Fast comprehension of embedded geometrical primitives and rules in human adults and preschoolers

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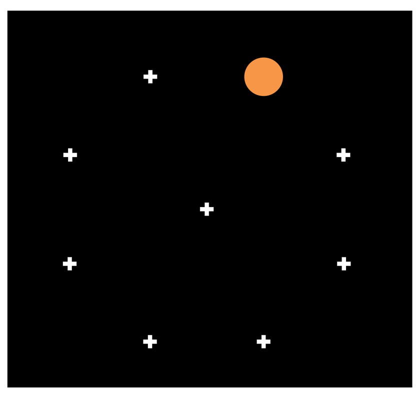
INTRODUCTION: ARE HUMANS ENDOWED WITH A GEOMETRICAL LANGUAGE?

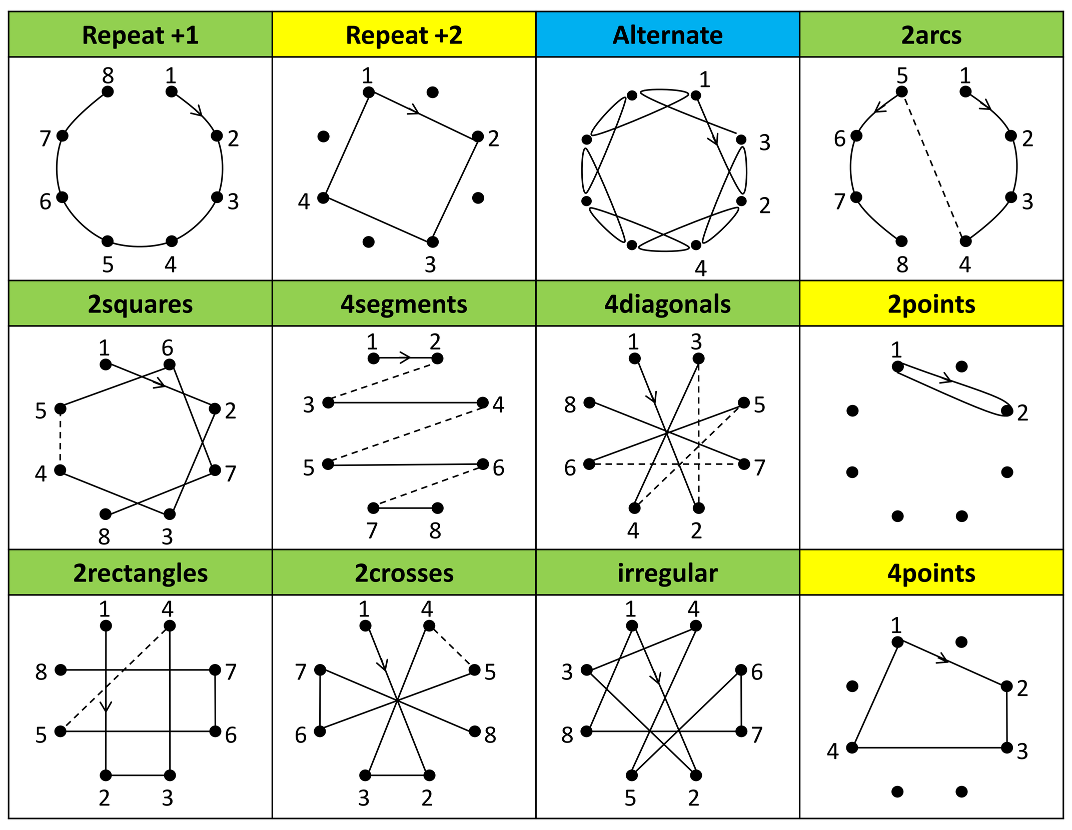
- Studies of sequence learning have outlined one possible mechanism by which complex mental representations are constructed out of simpler primitives: the human ability to **extract complex nested structures from sequential inputs**.
- Experiments in infants, preschoolers, and adults without access to education have demonstrated the existence of innate **"core knowledge" for space**, endowing humans with spontaneous intuitions of geometry.
- The question therefore arises whether a capacity for the internal representation and manipulation of nested sequences also underlies the acquisition of mathematics.
- We propose to formalize the human sensitivity to mathematical rules through a **"language of thought"** that allows the formation of complex representations from a small repertoire of primitives.

