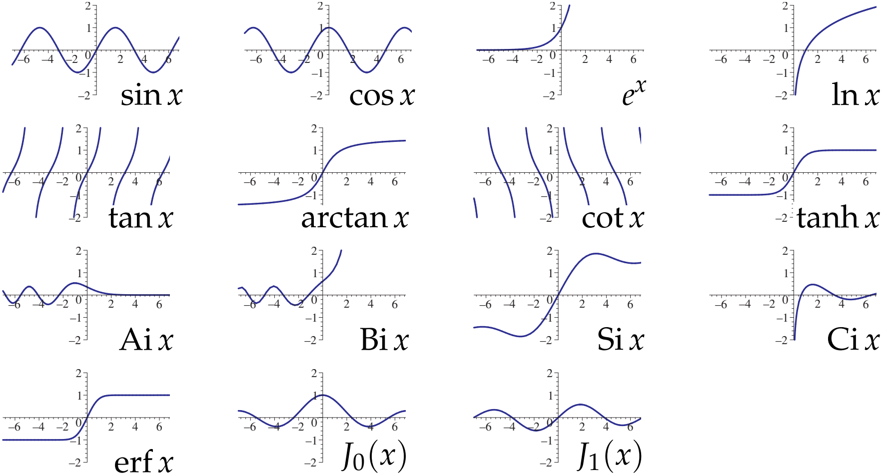

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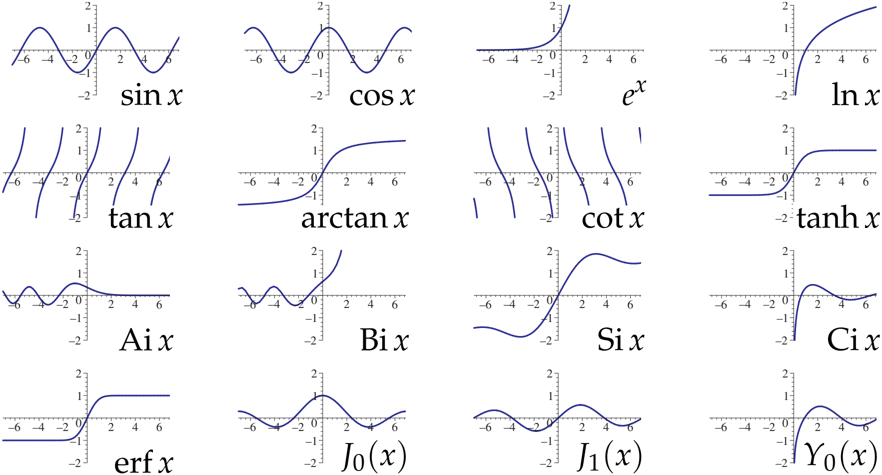

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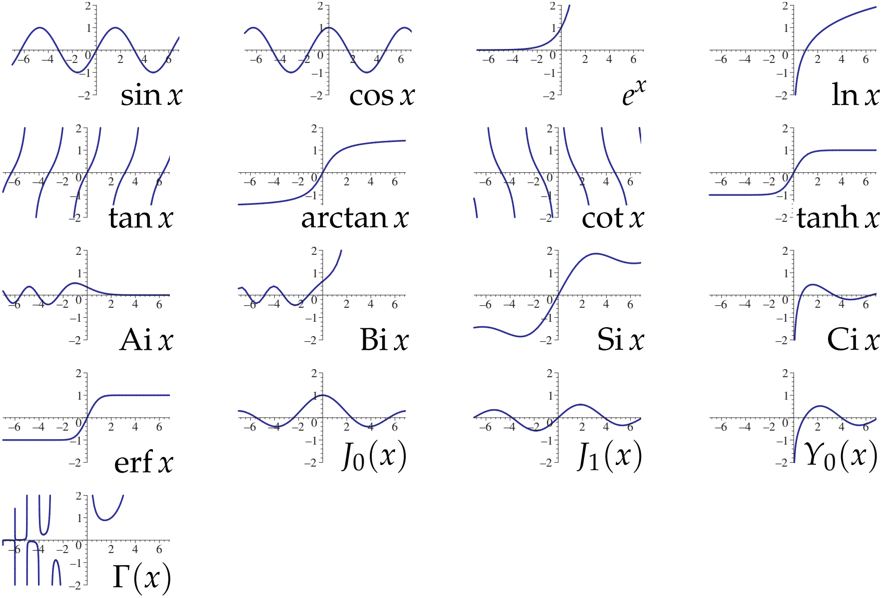

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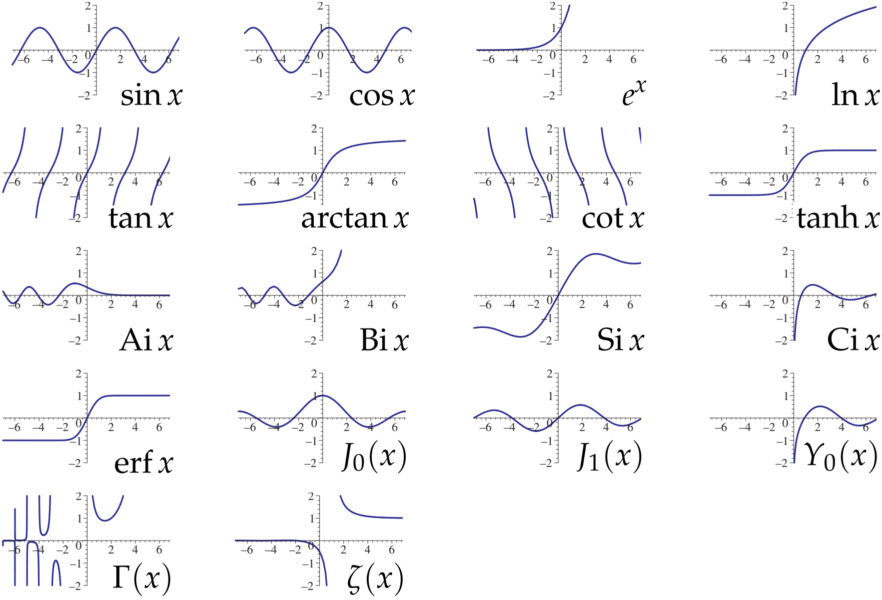

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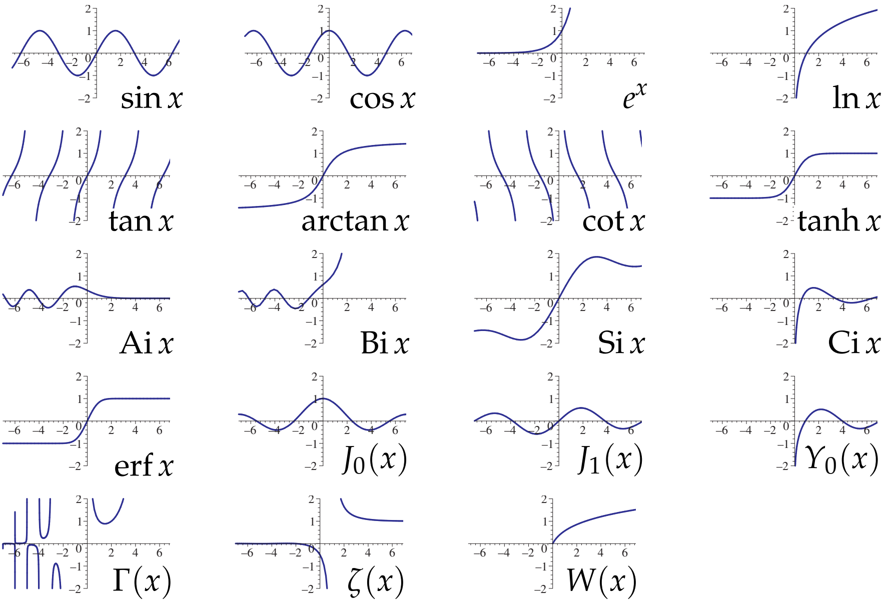

