# Dissociation between language- and math-responsive networks in the brain


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## INTRODUCTION: WHY SCAN MATHEMATICIANS ?

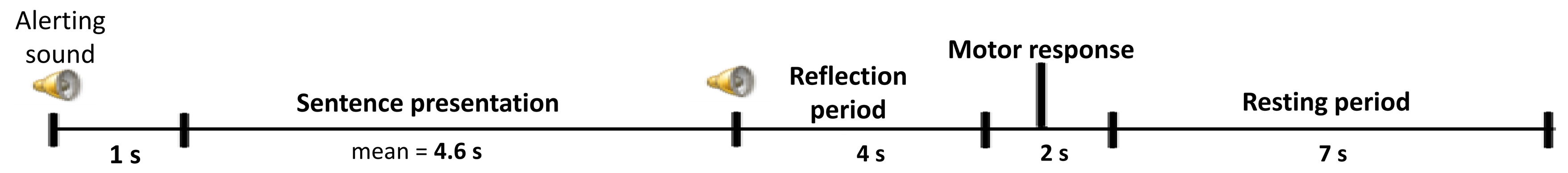
- ❖ Many scientists, like Hadamard or Einstein, share the intuition that, although mathematics is organized as a language, this language differs from, and even **dispenses with**, the structures of natural spoken language.
- ❖ This view, however, has been highly debated and disagrees with an influential view in cognitive science that considers mathematics as an **offshoot of the human capacity for language**. According to Noam Chomsky, "the origin of the mathematical capacity [lies in] an abstraction from linguistic operations".
- ❖ Does the human brain represent advanced mathematical concepts through **language**? Or does the acquisition of advanced mathematics rely mainly on a "neuronal recycling"[4] of brain regions involved in **number sense, spatial coding, and number recognition**?
- ❖ Is mathematical language similar to natural language? Are language areas used by mathematicians when they do mathematics? And does the brain comprise a **generic semantic system** that stores mathematical knowledge alongside knowledge of history, geography, or famous people?
- ❖ Here, **we refute those views** by reviewing three functional MRI studies of the representation and manipulation of high-level mathematical knowledge in professional mathematicians.

## METHOD

### Experiment 1: complex mathematical reflection

We scanned 15 professional mathematicians and 15 control subjects devoid of any mathematical training.

__6 Auditory runs__ in which participants were asked to perform fast intuitive semantic judgments on spoken mathematical and non mathematical statements (classify them as true, false, or meaningless). Four domains were studied: **analysis, algebra, topology and geometry**.




Exemplar statements of experiment 1:

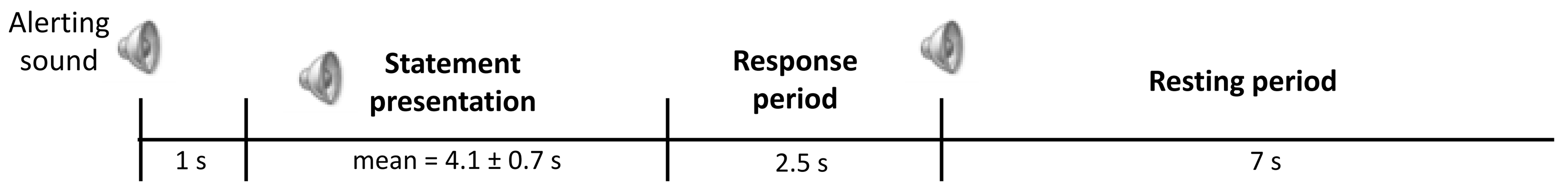
1) **Analysis**: A smooth function whose derivatives are all non-negative is analytic. (true)
2) **Algebra**: Any square matrix is equivalent to a permutation matrix. (false)
3) **Topology**: The graph of the completion of a compact group is dense in a partially connected open set. (meaningless)
4) **Geometry**: The euclidean orthogonal group has exactly two connected components. (true)
5) **Nonmath**: The end of the Council of Trent coincides with the fall of the Western Roman Empire. (false)

### fMRI acquisition and analysis :

- High resolution multiband fMRI sequence: TR = 1.5 s, voxel size = 1.5*1.5*1.5 mm$^3$
- Standard pre-processing and 2 mm smoothing
- General linear model computed in SPM8 at single and group levels.

