Characterization of laser quality for atomic physics

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Atomic physics is usually limited by the temperature of the atoms and the quality of the lasers. It's not easy within a sophisticated experiment to know what should be improved first. I've worked on a technique to characterize the frequency properties of a laser light: a low finesse cavity is used to convert frequency into intensity fluctuation. Then it's converted into an electrical signal by a photodiode. This signal is analysed by a spectrum analyser. This measurement also provides information about the opportunities of improving the control system of the lasers.

I. INTRODUCTION

As building universal quantum computers doesn't seem absolutely impossible nowadays, huge work is being done to slowly get closer from this goal. In fact such a machine is expected to be completely different from classical computers. A 2 bits classical memory can only be found in 4 different states: (00), (01), (10) and (11) whereas a quantum 2 bits memory can be in any normalised linear superposition of those same 4 states. On the other hand you can always read the state of a classical computer whereas for quantum computers, you have to choose an orthogonal base. In our example, once this choice is done you can have only 4 different results. As usual the complexity also gives more possibilities and quantum computers are supposed to be very efficient for some task. For instance Shor Algorithm[1] can find a number's prime factors in polynomial time whereas no polynomial time classical algorithm is known.

But from a physicist point of view, the main benefit of a quantum computer could be to simulate quantum systems. In fact classical computers are very inefficient in representing quantum systems. In classical physics when you study two separate vectorial spaces, the total space is described by a vectorial structure whose dimension is the sum of the dimensions of its components. In quantum physics if you want to describe a general state of the total system, you need to consider the tensorial product of the sub-spaces; the dimension is now the product of the dimensions of its components. Capacities of quantum computers grow with the number of bits as fast as the complexity of the problems with the number of particles. That's why quantum computers are expected to be more efficient than classical ones for simulating quantum physics.

To implement quantum computers, the main difficulty is to overcome decoherence. Photons can be good candidates for quantum information: the coherence time can be as long as few milliseconds (to be compared with nanoseconds in condensed matter). But two photons in vacuum don't interact together. As atoms can interact together and a photons be coupled with an atom, it is possible to make (indirectly) interactions between photons. That's what creates optical non-linearities in some crystals. But those non-linearities are usually very weak and appear at very high intensity; it is to say not at the level of the photon as needed for quantum information processing. Some research groups managed to amplify significantly the coupling between atoms and photos by using some high finesse cavity[2].

The experiment on which I worked shows that it is possible to have in free space (without optical cavity) non-linear effects at the level of photons. A cold atom gas is prepared so that the medium induces significant two-photon attenuation while it remains transparent for a single photon.

This is achieved by using three levels of the atoms: a fundamental (stable), an excited state (unstable) and a stable higher level (Rydberg state). A laser shining the cloud of cold atoms is tuned to drive the transition between the excited state and the Rydberg state. The probe laser is tuned to drive the transition between the fundamental to the excited state.

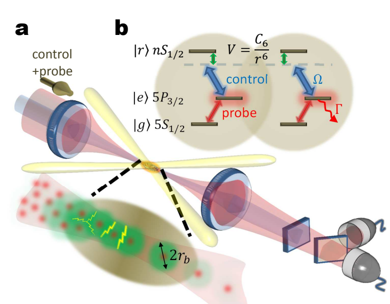

FIG. 1. Principle of the photon-photon interactions in the main experiment of the lab in which I worked

When a single photon comes from the probe laser, the control laser prevent the excited atom from spontaneous emission (in a random direction: the photon is lost if it happens) and the photon can travel through the medium as a coupled excitation of light and matter (EIT[3]). When two photons come together, the two Rydberg atoms interact (Rydberg atoms really look like a classical electric dipole and can interact at very long range). The interaction shifts the internal level so that the control laser is now detuned from the transition from the excited state to the Rydberg. At least one photon is scattered. This phenomenon is known in the literature as "Rydberg excitation blockade"[4].