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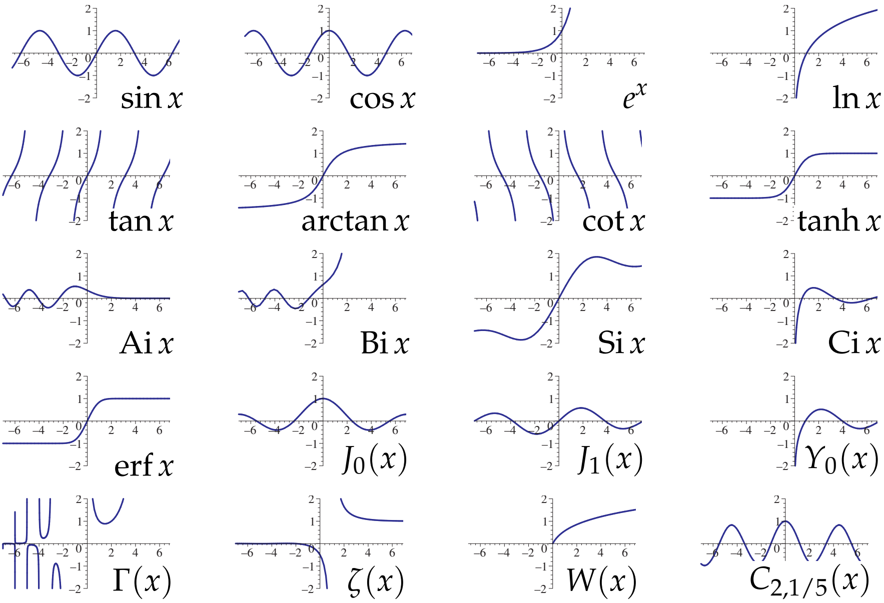

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10.4. Airy Functions

Definitions and Elementary Properties

Differential Equation

10.4.1 $w''-zw=0$

Pairs of linearly independent solutions are

$$\mathrm{Ai}\,(z),\ \mathrm{Bi}\,(z),$$
$$\mathrm{Ai}\,(z),\ \mathrm{Ai}\,(ze^{2\pi i/3}),$$
$$\mathrm{Ai}\,(z),\ \mathrm{Ai}\,(ze^{-2\pi i/3}).$$

Ascending Series

10.4.2 $\mathrm{Ai}\,(z)=c_1f(z)-c_2g(z)$

10.4.3 $\mathrm{Bi}\,(z)=\sqrt{3}\,[c_1f(z)+c_2g(z)]$

$$f(z)=1+\frac{1}{3!}z^3+\frac{1\cdot 4}{6!}z^6+\frac{1\cdot 4\cdot 7}{9!}z^9+\ldots$$
$$=\sum_0^\infty 3^k\left(\frac{1}{3}\right)_k\frac{z^{3k}}{(3k)!}$$
$$g(z)=z+\frac{2}{4!}z^4+\frac{2\cdot 5}{7!}z^7+\frac{2\cdot 5\cdot 8}{10!}z^{10}+\ldots$$
$$=\sum_0^\infty 3^k\left(\frac{2}{3}\right)_k\frac{z^{3k+1}}{(3k+1)!}$$
$$\left(\alpha+\frac{1}{3}\right)_0=1$$
$$3^k\left(\alpha+\frac{1}{3}\right)_k=(3\alpha+1)(3\alpha+4)\ldots(3\alpha+3k-2)$$
(α arbitrary; $k=1, 2, 3, \ldots$)

(See **6.1.22.**)

10.4.4

$$c_1=\mathrm{Ai}\,(0)=\mathrm{Bi}\,(0)/\sqrt{3}=3^{-2/3}/\Gamma(2/3)$$
$$=.35502\ 80538\ 87817$$

10.4.5

$$c_2=-\mathrm{Ai}'\,(0)=\mathrm{Bi}'\,(0)/\sqrt{3}=3^{-1/3}/\Gamma(1/3)$$
$$=.25881\ 94037\ 92807$$

Relations Between Solutions

10.4.6 $\mathrm{Bi}\,(z)=e^{\pi i/6}\,\mathrm{Ai}\,(ze^{2\pi i/3})+e^{-\pi i/6}\,\mathrm{Ai}\,(ze^{-2\pi i/3})$

10.4.7

$$\mathrm{Ai}\,(z)+e^{2\pi i/3}\,\mathrm{Ai}\,(ze^{2\pi i/3})+e^{-2\pi i/3}\,\mathrm{Ai}\,(ze^{-2\pi i/3})=0$$

10.4.8

$$\mathrm{Bi}\,(z)+e^{2\pi i/3}\,\mathrm{Bi}\,(ze^{2\pi i/3})+e^{-2\pi i/3}\,\mathrm{Bi}\,(ze^{-2\pi i/3})=0$$

10.4.9 $\mathrm{Ai}\,(ze^{\pm 2\pi i/3})=\frac{1}{2}e^{\pm\pi i/3}[\mathrm{Ai}\,(z)\mp i\,\mathrm{Bi}\,(z)]$

Wronskians

10.4.10 $W\{\mathrm{Ai}\,(z),\mathrm{Bi}\,(z)\}=\pi^{-1}$

10.4.11 $W\{\mathrm{Ai}\,(z),\mathrm{Ai}\,(ze^{2\pi i/3})\}=\frac{1}{2}\pi^{-1}e^{-\pi i/6}$

10.4.12 $W\{\mathrm{Ai}\,(z),\mathrm{Ai}\,(ze^{-2\pi i/3})\}=\frac{1}{2}\pi^{-1}e^{\pi i/6}$

10.4.13 $W\{\mathrm{Ai}\,(ze^{2\pi i/3}),\mathrm{Ai}\,(ze^{-2\pi i/3})\}=\frac{1}{2}i\pi^{-1}$

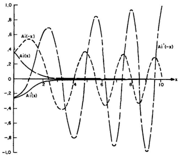

FIGURE 10.6. $\mathrm{Ai}\,(\pm x)$, $\mathrm{Ai}'\,(\pm x)$.

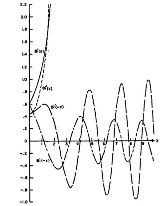

FIGURE 10.7. $\mathrm{Bi}\,(\pm x)$, $\mathrm{Bi}'\,(\pm x)$.

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Representations in Terms of Bessel Functions

$$\zeta=\tfrac{2}{3}z^{3/2}$$

10.4.14
$$\mathrm{Ai}\,(z)=\tfrac{1}{3}\sqrt{z}[I_{-1/3}(\zeta)-I_{1/3}(\zeta)]=\pi^{-1}\sqrt{z/3}K_{1/3}(\zeta)$$

10.4.15
$$\mathrm{Ai}\,(-z)=\tfrac{1}{3}\sqrt{z}[J_{1/3}(\zeta)+J_{-1/3}(\zeta)]$$
$$=\tfrac{1}{2}\sqrt{z/3}[e^{\pi i/6}H^{(1)}_{1/3}(\zeta)+e^{-\pi i/6}H^{(2)}_{1/3}(\zeta)]$$

10.4.16
$$-\mathrm{Ai}'(z)=\tfrac{1}{3}z[I_{-2/3}(\zeta)-I_{2/3}(\zeta)]=\pi^{-1}(z/\sqrt{3})K_{2/3}(\zeta)$$ *

10.4.17
$$\mathrm{Ai}'\,(-z)=-\tfrac{1}{3}z[J_{-2/3}(\zeta)-J_{2/3}(\zeta)]$$
$$=\tfrac{1}{2}(z/\sqrt{3})[e^{-\pi i/6}H^{(1)}_{2/3}(\zeta)+e^{\pi i/6}H^{(2)}_{2/3}(\zeta)]$$

10.4.18 $\mathrm{Bi}\,(z)=\sqrt{z/3}[I_{-1/3}(\zeta)+I_{1/3}(\zeta)]$

10.4.19
$$\mathrm{Bi}\,(-z)=\sqrt{z/3}[J_{-1/3}(\zeta)-J_{1/3}(\zeta)]$$
$$=\tfrac{1}{2}i\sqrt{z/3}[e^{\pi i/6}H^{(1)}_{1/3}(\zeta)-e^{-\pi i/6}H^{(2)}_{1/3}(\zeta)]$$

10.4.20 $\mathrm{Bi}'\,(z)=(z/\sqrt{3})[I_{-2/3}(\zeta)+I_{2/3}(\zeta)]$

10.4.21
$$\mathrm{Bi}'\,(-z)=(z/\sqrt{3})[J_{-2/3}(\zeta)+J_{2/3}(\zeta)]$$
$$=\tfrac{1}{2}i(z/\sqrt{3})[e^{-\pi i/6}H^{(1)}_{2/3}(\zeta)-e^{\pi i/6}H^{(2)}_{2/3}(\zeta)]$$

Representations of Bessel Functions in Terms of Airy Functions

$$z=\left(\frac{3}{2}\zeta\right)^{2/3}$$

10.4.22 $J_{\pm 1/3}(\zeta)=\tfrac{1}{2}\sqrt{3/z}[\sqrt{3}\,\mathrm{Ai}\,(-z)\mp\mathrm{Bi}\,(-z)]$

***10.4.23** $H^{(1)}_{\pm 1/3}(\zeta)=e^{\mp\pi i/6}\sqrt{3/z}[\mathrm{Ai}\,(-z)-i\,\mathrm{Bi}\,(-z)]$

10.4.24 $H^{(2)}_{\pm 1/3}(\zeta)=e^{\pm\pi i/6}\sqrt{3/z}[\mathrm{Ai}\,(-z)+i\,\mathrm{Bi}\,(-z)]$