METHOD: COMPLETION TASK

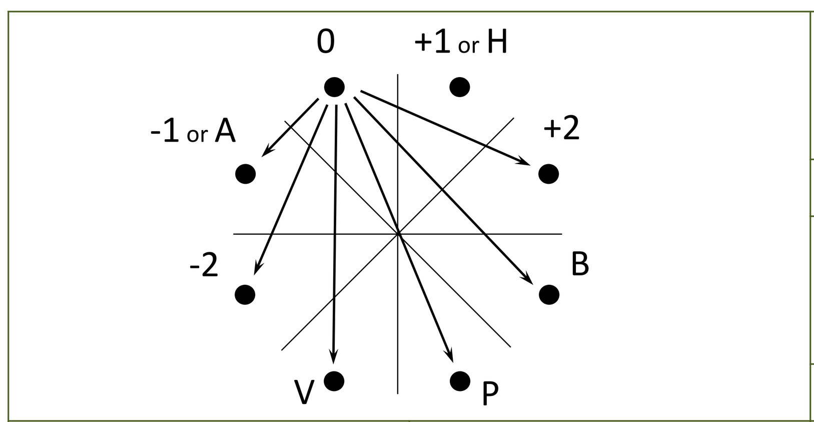


Subjects saw the first locations of a given sequence and had to point to the next ones. If they were mistaken, the sequence restarted.

Participants:
23 French adults and 47 5-years-old children, and 14 Munduruku teenagers and adults.



