# **103-Numerical Cognition #1**

# Inserm



# **REPRESENTATIONS MATTER IN** CHILDREN'S UNDERSTANDING THAT " $2 \times 3 = 3 = 2$ "

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## **INTRODUCTION: Commutativity is a fundamental case of compositionality**

What is commutativity?

The principle of an operation by which switching the operands order does not change the result. Most operations are not commutative, but the primordial operations of addition, multiplication, translation, or rotation all are.

#### 2 + 3 = 3 + 2 $2 \times 3 = 3 \times 2$ $A \cap B$ Α

Why is it essential to children's arithmetic learning?

Commutativity is an important aspect of understanding number compositionality, i.e. understanding that a given number can be obtained by composing smaller numbers in various ways.

The present question:

Children seem to learn about additive commutativity from concrete situations first and then generalize it to symbols. Here, we ask whether it is also the case for multiplication, and what kinds of representations best convey the commutative principle in this case.

## **EXPERIMENT 1: Symbols and commutativity**



### **EXPERIMENT 2: Geometric vs linguistic cues**

Session 1: Non-symbolic judgments of commutativity – geometric cues

Participants: 42 preschoolers (5-year-olds) in the USA.







non-symbolic trials. It can be a first indication that commutativity of multiplication is learned symbolically first.

A regression analysis of children's performance on non-symbolic trials showed an influence of their symbolic performance but not their numeric weber fraction (i.e., an estimate of acuity of their number sense; evaluated in a separate block of the Panamath game at the end of the session).



-0.2

-0.2 -0.1 0.0 0.1 0.2 0.3

Weber fraction | others

Ungrouped

Baseline

\*\*\*

Added-variable plots

-0.2

Symbolic performance | others

0.2

0.6

Session 2: Math training on the commutative principle of multiplication

-0.6

0.4

0.2

0.0

-0.2

-04

**Participants:** 44 children agreed to participate in this second session.

#### Video lesson



#### Session 2: Non-symbolic judgments of commutativity – verbal cues

Participants: Same as session 1.

Charlie and Jessie are getting apples at the apple tree. Charlie got 2 baskets of 3 apples each,





Performance of: (1) Children with low pre-intervention level of symbolic mastery improved only on symbolic multiplicative commutative trials; (2) Children with high pre-intervention level of symbolic mastery improved only on non-symbolic multiplicative commutative trials.



#### **DISCUSSION and CONCLUSIONS**

- Our findings suggest that children have a limited understanding of the commutative principle of multiplication.
- To perceive quickly and accurately that 2 groups of 3 dots and 3 groups of 2 dots have the same number of dots, children may need to master multiplication and commutativity symbolically first.
- While the symmetry inherent to commutativity is obvious in symbolic expressions that display the same mirrored numbers, it is not in sets arranged in equal subgroups, likely because the intrinsic bidimensionality of multiplication that appears only in rectangular displays.
- Less than children's inability to grasp commutativity in multiplication before learning it formally, this result might
- reflect the fact that simple grouping is inefficient at conveying the idea of commutativity.
- Whether training commutativity with rectangular displays would readily transfer to other representations and yield generalization to other commutative situations remains to be investigated.

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