In practical the lasers are not monochromatic and the difference of frequency between the two stable states is broadened by Doppler Effect. As the blockade happens when the Rydberg state frequency moves out of resonance with the laser, the broader the lasers spectra will be, the stronger the Rydberg atoms will need to interact before running out of the transparency window. That's why the atoms need to be cooled and the lasers want narrow spectra. Therefore the easier to improve interactions between photons is to identify the broadest spectrum. Once you have identified a guilty laser it is very interesting to know whether it will be useful to improve the electronic control of the laser or not. My work for the internship was to characterise the control and probe lasers.

II. MEASUREMENT PRINCIPLE

The most important characteristic of the light we want to measure is the linewidth – because it impacts directly the intensity of the photon-photon interaction. As we want a very narrow laser spectrum, the measurement will be difficult. Using a grating or a prism will obviously not have enough precision here. Other techniques are using a high finesse Fabry–Pérot interferometer or beating the light with a known laser and then measure the radio frequency signals. An alternative to all those sophisticated techniques starts from the idea that we want to measure frequency fluctuations.

The concept will be to convert the fluctuations of frequency into fluctuations of intensity with a Fabry-Pérot cavity used at half resonance and then convert those fluctuations of luminous power into fluctuations of voltage with a photo-diode. Those can then be analysed by a spectrum analyser. From the spectrum analyser, we should be able to recover the linewidth of the laser and even more: by knowing the frequencies of the frequency shifts, we should be able to know whether we will be able to cancel them with an electronic control of the laser (low frequency) or not (high frequency).

Before going further with the experimental part, let's define the words like "spectrum" that we use without having a good definition and find the link between what is going to be read on the spectrum analyser and the spectrum of the laser.

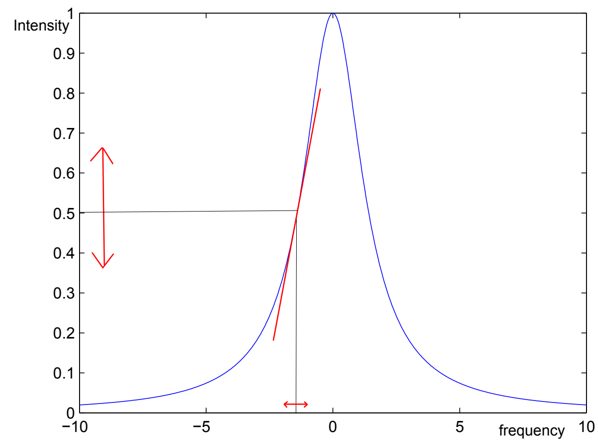

FIG. 2. Idea of the measurement: the laser is at half the resonance of the cavity so that the frequency fluctuations are converted into intensity fluctuation that we can detect with a photo-diode.

III. THEORY OF THE MEASUREMENT

A. Definitions

What is a spectrum? The intuitive answer would be: "The Fourier transform of what we study". But just talking about Fourier transform is not precise enough: There is a lot of different conventions, and depending on the properties of the function one studies, the Fourier transform will be different. That's why saying "the Fourier transform" is not sufficient. As you don't expect noise to be periodic, Fourier series will not work. Noise is not supposed to be square-integrable so standard Fourier transforms (with Lebesgue integrals and functions) will not work either. Hopefully a good normalization will gives intuitive results. Let's first define the kind of object we want to study: noises.

1. Noise

Intuitively noises are random functions of time. So we will study continuous (time variable) stationary stochastic processes. Let's write Ω a space of probability, T the time space ($\simeq \mathbb{R}$) and X a physical variable space ($\simeq \mathbb{R}$). The stochastic process N is a function from Ω to the set of the time dependent x:

$$\begin{aligned} N:\ \Omega &\longrightarrow (T \to X) \\ \omega &\longmapsto (t \mapsto x_\omega(t)) \end{aligned}$$

Remarks:

- At that point we don't add any other assumption.
- In the lab we have access only to one realization of

the random process. The ergodic hypothesis will have to be used (but we don't assume it now).