10.4.25 $I_{\pm 1/3}(\zeta)=\tfrac{1}{2}\sqrt{3/z}[\mp\sqrt{3}\,\mathrm{Ai}\,(z)+\mathrm{Bi}\,(z)]$

10.4.26 $K_{\pm 1/3}(\zeta)=\pi\sqrt{3/z}\,\mathrm{Ai}\,(z)$

10.4.27 $J_{\pm 2/3}(\zeta)=(\sqrt{3}/2z)[\pm\sqrt{3}\,\mathrm{Ai}'\,(-z)+\mathrm{Bi}'\,(-z)]$

10.4.28
$$H^{(1)}_{2/3}(\zeta)=e^{-2\pi i/3}H^{(1)}_{-2/3}(\zeta)$$
$$=e^{\pi i/6}(\sqrt{3}/z)[\mathrm{Ai}'\,(-z)-i\,\mathrm{Bi}'\,(-z)]$$

10.4.29
$$H^{(2)}_{2/3}(\zeta)=e^{2\pi i/3}H^{(2)}_{-2/3}(\zeta)$$
$$=e^{-\pi i/6}(\sqrt{3}/z)[\mathrm{Ai}'\,(-z)+i\,\mathrm{Bi}'\,(-z)]$$

*See page II.

10.4.30 $I_{\pm 2/3}(\zeta)=(\sqrt{3}/2z)[\pm\sqrt{3}\,\mathrm{Ai}'\,(z)+\mathrm{Bi}'\,(z)]$

10.4.31 $K_{\pm 2/3}(\zeta)=-\pi(\sqrt{3}/z)\,\mathrm{Ai}'\,(z)$

Integral Representations

10.4.32
$$(3a)^{-1/3}\pi\,\mathrm{Ai}\,[\pm(3a)^{-1/3}x]=\int_0^\infty\cos\,(at^3\pm xt)dt$$

10.4.33
$$(3a)^{-1/3}\pi\,\mathrm{Bi}\,[\pm(3a)^{-1/3}x]$$
$$=\int_0^\infty[\exp\,(-at^3\pm xt)+\sin\,(at^3\pm xt)]dt$$

The Integrals $\int_0^z \mathrm{Ai}\,(\pm t)dt$, $\int_0^z \mathrm{Bi}\,(\pm t)dt$

$$\zeta=\tfrac{2}{3}z^{3/2}$$

10.4.34 $\int_0^z \mathrm{Ai}\,(t)dt=\frac{1}{3}\int_0^\zeta[I_{-1/3}(t)-I_{1/3}(t)]dt$

10.4.35 $\int_0^z \mathrm{Ai}\,(-t)dt=\frac{1}{3}\int_0^\zeta[J_{-1/3}(t)+J_{1/3}(t)]dt$

10.4.36 $\int_0^z \mathrm{Bi}\,(t)dt=\frac{1}{\sqrt{3}}\int_0^\zeta[I_{-1/3}(t)+I_{1/3}(t)]dt$

10.4.37 $\int_0^z \mathrm{Bi}\,(-t)dt=\frac{1}{\sqrt{3}}\int_0^\zeta[J_{-1/3}(t)-J_{1/3}(t)]dt$

Ascending Series for $\int_0^z \mathrm{Ai}\,(\pm t)dt$, $\int_0^z \mathrm{Bi}\,(\pm t)dt$

10.4.38 $\int_0^z \mathrm{Ai}\,(t)dt=c_1F(z)-c_2G(z)$

(See **10.4.2.**)

10.4.39 $\int_0^z \mathrm{Ai}\,(-t)dt=-c_1F(-z)+c_2G(-z)$

10.4.40 $\int_0^z \mathrm{Bi}\,(t)dt=\sqrt{3}[c_1F(z)+c_2G(z)]$

(See **10.4.3.**)

10.4.41
$$\int_0^z \mathrm{Bi}\,(-t)dt=-\sqrt{3}[c_1F(-z)+c_2G(-z)]$$
$$F(z)=z+\frac{1}{4!}z^4+\frac{1\cdot 4}{7!}z^7+\frac{1\cdot 4\cdot 7}{10!}z^{10}+\ldots$$
$$=\sum_0^\infty 3^k\left(\frac{1}{3}\right)_k\frac{z^{3k+1}}{(3k+1)!}$$
$$G(z)=\frac{1}{2!}z^2+\frac{2}{5!}z^5+\frac{2\cdot 5}{8!}z^8+\frac{2\cdot 5\cdot 8}{11!}z^{11}+\ldots$$
$$=\sum_0^\infty 3^k\left(\frac{2}{3}\right)_k\frac{z^{3k+2}}{(3k+2)!}$$

The constants c_1, c_2 are given in **10.4.4, 10.4.5.**

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AIRY FUNCTIONS **Table 10.11**