### Control experiments 2 & 3: simpler facts, immediate response

We scanned 14 professional mathematicians.




Exemplar statements of experiment 2:

1) **Algebraic facts**: $(a+b)(a-b) = a^2 - b^2$
2) **Algebraic calculation**: $(x-1)(x+1) = x^2 - 1$
3) **Trigonometry**: $\sin(x+3\pi/2) = -\cos x$
4) **Complex numbers**: $Re(e^{i\pi/4}) = Im(e^{i\pi/4})$
5) **Euclidean geometry**: The section of a sphere by a plane is always a point
6) **General knowledge**: Rock'n'roll is a musical style characterized by a slow tempo

Exemplar statements of experiment 3:

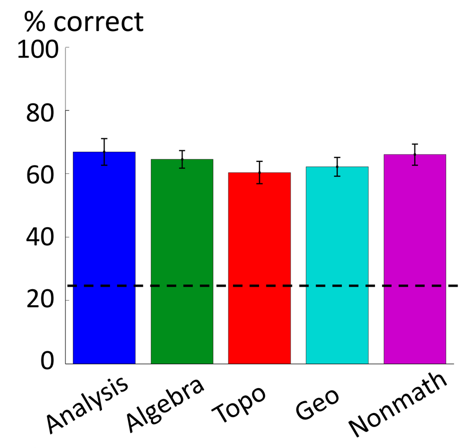
| | Math | vs | Non-math |
|---|---|---|---|
| 1) **Declaratives**: | The sine function is periodical | | Londonian buses are red |
| 2) **Quantified declaratives**: | Some matrices are diagonalizable | | Some ocean currents are warm |
| 3) **Negatives** : | Hyperboloids are not connected | | Tigers are not carnivores |
| 4) **Quantified negatives**: | Some infinite sets are not countable | | Some green plants are not climbing |

## RESULTS

### Behavior

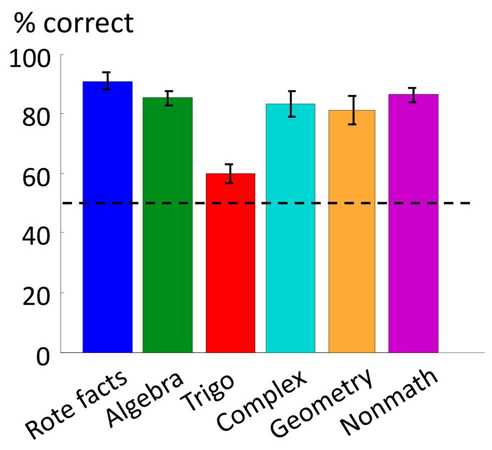
**Experiment 1**

- Mathematicians performed better than chance with math and non-math sentences.
- Mathematicians easily rejected meaningless math statements, but found it harder to judge the truth value of meaningful statements (55 % correct).



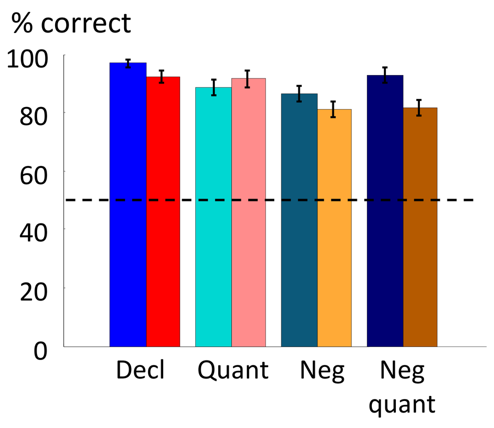

**Experiment 2**

- Mathematicians found these statements simpler than those of experiment 1.
- No difference was seen between math and nonmath statements.
- Trigonometry was significantly harder than other math.