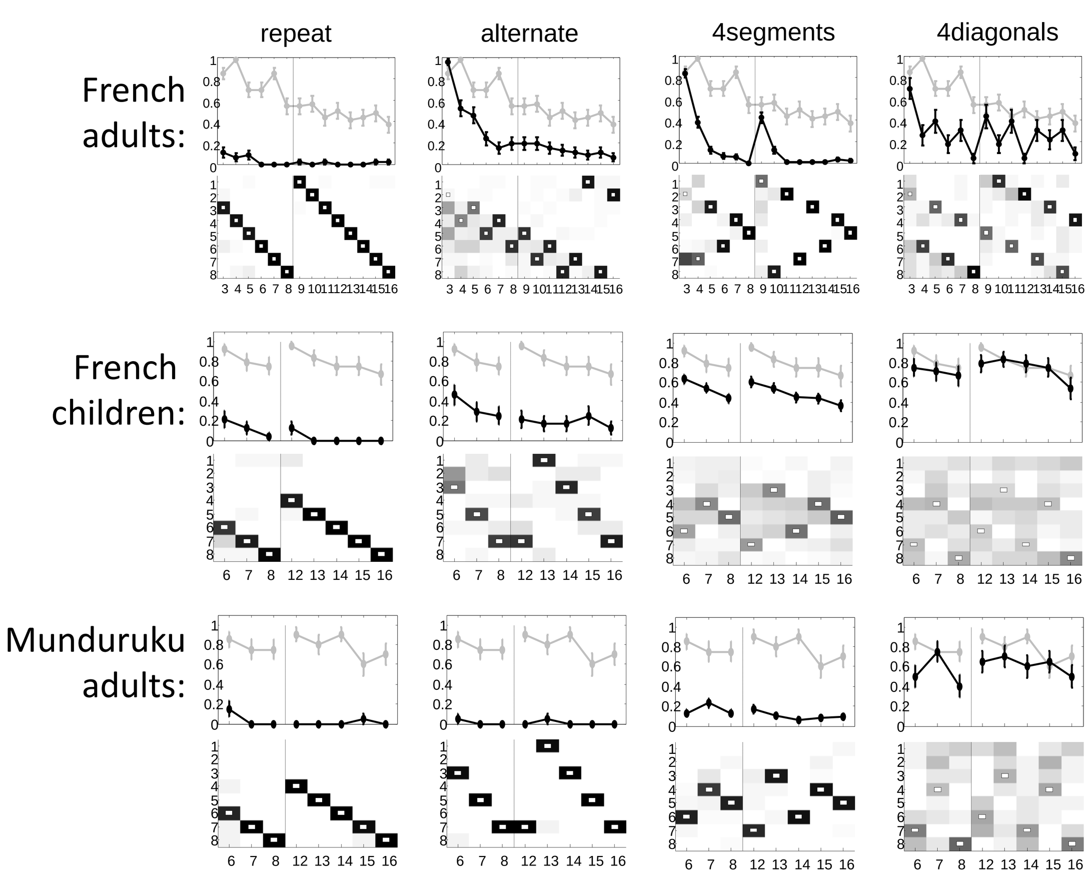
GEOMETRICAL LANGUAGE

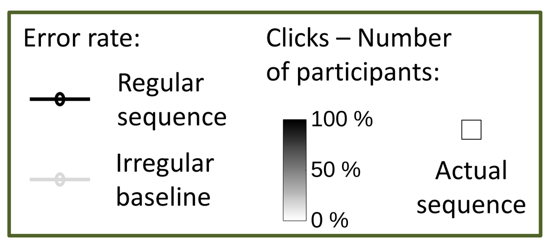


		Repeat +1 01234567 K=5	**Repeat +2** 02460246 K=5
		$[+0, [+1]^7]$	$[+0, [+2]^7]$
		Alternate 0213243546 K=8	**2arcs** 01237654 K=8
		$[+0, +2]^8$ {+1}]	$[[+0, [+1]^3]^2$ <V>]
2squares 02467135 K=8	**4segments** 01726354 K=7	**4diagonals** 04152637 K=7	**2points** 02020202 K=7
$[[+0, [+2]^3]^2$ <−1>]	$[[+0, H]^4$ {−1}]	$[[+0, P]^4$ {+1}]	$[[+0, +2]^4$ {+0}]
2rectangles 05416327 K = 10	**2crosses** 04512673 K=7	**Irregular** 04715236 K = 16	**4points** 02360236 K = 11
$[[[+0, −3]^2${P}]^2 <−2>]	$[[+0, P]^4$ {−3}]	[+0,P,B,+2,P,B,+1,H]	$[[+0,+2,+1,+3]^2$ <+0>]

RESULTS

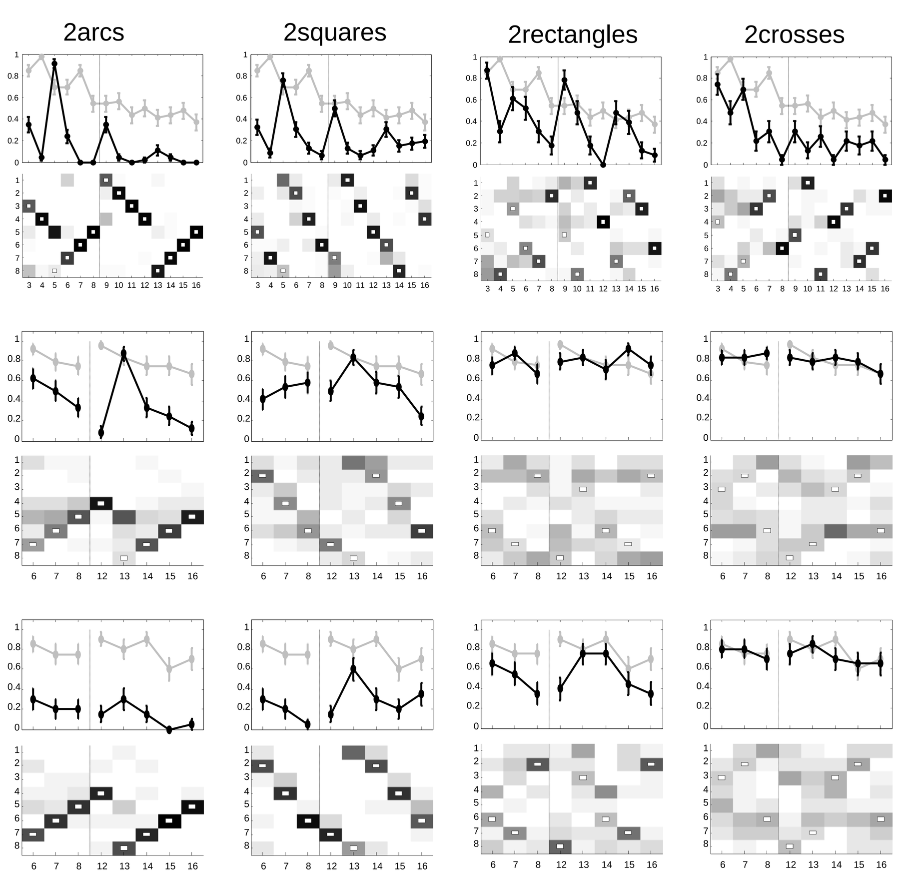
Knowledge of geometrical primitive rules