x	$\mathrm{Ai}(x)$	$\mathrm{Ai}'(x)$	$\mathrm{Bi}(x)$	$\mathrm{Bi}'(x)$	x	$\mathrm{Ai}(x)$	$\mathrm{Ai}'(x)$	$\mathrm{Bi}(x)$	$\mathrm{Bi}'(x)$
0.00	0.35502 805	−0.25881 940	0.61492 663	0.44828 836	0.50	0.23169 361	−0.22491 053	0.85427 704	0.54457 256
0.01	0.35243 992	−0.25880 174	0.61940 962	0.44831 926	0.51	0.22945 031	−0.22374 617	0.85974 431	0.54890 049
0.02	0.34985 214	−0.25874 909	0.62389 322	0.44841 254	0.52	0.22721 872	−0.22257 027	0.86525 543	0.55334 239
0.03	0.34726 505	−0.25866 197	0.62837 808	0.44856 911	0.53	0.22499 894	−0.22138 322	0.87081 154	0.55789 959
0.04	0.34467 901	−0.25854 090	0.63286 482	0.44878 987	0.54	0.22279 109	−0.22018 541	0.87641 381	0.56257 345
0.05	0.34209 435	−0.25838 640	0.63735 409	0.44907 570	0.55	0.22059 527	−0.21897 720	0.88206 341	0.56736 532
0.06	0.33951 139	−0.25819 898	0.64184 655	0.44942 752	0.56	0.21841 158	−0.21775 898	0.88776 152	0.57227 662
0.07	0.33693 047	−0.25797 916	0.64634 286	0.44984 622	0.57	0.21624 012	−0.21653 112	0.89350 934	0.57730 873
0.08	0.33435 191	−0.25772 745	0.65084 370	0.45033 270	0.58	0.21408 099	−0.21529 397	0.89930 810	0.58246 311
0.09	0.33177 603	−0.25744 437	0.65534 975	0.45088 787	0.59	0.21193 427	−0.21404 790	0.90515 902	0.58774 120
0.10	0.32920 313	−0.25713 042	0.65986 169	0.45151 263	0.60	0.20980 006	−0.21279 326	0.91106 334	0.59314 448
0.11	0.32663 352	−0.25678 613	0.66438 023	0.45220 789	0.61	0.20767 844	−0.21153 041	0.91702 233	0.59867 447
0.12	0.32406 751	−0.25641 200	0.66890 609	0.45297 457	0.62	0.20556 948	−0.21025 970	0.92303 726	0.60433 267
0.13	0.32150 538	−0.25600 854	0.67343 997	0.45381 357	0.63	0.20347 327	−0.20898 146	0.92910 941	0.61012 064
0.14	0.31894 743	−0.25557 625	0.67798 260	0.45472 582	0.64	0.20138 987	−0.20769 605	0.93524 011	0.61603 997
0.15	0.31639 395	−0.25511 565	0.68253 473	0.45571 223	0.65	0.19931 937	−0.20640 378	0.94143 066	0.62209 226
0.16	0.31384 521	−0.25462 724	0.68709 709	0.45677 373	0.66	0.19726 182	−0.20510 500	0.94768 241	0.62827 912
0.17	0.31130 150	−0.25411 151	0.69167 046	0.45791 125	0.67	0.19521 729	−0.20380 004	0.95399 670	0.63460 222
0.18	0.30876 307	−0.25356 898	0.69625 558	0.45912 572	0.68	0.19318 584	−0.20248 920	0.96037 491	0.64106 324
0.19	0.30623 020	−0.25300 013	0.70085 323	0.46041 808	0.69	0.19116 752	−0.20117 281	0.96681 843	0.64766 389
0.20	0.30370 315	−0.25240 547	0.70546 420	0.46178 928	0.70	0.18916 240	−0.19985 119	0.97332 866	0.65440 592
0.21	0.30118 218	−0.25178 548	0.71008 928	0.46324 026	0.71	0.18717 052	−0.19852 464	0.97990 703	0.66129 109
0.22	0.29866 753	−0.25114 067	0.71472 927	0.46477 197	0.72	0.18519 192	−0.19719 347	0.98655 496	0.66832 121
0.23	0.29615 945	−0.25047 151	0.71938 499	0.46638 539	0.73	0.18322 666	−0.19585 798	0.99327 394	0.67549 810
0.24	0.29365 818	−0.24977 850	0.72405 726	0.46808 147	0.74	0.18127 478	−0.19451 846	1.00006 542	0.68282 363
0.25	0.29116 395	−0.24906 211	0.72874 690	0.46986 119	0.75	0.17933 631	−0.19317 521	1.00693 091	0.69029 970
0.26	0.28867 701	−0.24832 284	0.73345 477	0.47172 554	0.76	0.17741 128	−0.19182 851	1.01387 192	0.69792 824
0.27	0.28619 757	−0.24756 115	0.73818 170	0.47367 549	0.77	0.17549 975	−0.19047 865	1.02088 999	0.70571 121
0.28	0.28372 586	−0.24677 753	0.74292 857	0.47571 205	0.78	0.17360 172	−0.18912 591	1.02798 667	0.71365 062
0.29	0.28126 209	−0.24597 244	0.74769 624	0.47783 623	0.79	0.17171 724	−0.18777 055	1.03516 353	0.72174 849
0.30	0.27880 648	−0.24514 636	0.75248 559	0.48004 903	0.80	0.16984 632	−0.18641 286	1.04242 217	0.73000 690
0.31	0.27635 923	−0.24429 976	0.75729 752	0.48235 148	0.81	0.16798 899	−0.18505 310	1.04976 421	0.73842 795
0.32	0.27392 055	−0.24343 309	0.76213 292	0.48474 462	0.82	0.16614 526	−0.18369 153	1.05719 128	0.74701 380
0.33	0.27149 064	−0.24254 682	0.76699 272	0.48722 948	0.83	0.16431 516	−0.18232 840	1.06470 504	0.75576 663
0.34	0.26906 968	−0.24164 140	0.77187 782	0.48980 713	0.84	0.16249 870	−0.18096 398	1.07230 717	0.76468 865
0.35	0.26665 787	−0.24071 730	0.77678 917	0.49247 861	0.85	0.16069 588	−0.17959 851	1.07999 939	0.77378 215
0.36	0.26425 540	−0.23977 495	0.78172 770	0.49524 501	0.86	0.15890 673	−0.17823 223	1.08778 340	0.78304 942
0.37	0.26186 243	−0.23881 481	0.78669 439	0.49810 741	0.87	0.15713 124	−0.17686 539	1.09566 096	0.79249 282
0.38	0.25947 916	−0.23783 731	0.79169 018	0.50106 692	0.88	0.15536 942	−0.17549 823	1.10363 385	0.80211 473
0.39	0.25710 574	−0.23684 291	0.79671 605	0.50412 463	0.89	0.15362 128	−0.17413 097	1.11170 386	0.81191 759
0.40	0.25474 235	−0.23583 203	0.80177 300	0.50728 168	0.90	0.15188 680	−0.17276 384	1.11987 281	0.82190 389
0.41	0.25238 916	−0.23480 512	0.80686 202	0.51053 920	0.91	0.15016 600	−0.17139 708	1.12814 255	0.83207 615
0.42	0.25004 630	−0.23376 259	0.81198 412	0.51389 833	0.92	0.14845 886	−0.17003 090	1.13651 496	0.84243 695
0.43	0.24771 395	−0.23270 487	0.81714 033	0.51736 025	0.93	0.14676 538	−0.16866 551	1.14499 193	0.85298 891
0.44	0.24539 226	−0.23163 239	0.82233 167	0.52092 614	0.94	0.14508 555	−0.16730 113	1.15357 539	0.86373 470
0.45	0.24308 135	−0.23054 556	0.82755 920	0.52459 717	0.95	0.14341 935	−0.16593 797	1.16226 728	0.87467 704
0.46	0.24078 139	−0.22944 479	0.83282 397	0.52837 457	0.96	0.14176 678	−0.16457 623	1.17106 959	0.88581 871
0.47	0.23849 250	−0.22833 050	0.83812 705	0.53225 956	0.97	0.14012 782	−0.16321 611	1.17998 433	0.89716 253
0.48	0.23621 482	−0.22720 310	0.84346 952	0.53625 338	0.98	0.13850 245	−0.16185 781	1.18901 352	0.90871 137
0.49	0.23394 848	−0.22606 297	0.84885 248	0.54035 729	0.99	0.13689 066	−0.16050 153	1.19815 925	0.92046 818
0.50	0.23169 361	−0.22491 053	0.85427 704	0.54457 256	1.00	0.13529 242	−0.15914 744	1.20742 359	0.93243 593
	$\left[{(-6)1 \atop 4}\right]$	$\left[{(-6)4 \atop 4}\right]$	$\left[{(-6)5 \atop 4}\right]$	$\left[{(-5)1 \atop 4}\right]$		$\left[{(-6)2 \atop 4}\right]$	$\left[{(-6)1 \atop 4}\right]$	$\left[{(-5)1 \atop 4}\right]$	$\left[{(-5)3 \atop 4}\right]$

AIRY FUNCTIONS—AUXILIARY FUNCTIONS FOR LARGE POSITIVE ARGUMENTS

ζ^{-1}	x	$f(-\zeta)$	$f(\zeta)$	$g(-\zeta)$	$g(\zeta)$	ζ^{-1}	x	$f(-\zeta)$	$f(\zeta)$	$g(-\zeta)$	$g(\zeta)$
1.5	1.000000	0.527027	0.619912	0.619954	0.478728	0.50	2.080084	0.548230	0.593015	0.587245	0.526011
1.4	1.047069	0.528783	0.620335	0.617156	0.479925	0.45	2.231443	0.549584	0.589451	0.585235	0.530678
1.3	1.100099	0.530601	0.620327	0.614275	0.481658	0.40	2.413723	0.550980	0.585855	0.583174	0.535345
1.2	1.160397	0.532488	0.619799	0.611305	0.484018	0.35	2.638450	0.552421	0.582330	0.581056	0.539902
1.1	1.229700	0.534448	0.618649	0.608239	0.487107	0.30	2.924018	0.553912	0.578985	0.578878	0.544235
1.0	1.310371	0.536489	0.616764	0.605068	0.491037	0.25	3.301927	0.555456	0.575908	0.576635	0.548255
0.9	1.405721	0.538618	0.614022	0.601782	0.495921	0.20	3.831547	0.557058	0.573135	0.574320	0.551930
0.8	1.520550	0.540844	0.610309	0.598372	0.501859	0.15	4.641589	0.558724	0.570636	0.571927	0.555296
0.7	1.662119	0.543180	0.605543	0.594823	0.508909	0.10	6.082202	0.560462	0.568343	0.569448	0.558428
0.6	1.842016	0.545636	0.599723	0.591120	0.517032	0.05	9.654894	0.562280	0.566204	0.566873	0.561382
0.5	2.080084	0.548230	0.593015	0.587245	0.526011	0.00	∞	0.564190	0.564190	0.564190	0.564190
	$\left[{(-3)7 \atop 9}\right]$	$\left[{(-5)2 \atop 4}\right]$	$\left[{(-4)1 \atop 6}\right]$	$\left[{(-5)2 \atop 4}\right]$	$\left[{(-4)1 \atop 6}\right]$			$\left[{(-5)1 \atop 4}\right]$	$\left[{(-5)4 \atop 6}\right]$	$\left[{(-5)1 \atop 4}\right]$	$\left[{(-5)4 \atop 6}\right]$

$\mathrm{Ai}(x)=\frac{1}{2}x^{-\frac{1}{4}}e^{-\zeta}f(-\zeta)$ $\mathrm{Bi}(x)=x^{-\frac{1}{4}}e^{\zeta}f(\zeta)$ $\mathrm{Ai}'(x)=-\frac{1}{2}x^{\frac{1}{4}}e^{-\zeta}g(-\zeta)$ $\mathrm{Bi}'(x)=x^{\frac{1}{4}}e^{\zeta}g(\zeta)$ $\zeta=\frac{2}{3}x^{\frac{3}{2}}$

From J. C. P. Miller, The Airy integral, British Assoc. Adv. Sci. Mathematical Tables, Part-vol. B. Cambridge Univ. Press, Cambridge, England, 1946 (with permission).