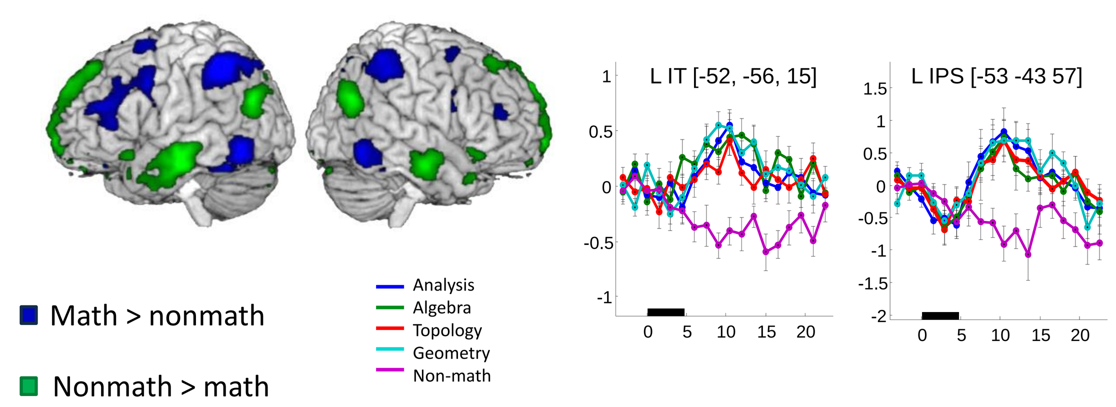

**Experiment 3**

- Mathematicians performed almost perfectly.
- No difference was seen between math and nonmath statements (except for neg quant).
- Negatives were significantly harder than other statements.