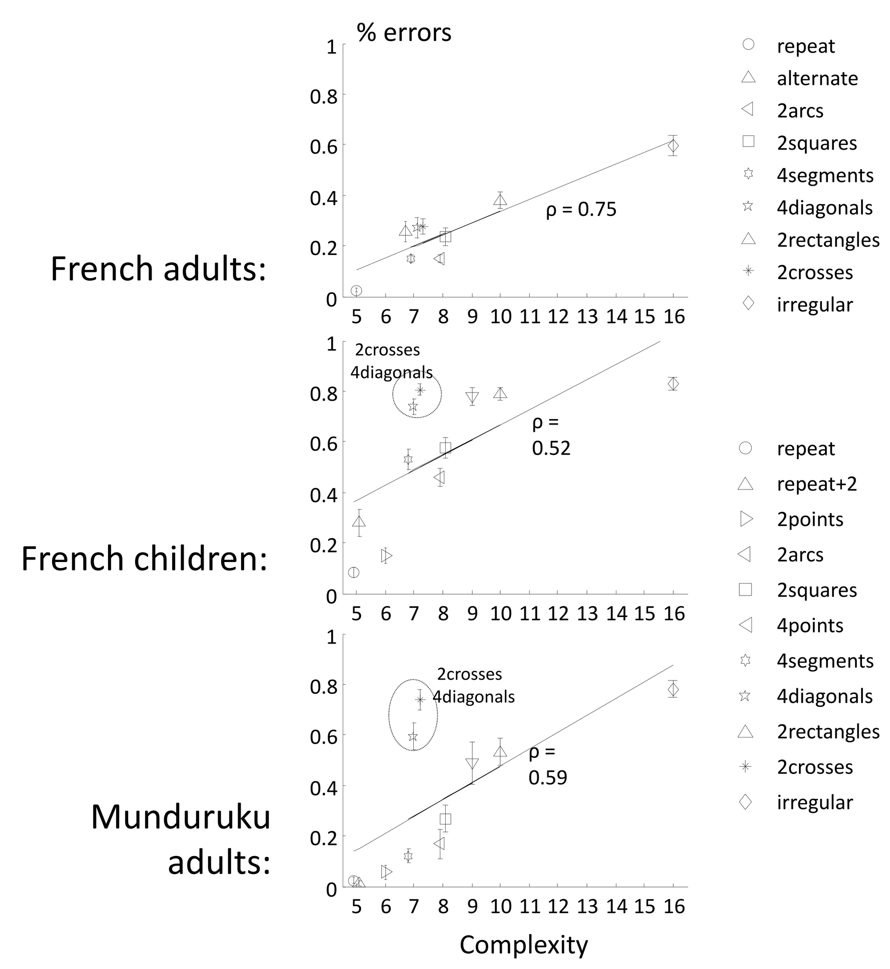


- In comparison to irregular baseline, most of the regular sequences were well learnt.
- Error pattern directly reflects the hierarchical internal representation of sequences.
- Error rate at specific data point indicates how well a given rule is understood: e.g. in "4segments", even data points reveal that all axial symmetries are detected by all groups of participants.