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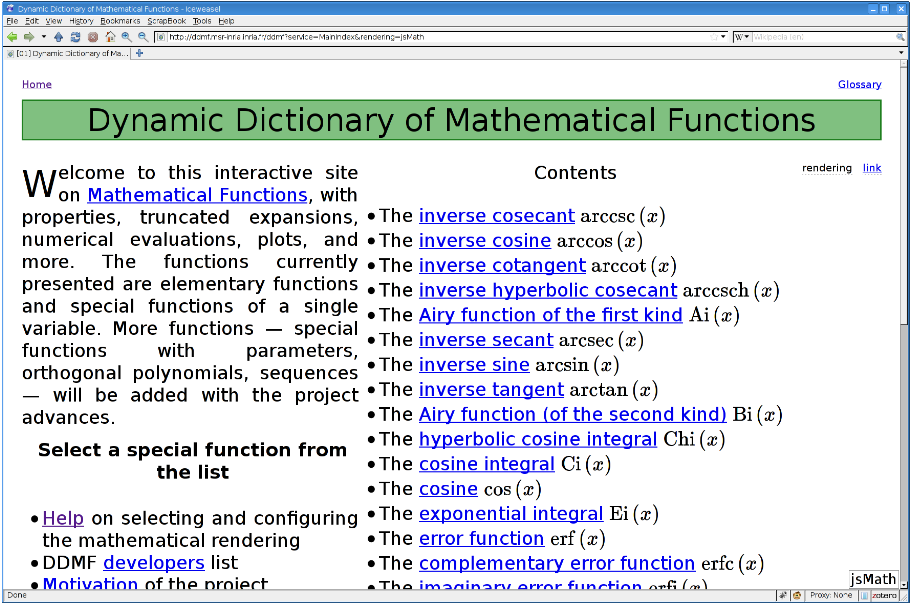
Table 10.11 AIRY FUNCTIONS

x	$\mathrm{Ai}(-x)$	$\mathrm{Ai}'(-x)$	$\mathrm{Bi}(-x)$	$\mathrm{Bi}'(-x)$	x	$\mathrm{Ai}(-x)$	$\mathrm{Ai}'(-x)$	$\mathrm{Bi}(-x)$	$\mathrm{Bi}'(-x)$
0.00	0.35502 805	−0.25881 940	0.61492 663	0.44828 836	0.50	0.47572 809	−0.20408 167	0.38035 266	0.50593 371
0.01	0.35761 619	−0.25880 157	0.61044 364	0.44831 896	0.51	0.47775 692	−0.20167 409	0.37528 379	0.50784 166
0.02	0.36020 397	−0.25874 771	0.60596 005	0.44841 015	0.52	0.47976 138	−0.19920 846	0.37019 579	0.50976 123
0.03	0.36279 102	−0.25865 731	0.60147 524	0.44856 104	0.53	0.48174 089	−0.19668 449	0.36508 853	0.51169 132
0.04	0.36537 699	−0.25852 986	0.59698 863	0.44877 074	0.54	0.48369 487	−0.19410 192	0.35996 193	0.51363 080
0.05	0.36796 149	−0.25836 484	0.59249 963	0.44903 833	0.55	0.48562 274	−0.19146 050	0.35481 589	0.51557 853
0.06	0.37054 416	−0.25816 173	0.58800 767	0.44936 293	0.56	0.48752 389	−0.18875 999	0.34965 033	0.51753 339
0.07	0.37312 460	−0.25792 001	0.58351 218	0.44974 364	0.57	0.48939 774	−0.18600 016	0.34446 520	0.51949 424
0.08	0.37570 243	−0.25763 918	0.57901 261	0.45017 955	0.58	0.49124 369	−0.18318 078	0.33926 043	0.52145 991
0.09	0.37827 725	−0.25731 872	0.57450 841	0.45066 976	0.59	0.49306 115	−0.18030 166	0.33403 599	0.52342 927
0.10	0.38084 867	−0.25695 811	0.56999 904	0.45121 336	0.60	0.49484 953	−0.17736 260	0.32879 184	0.52540 115
0.11	0.38341 628	−0.25655 685	0.56548 397	0.45180 945	0.61	0.49660 821	−0.17436 341	0.32352 796	0.52737 438
0.12	0.38597 967	−0.25611 443	0.56096 268	0.45245 712	0.62	0.49833 659	−0.17130 392	0.31824 435	0.52934 780
0.13	0.38853 843	−0.25563 033	0.55643 466	0.45315 546	0.63	0.50003 408	−0.16818 399	0.31294 101	0.53132 022
0.14	0.39109 213	−0.25510 406	0.55189 940	0.45390 355	0.64	0.50170 007	−0.16500 345	0.30761 795	0.53329 046
0.15	0.39364 037	−0.25453 511	0.54735 642	0.45470 047	0.65	0.50333 395	−0.16176 218	0.30227 521	0.53525 733
0.16	0.39618 269	−0.25392 297	0.54280 523	0.45554 530	0.66	0.50493 511	−0.15846 007	0.29691 282	0.53721 964
0.17	0.39871 868	−0.25326 716	0.53824 536	0.45643 713	0.67	0.50650 295	−0.15509 701	0.29153 084	0.53917 618
0.18	0.40124 789	−0.25256 716	0.53367 634	0.45737 503	0.68	0.50803 685	−0.15167 290	0.28612 932	0.54112 575
0.19	0.40376 987	−0.25182 250	0.52909 771	0.45835 806	0.69	0.50953 620	−0.14818 768	0.28070 835	0.54306 714
0.20	0.40628 419	−0.25103 267	0.52450 903	0.45938 529	0.70	0.51100 040	−0.14464 129	0.27526 801	0.54499 912
0.21	0.40879 038	−0.25019 720	0.51990 986	0.46045 578	0.71	0.51242 882	−0.14103 366	0.26980 840	0.54692 048
0.22	0.41128 798	−0.24931 559	0.51529 977	0.46156 860	0.72	0.51382 087	−0.13736 479	0.26432 964	0.54883 000
0.23	0.41377 653	−0.24838 737	0.51067 835	0.46272 279	0.73	0.51517 591	−0.13363 464	0.25883 185	0.55072 642
0.24	0.41625 557	−0.24741 206	0.50604 518	0.46391 740	0.74	0.51649 336	−0.12984 322	0.25331 516	0.55260 852
0.25	0.41872 461	−0.24638 919	0.50139 987	0.46515 148	0.75	0.51777 258	−0.12599 055	0.24777 973	0.55447 506
0.26	0.42118 319	−0.24531 828	0.49674 203	0.46642 408	0.76	0.51901 296	−0.12207 665	0.24222 571	0.55632 480
0.27	0.42363 082	−0.24419 888	0.49207 127	0.46773 423	0.77	0.52021 390	−0.11810 157	0.23665 329	0.55815 647
0.28	0.42606 701	−0.24303 053	0.48738 722	0.46908 095	0.78	0.52137 479	−0.11406 538	0.23106 265	0.55996 884
0.29	0.42849 126	−0.24181 276	0.48268 953	0.47046 327	0.79	0.52249 501	−0.10996 815	0.22545 398	0.56176 063
0.30	0.43090 310	−0.24054 513	0.47797 784	0.47188 022	0.80	0.52357 395	−0.10580 999	0.21982 751	0.56353 059
0.31	0.43330 200	−0.23922 719	0.47325 181	0.47333 081	0.81	0.52461 101	−0.10159 101	0.21418 345	0.56527 745
0.32	0.43568 747	−0.23785 851	0.46851 112	0.47481 405	0.82	0.52560 557	−0.09731 134	0.20852 204	0.56699 994
0.33	0.43805 900	−0.23643 865	0.46375 543	0.47632 895	0.83	0.52655 703	−0.09297 113	0.20284 354	0.56869 679
0.34	0.44041 607	−0.23496 718	0.45898 443	0.47787 450	0.84	0.52746 479	−0.08857 055	0.19714 820	0.57036 671
0.35	0.44275 817	−0.23344 368	0.45419 784	0.47944 970	0.85	0.52832 824	−0.08410 979	0.19143 630	0.57200 845
0.36	0.44508 477	−0.23186 773	0.44939 534	0.48105 354	0.86	0.52914 678	−0.07958 904	0.18570 813	0.57362 071
0.37	0.44739 535	−0.23023 893	0.44457 667	0.48268 500	0.87	0.52991 982	−0.07500 854	0.17996 399	0.57520 220
0.38	0.44968 937	−0.22855 687	0.43974 156	0.48434 307	0.88	0.53064 676	−0.07036 852	0.17420 419	0.57675 165
0.39	0.45196 631	−0.22682 116	0.43488 973	0.48602 670	0.89	0.53132 700	−0.06566 925	0.16842 906	0.57826 777
0.40	0.45422 561	−0.22503 141	0.43002 094	0.48773 486	0.90	0.53195 995	−0.06091 100	0.16263 895	0.57974 926
0.41	0.45646 675	−0.22318 723	0.42513 495	0.48946 652	0.91	0.53254 502	−0.05609 407	0.15683 420	0.58119 484
0.42	0.45868 918	−0.22128 826	0.42023 153	0.49122 062	0.92	0.53308 163	−0.05121 879	0.15101 518	0.58260 321
0.43	0.46089 233	−0.21933 412	0.41531 047	0.49299 611	0.93	0.53356 920	−0.04628 549	0.14518 226	0.58397 309
0.44	0.46307 567	−0.21732 447	0.41037 154	0.49479 193	0.94	0.53400 715	−0.04129 452	0.13933 585	0.58530 317
0.45	0.46523 864	−0.21525 894	0.40541 457	0.49660 702	0.95	0.53439 490	−0.03624 628	0.13347 634	0.58659 217
0.46	0.46738 066	−0.21313 721	0.40043 934	0.49844 031	0.96	0.53473 189	−0.03114 116	0.12760 415	0.58783 879
0.47	0.46950 119	−0.21095 893	0.39544 570	0.50029 070	0.97	0.53501 754	−0.02597 957	0.12171 971	0.58904 174
0.48	0.47159 965	−0.20872 379	0.39043 348	0.50215 713	0.98	0.53525 129	−0.02076 197	0.11582 346	0.59019 973
0.49	0.47367 548	−0.20643 147	0.38540 251	0.50403 850	0.99	0.53543 259	−0.01548 880	0.10991 587	0.59131 145
0.50	0.47572 809	−0.20408 167	0.38035 266	0.50593 371	1.00	0.53556 088	−0.01016 057	0.10399 739	0.59237 563
	$\left[{(-6)3 \atop 4}\right]$	$\left[{(-6)7 \atop 4}\right]$	$\left[{(-6)2 \atop 4}\right]$	$\left[{(-6)8 \atop 4}\right]$		$\left[{(-6)7 \atop 4}\right]$	$\left[{(-6)8 \atop 4}\right]$	$\left[{(-6)2 \atop 4}\right]$	$\left[{(-6)6 \atop 4}\right]$

