TECHNISCHE UNIVERSITÄT WIEN

Kinetically Constrained Models Out of Equilibrium

Ivailo Hartarsky (joint with Fabio Toninelli)

Inn'formal probability seminar

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FWF

Der Wissenschaftsfonds.

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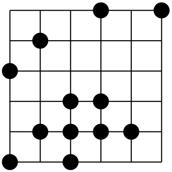

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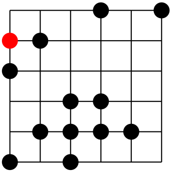

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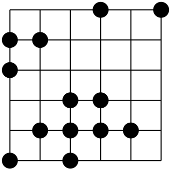

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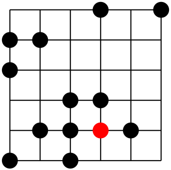

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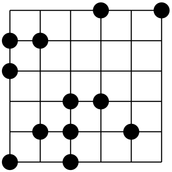

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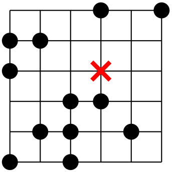

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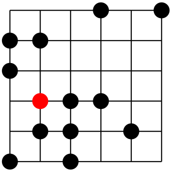

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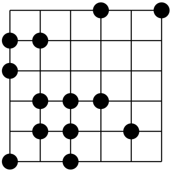

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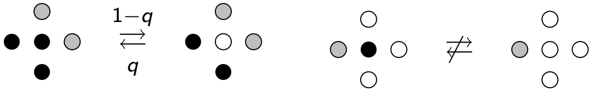

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$$\max_\omega d_{\mathrm{TV}}\left(\mu^\omega_{n/C}, Ber(q)^{\otimes\Lambda}\right) \geq 1 - \varepsilon, \quad \max_\omega d_{\mathrm{TV}}\left(\mu^\omega_{Cn}, Ber(q)^{\otimes\Lambda}\right) \leq \varepsilon.$$

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Remark

Holds for any kinetically constrained model, dimension and domain.

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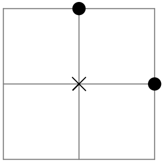

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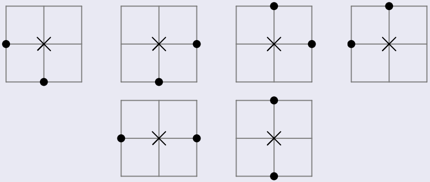

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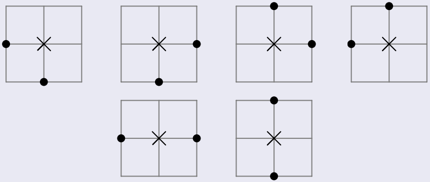

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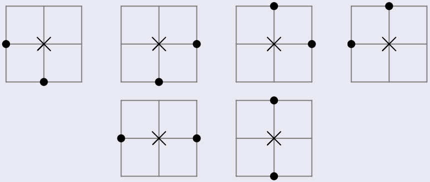

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- *subcritical trivial:* $\mathbb{P}_\pi(\tau = \infty) > 0$ *for any* $q < 1$.

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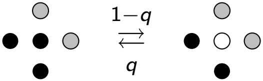
- East, $d = 1$, any $q > 0$: cutoff. [Ganguly, Lubetzky, Martinelli'15]
- East, $d > 1$, q small enough: cutoff for an axis-parallel box with special boundary condition. [Couzinié, Martinelli'22]

FA-2f



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Observation

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Make all ◦ initially ◦.

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CCP with $\circ$ initial condition has at most as much $\bullet$ as FA-2f.

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Lemma

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All ○ form a single space-time connected component.

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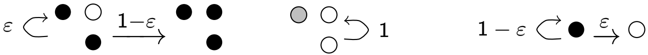

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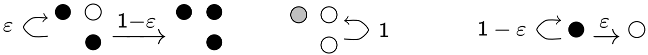

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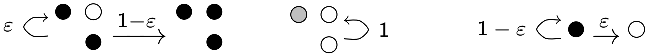
For any $\varepsilon > 0$*, if q is close enough to 1, then* ε*-BPwD has at most as much* $\bullet$ *as q-CCP sped up by a factor* $1/\sqrt{1-q}$*.*

Bootstrap percolation with death

Discrete time, parallel updates (probabilistic cellular automaton).



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Theorem (Toom'80)

Let $\delta > 0$. For $\varepsilon > 0$ small enough, for ε-BPwD with $\bullet$ initial condition on $\mathbb{Z}^2$, $\mathbb{P}(0$ is $\circ$ at time $t) < \delta$ for all $t \geq 0$.

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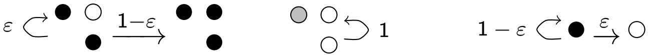

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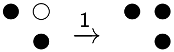
Proposition

For $C > 0$ there exists $\varepsilon > 0$ such that for any $x \in \mathbb{Z}^2 \times \mathbb{R}_+$, in ε-BPwD with $\bullet$ initial condition on $\mathbb{Z}^2$,

$$\mathbb{P}(x \in \text{space-time } \circ\text{-connected component of diameter } n) \leq e^{-Cn}.$$

Last passage percolation

Last passage percolation



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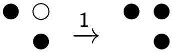

Observation

All • sites in LPP sped up by a factor $1-q$ *are coupled in* q*-CPP.*

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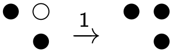

Observation

All • sites in LPP sped up by a factor $1 - q$ are coupled in q-CPP.

Theorem (Greenberg, Pascoe, Randall'09)

LPP on a box with • boundary condition reaches the • configuration in linear time.

Recap of the proof scheme for FA-2f

Recap of the proof scheme for FA-2f

FA-2f

Recap of the proof scheme for FA-2f

FA-2f $\rightarrow \mathrm{CCP}$

Recap of the proof scheme for FA-2f

FA-2f $\to \mathrm{CCP} \to \mathrm{LPP}$

Recap of the proof scheme for FA-2f

$$\text{FA-2f} \to \mathrm{CCP} \to \mathrm{LPP}$$
$$\to \mathrm{BPwD}$$

Open questions

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Both questions are open even for FA-1f in $d = 2$.

Open questions

- Prove the same for low temperatures (any $q > 0$).
- Prove cutoff.
- What about other graphs?

?