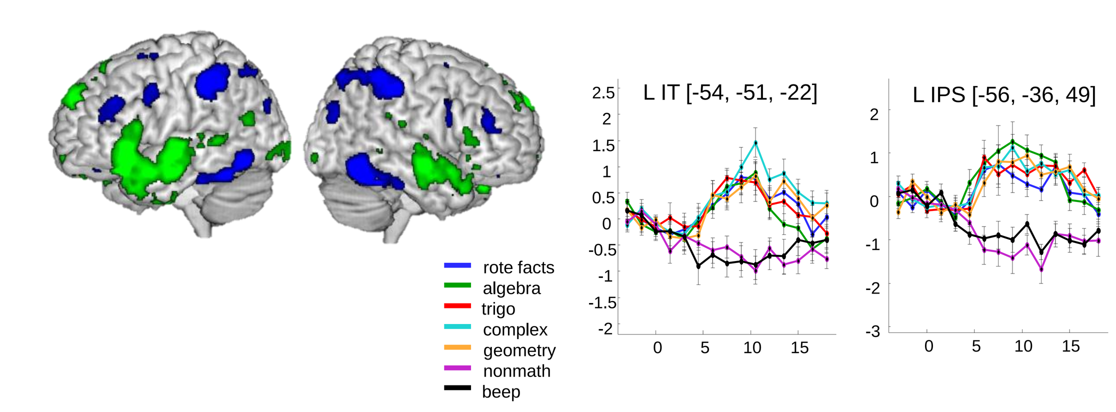



### Separation of brain networks

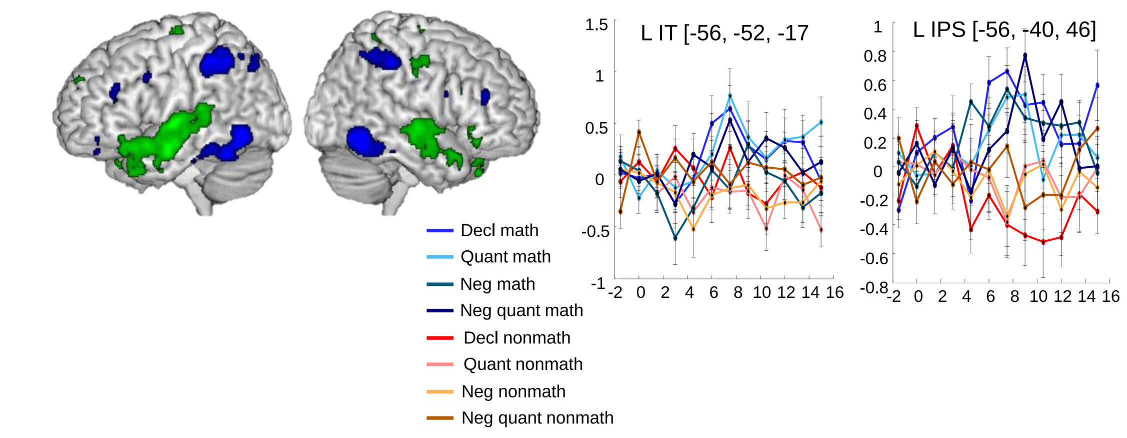
**Experiment 1**




**Experiment 2**

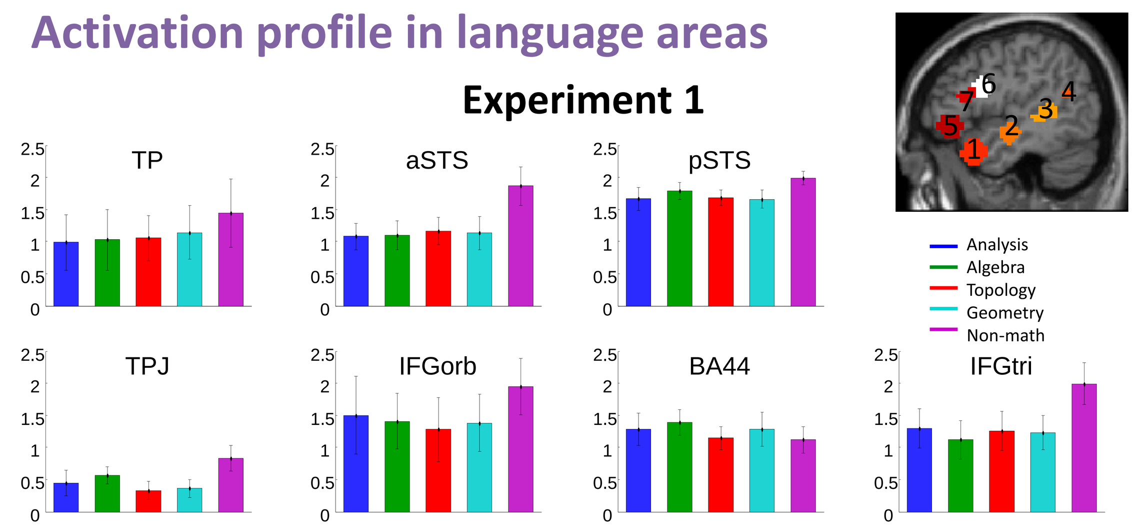



**Experiment 3**

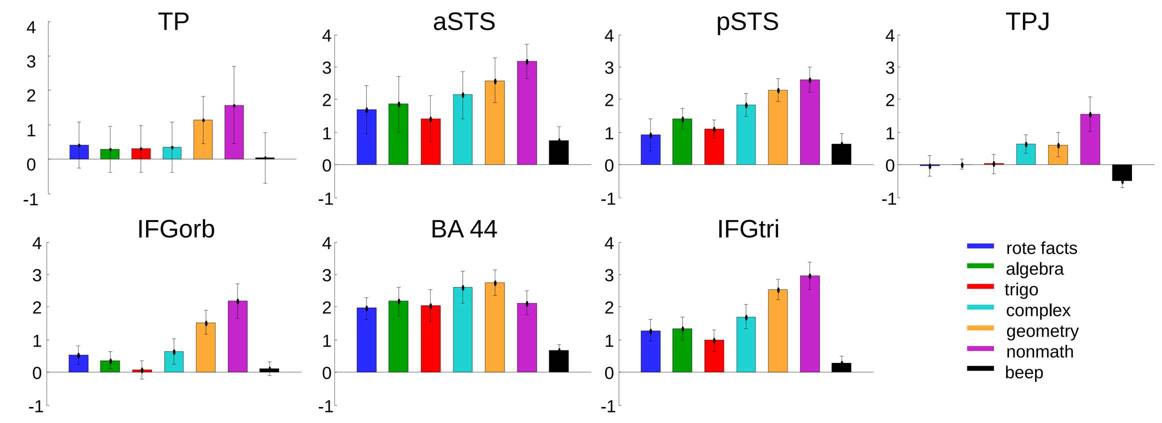



### Activation profile in language areas

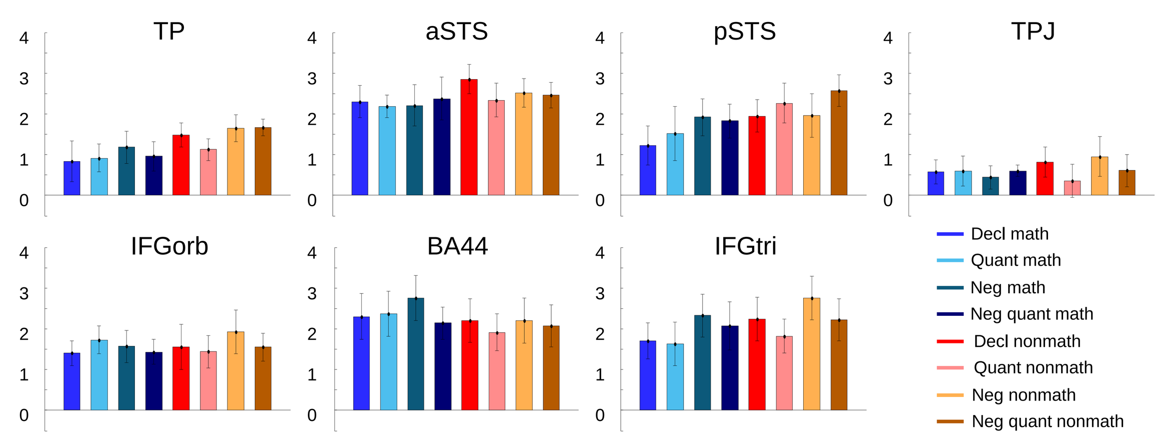
**Experiment 1**




**Experiment 2**




**Experiment 3**




## CONCLUSIONS

- ❖ High-level mathematics does not recruit the classical language semantic network.
- ❖ On the contrary, advanced mathematics makes use of the very same regions involved in elementary arithmetic and number recognition such as the intraparietal sulcus and the visual number form areas.
- ❖ Math-responsive regions are only concerned with mathematics and activates regardless or domain or difficulty.
- ❖ Together with other fMRI and recent intracranial studies, our results indicated a major separation between two brain networks for mathematical and non-mathematical semantics, which goes a long way to explain a variety of facts in neuroimaging, neuropsychology, and developmental disorders, such as evidence that severe aphasics may still understand and perform algebraic operations.
- ❖ Our fMRI findings raise many questions regarding the operational definition and intrinsic characteristics of the fields of "mathematics" and "language" that activate those two gross circuits. First, what is the exact extension of the domain of mathematics? Second, where does language stop and mathematics begin? Identifying exactly what the different mathematical domains share and delimiting, within natural language, the nature of the processes and concepts that do or do not activate the math-responsive network, are open questions that remain to be thoroughly investigated.

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