Demo Time!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Wikipedia (en)
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Dynamic Dictionary of Mathematical Functions
Welcome to this interactive site on Mathematical Functions, with properties, truncated expansions, numerical evaluations, plots, and more. The functions currently presented are elementary functions and special functions of a single variable. More functions — special functions with parameters, orthogonal polynomials, sequences — will be added with the project advances.
Select a special function from the list
• Help on selecting and configuring the mathematical rendering
• DDMF developers list
• Motivation of the project
Contents
rendering link
• The inverse cosecant arccsc (x)
• The inverse cosine arccos (x)
• The inverse cotangent arccot (x)
• The inverse hyperbolic cosecant arccsch (x)
• The Airy function of the first kind Ai (x)
• The inverse secant arcsec (x)
• The inverse sine arcsin (x)
• The inverse tangent arctan (x)
• The Airy function (of the second kind) Bi (x)
• The hyperbolic cosine integral Chi (x)
• The cosine integral Ci (x)
• The cosine cos (x)
• The exponential integral Ei (x)
• The error function erf (x)
• The complementary error function erfc (x)
• The imaginary error function erfi (x)
jsMath
Done
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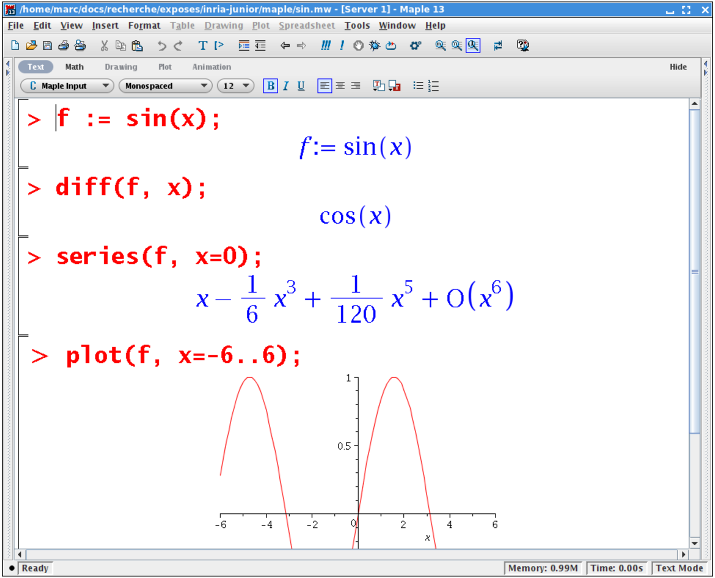
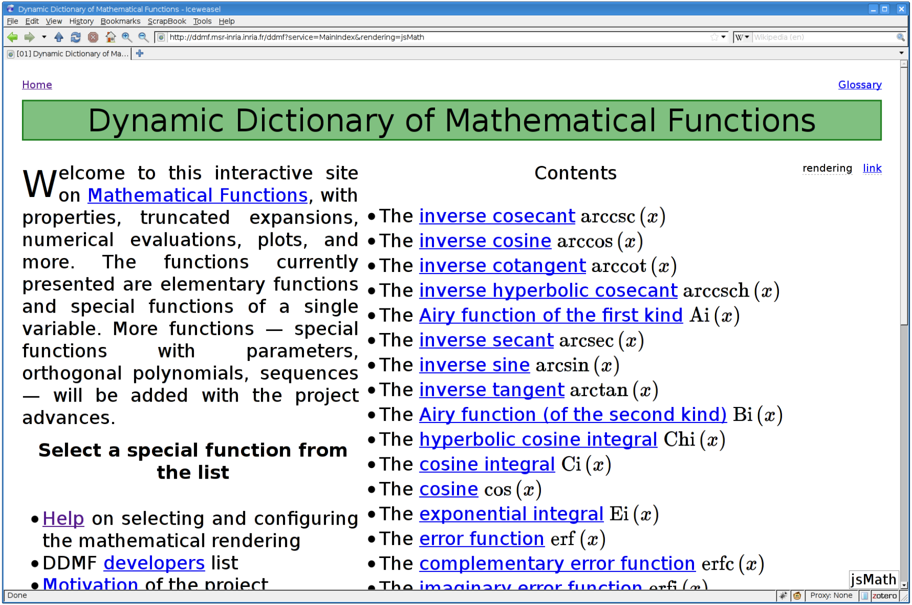
Computer Algebra



A Unified Approach to Special Functions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- The exponential integral $\mathrm{Ei}(x)$
- The error function $\operatorname{erf}(x)$
- The complementary error function $\operatorname{erfc}(x)$
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jsMath

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A Unified Approach to Special Functions

The Special Function Ai(x) - Iceweasel

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The Special Function $\mathrm{Ai}\,(x)$

1. Differential equation

rendering link

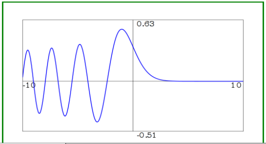
The function $\mathrm{Ai}\,(x)$ satisfies

$$\frac{d^2}{dx^2}y\,(x) - xy\,(x) = 0$$

with initial values $y\,(0) = 1/3\,\frac{\sqrt[3]{3}}{\Gamma(2/3)},\ (y')\,(0) = -1/2\,\frac{\sqrt[6]{3}\Gamma(2/3)}{\pi}$.

metadata

2. Plot of $\mathrm{Ai}\,(x)$



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The Special Function Ai(x) - Iceweasel

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[01] Loading...