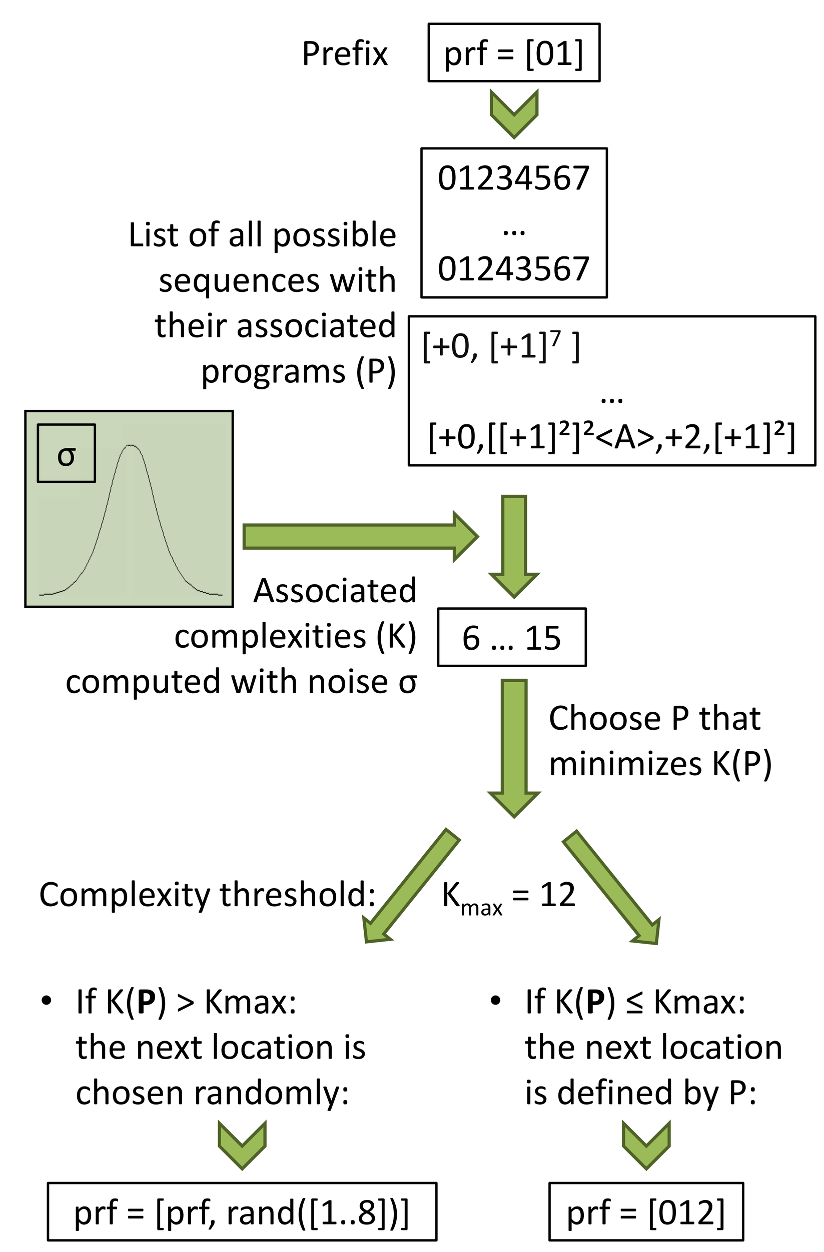
Learning of embedded rules

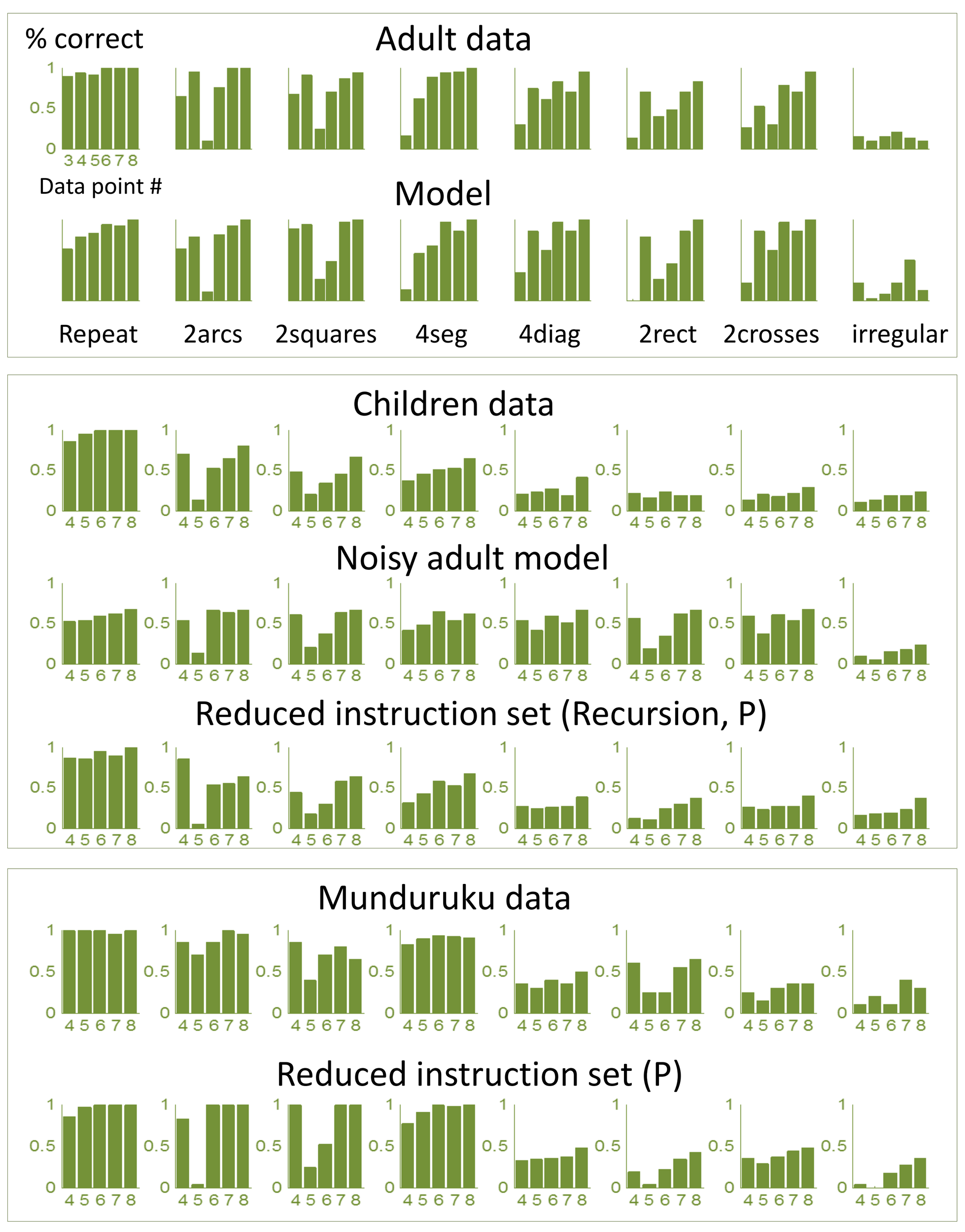


Kolmogorov complexity well predicts subjects' performance



MODEL FITS





CONCLUSIONS

- **Simple rotations and axial symmetries were all detected** and quickly used by human adults and 5-years-old children. Point symmetry was more challenging for French preschoolers or Munduruku adults than for French adults.
- Human subjects were able to detect most of the embedded expressions we used to define our visuospatial sequences such as simple repetition, concatenation, and some repetition with variation.
- **The analysis of error patterns provided direct evidence for hierarchical embedding**: superficial rules were acquired more quickly and induced fewer errors than deeper rules.
- **With age, a geometrical language endowed with nested rules seems to arise even in the absence of formal schooling**, as Munduruku adults who lacked school-based education, performed better than 5-years-old kids.
- In children, the failure with complex sequences could arise from limitations in working memory and not necessarily to a lack of understanding nested structures.
- **The theoretical complexity of a sequence was an excellent predictor of its mean error rate**, and we confirm that minimal description length is a reasonable approach of adult sequence learning capacity.
- Additional primitives, both geometrical and non-geometrical still need to be added to our model to complete its description of "core geometry".

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