Home

The Special

Our data structure:
LODE with polynomial coefficients
+ initial values
("D-finite functions")

1. Differential equation

The function $\mathrm{Ai}(x)$ satisfies

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metadata

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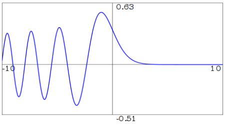

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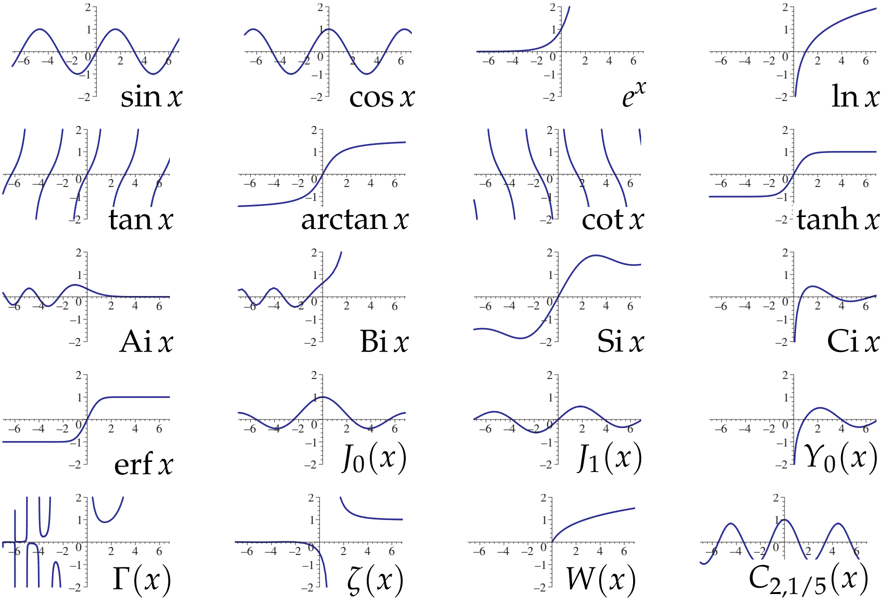
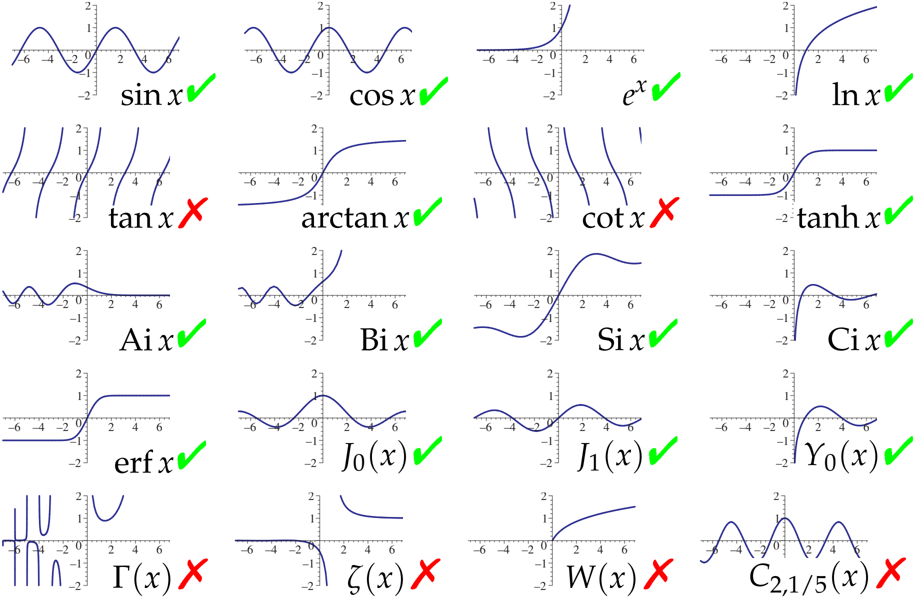
Mathematical Functions



Mathematical Functions



D-finite functions: An Example

$$(\sin x)^2 + (\cos x)^2 = 1$$

D-finite functions: An Example

1. $y(x) = \sin x \quad \Longleftrightarrow \quad y'' + y = 0 \quad y(0) = 0, y'(0) = 1$
   $y(x) = \cos x \quad \Longleftrightarrow \quad y'' + y = 0 \quad y(0) = 1, y'(0) = 0$
2. For any two solutions f, g of $y'' + y = 0$, the product fg satisfies $y''' + 3y' = 0$
3. The sum of two solutions of a linear homogeneous differential equation is still a solution
4. Hence, $y(x) = (\sin x)^2 + (\cos x)^2$ must satisfy $y''' + 3y' = 0$
5. Additionally,
   $$y'' + y = 0 \quad y(0) = 0, y'(0) = 1 \quad \Longrightarrow \quad y(x) = x - \frac{1}{6}x^3 + O(x^4)$$
   $$y'' + y = 0 \quad y(0) = 1, y'(0) = 0 \quad \Longrightarrow \quad y(x) = 1 - \frac{1}{2}x^2 + O(x^4)$$
   so that $(\sin x)^2 + (\cos x)^2 = 1 + O(x^4)$
6. Thus, $(\sin x)^2 + (\cos x)^2 = 1$.

Numerical Evaluation

The Special Function Ai(x) - Iceweasel

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The Special Function $\mathrm{Ai}\,(x)$

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metadata

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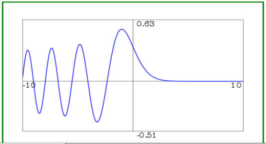

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Numerical Evaluation

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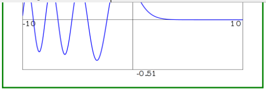

min = -10 max = 10 Submit Query

3. Numerical Evaluation

$$\mathrm{Ai}\,(1/4+1/4\,i) \approx 0.28881085 - 0.06285935\,i.$$

metadata

(Below, path may be either a point z or a broken-line path $[z_1, z_2, \ldots, z_n]$ along which to perform analytic continuation of the solution of the defining differential equation. Each z_k should be of the form x + y*i.)

path = 1/4+1/4*i precision = 8 Submit Query

4. Taylor expansion of $\mathrm{Ai}\,(x)$ at 0

- Expansion of AiryAi at 0:

$$\mathrm{Ai}\,(x) = \sum_{n=0}^{\infty} 1/3\,\frac{\sqrt[3]{3}x^{3\,n}}{9^n\Gamma\,(n+2/3)\,n!} - 1/9\,\frac{3^{2/3}x^{3\,n+1}}{9^n\Gamma\,(n+4/3)\,n!}.$$

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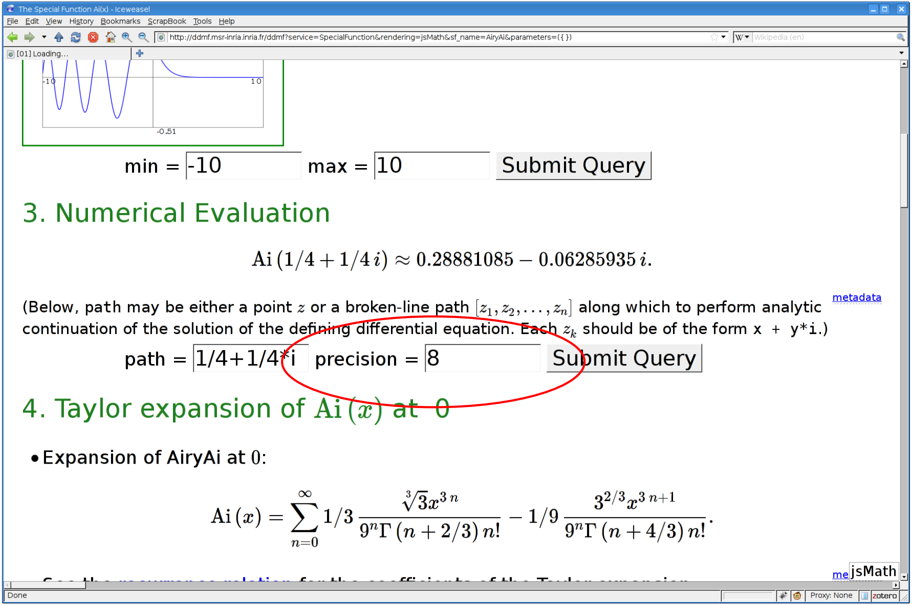
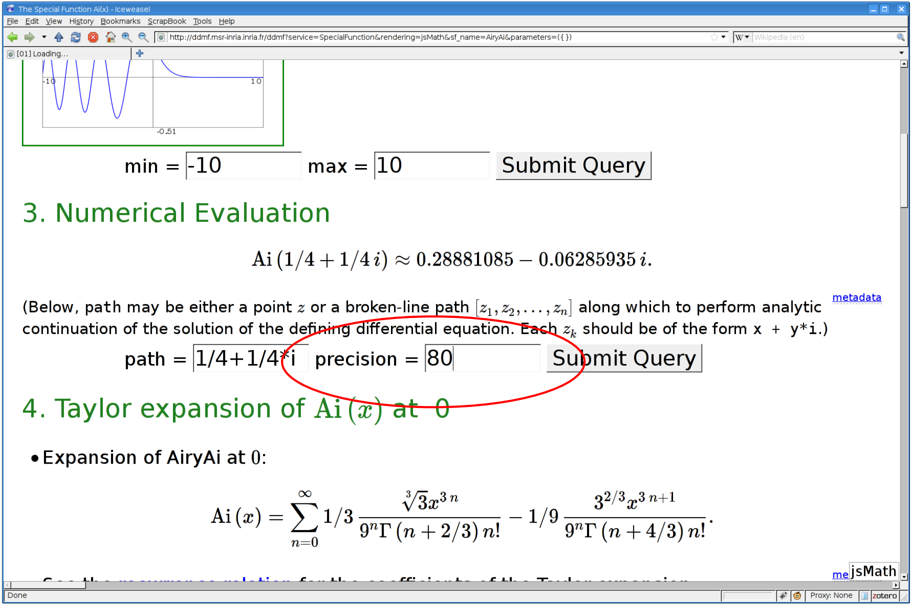
Numerical Evaluation



Numerical Evaluation



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-10 10 -0.51

min = -10 max = 10 Submit Query

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metadata

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path = 1/4+1/4*i precision = 80 Submit Query

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Numerical Evaluation

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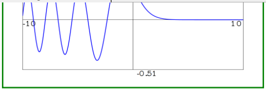

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$$\mathrm{Ai}\left(1/4+1/4\,i\right) \approx 0.2888108538482087217325648367140704681126252480580043686174937839264 7$$

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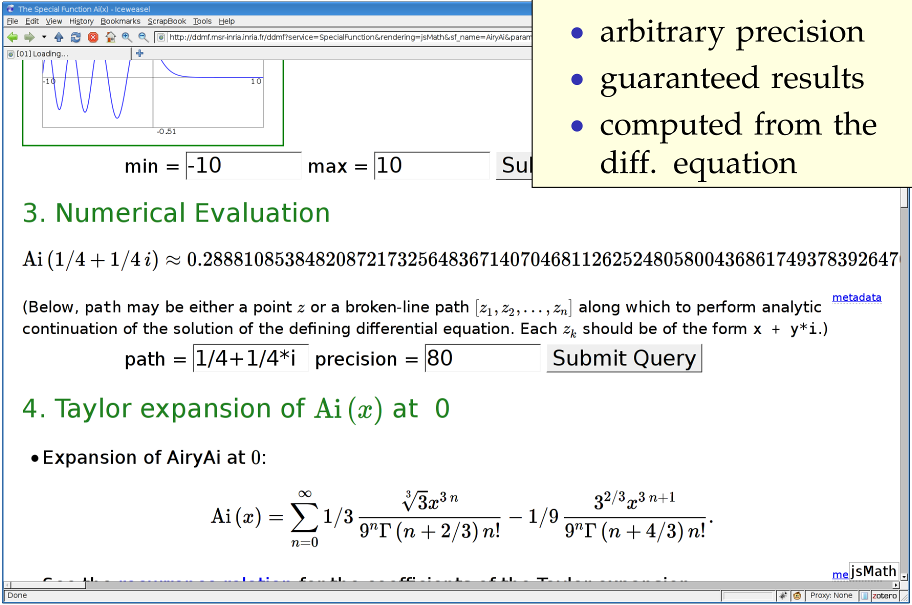
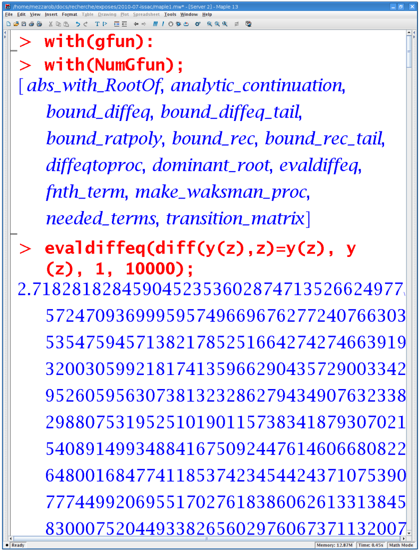
Numerical Evaluation

- arbitrary precision
- guaranteed results
- computed from the diff. equation



NumGfun



- Maple package behind the "analytic" features of the DDMF
- Features: numerical evaluation, bounds, and more
- Subpackage of Salvy & Zimmermann's gfun, LGPL.

Have a look at the DDMF

`http://ddmf.msr-inria.inria.fr/`

Get NumGfun from

`http://algo.inria.fr/libraries/`

Differential equations are a nice data structure

to work with their solutions

(Recurrence relations too, for sequences.)
(And more complicated functional equations as well.)

Have a look at the DDMF

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Differential equations are a … re

to work with their solutions

Thank you for your attention!

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Differential equations are a nice data structure

to work with their solutions

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High-Precision Numerical Evaluation

One Million Decimal Digits of π

$$\frac{1}{\pi} = 12 \sum_{k=0}^{\infty} \frac{(-1)^k \, (6k)! \, (13591409 + 545140134k)}{(3k)! \, (k!)^3 \, 640320^{3k+3/2}} \qquad \text{(Chudnovsky}^2 \text{ 1989)}$$

$\pi \simeq$ 3,141592653589793 238462643383279 502884197169399375 105820974944592307 81640628620899862 803482534211706798 214808651328230664 709384460955058 223172535940812848 111745028410270

. . .

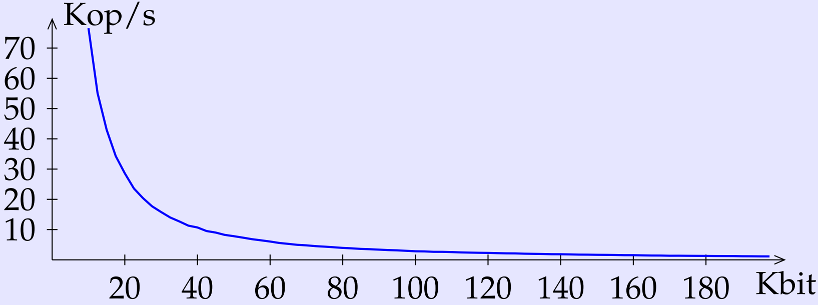
879 613 033 1164 6283 9963 46460 4220901061057794581 51

Fast Integer Multiplication

Theorem (Fürer)

The product of two n-bits integers may be computed in $O(n\,\log(n)\,2^{O(\log^* n)})$ bit operations.

In Practice (GMP 5.0.2)



Computing π

A Matrix Formula

1. $$s_n = 12 \sum_{k=0}^{n-1} \frac{(-1)^k \, (6k)! \, (13591409 + 545140134k)}{(3k)! \, (k!)^3 \, 640320^{3k+3/2}} = \sum_{k=0}^{n-1} t_k$$

   where $t_{k+1} = \mathrm{rat}(k) \, t_k$

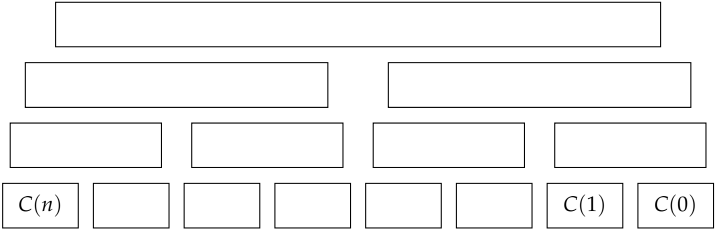
2. hence $$\begin{bmatrix} t_{k+1} \\ s_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathrm{rat}(k) & 0 \\ 1 & 1 \end{bmatrix}}_{C(k)} \begin{bmatrix} t_k \\ s_k \end{bmatrix}$$

3. $$\begin{bmatrix} t_n \\ s_n \end{bmatrix} = C(n-1) \cdots C(1) \, C(0) \begin{bmatrix} t_0 \\ s_0 \end{bmatrix} \to \begin{bmatrix} * \\ \pi \end{bmatrix} \quad \text{as } n \to \infty$$

Computing π

A Matrix Product Tree

$$C(n-1)\cdots C(1)\cdot C(0) = \big(C(n-1)\cdots C(\lfloor \tfrac{n}{2}\rfloor + 1)\big)\cdot\big(C(\lfloor \tfrac{n}{2}\rfloor)\cdots C(0)\big)$$



Overall complexity $\simeq$ area = $n \times$ (logarithmic factors)

Computing π

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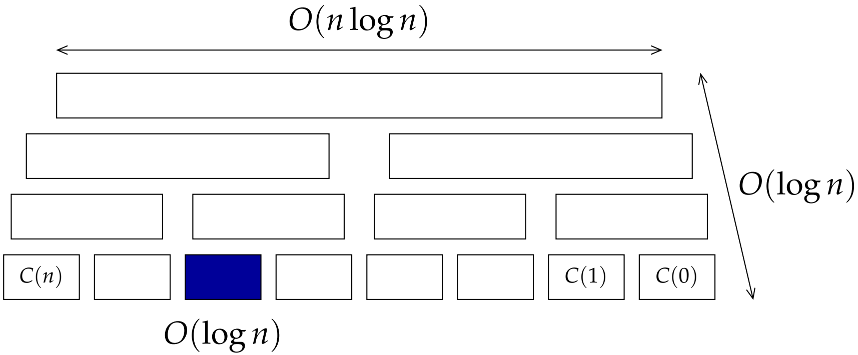

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