From now we use the signs <> for average over the probability space: $< x >= \int_\Omega x$

2. Fourier transform

This part is here only to give my conventions for Fourier transform. Given a function $t \mapsto x(t)$, if it exists I call Fourier transform of x the function

$$f \mapsto X(f) = \int_{-\infty}^{+\infty} x(t) e^{-2\pi i f t} dt$$

Remarks:

- With this convention f is a frequency.
- With this convention the invert Fourier transform is the same with just the sign in the exponential flipped: $f \mapsto x(t) = \int_{-\infty}^{+\infty} X(f) e^{2\pi i f t} dt$
- With this convention Parseval-Plancherel's equation is written: $\int_{-\infty}^{+\infty} x^2(t)dt = \int_{-\infty}^{+\infty} |X(f)|^2 df$

3. Spectral power density

Let's take one realization of the noise $x : t \mapsto x(t)$. "Usually" x is not suitable for a Fourier transformation (there no reason for it to be square-integrable). Given T a time (real number) we define x_T as the function:

$$x_T(t) = \begin{cases} x(t) & \text{for } |t| \le T \\ 0 & \text{for } |t| > T \end{cases}$$

For all T, x_T is Fourier transformable (non-zero only on a segment). Let's write $X_T(f)$ the Fourier transform of $x_T(t)$. At this point it's good to remember that $X_T(f)$ is a stochastic process. We can reasonably estimate that $X_T(f)$ scales with T at a speed of the order of $\sqrt{T}$. This estimation leads us to define:

When it exist, we call spectral power density:

$$S(f) = \lim_{T \to \infty} \frac{< |X_T(f)|^2 >}{2T} \qquad (1)$$

Remarks:

- We generally can't invert the limit and average because one realization of the random process is a common function and usually the limit does not converge.

4. Properties of the spectral power density

The properties of the standard Fourier transform have equivalents with the spectral power density. We keep the names used before ($x(t)$ the function and $S(f)$ the spectrum power density).

Parseval equation becomes:

$$< |x^2| >= \int_{-\infty}^{+\infty} S(f) df$$

Remark:

- We don't precise t because x is a stationary process.

Given a linear filter of transparency $h(f)$, the spectrum power density after the filter $S'(f)$ is:

$$S'(f) = S(f)|h(f)|^2$$

The Wiener-Khintchine theorem makes the link between the spectral power density and the autocorrelation function:

$$S(f) = \int_{-\infty}^{+\infty} < x(t)x(t+\tau) > e^{-2\pi i f \tau} d\tau$$

Now we know what a spectrum is and can start the theory of the measurement.

B. Recovering the laser spectrum from the frequency noise spectrum

The measurement I suggest is really useful only if we can recover at least some characteristics of the laser light. Let's consider a complex representation of the laser electric field (only the temporal part) which is almost a monochromatic field with the frequency ν_0:

$$E(t) = E_0 e^{2\pi i \int_0^t (\nu_0 + \Delta\nu(t))dt}$$

We call respectively $S_E(f)$ and $S_{\Delta\nu}(f)$ the spectrum power density of $E(t)$ and $\Delta\nu(t)$. The goal here is to express $S_E(f)$ with only $S_{\Delta\nu}(f)$ and ν_0.

We can start with Wiener-Khintchine's theorem for $E(t)$. It leads to:

$$S_E(f) = E_0^2 \int_{-\infty}^{+\infty} e^{2\pi i(\nu_0 - f)\tau} < e^{2\pi i \int_0^\tau \Delta\nu(t)dt} > d\tau$$

In this expression appears $< e^{2\pi i \int_0^\tau \Delta\nu(t)dt} >$. This can't be expressed easily from $S_{\Delta\nu}(f)$. At this point we introduce the strong hypothesis that for each τ, $\int_0^\tau \Delta\nu(t)dt$ (a random variable since E(t) is a random process) has a Gaussian probability law. This is a strong hypothesis but since we have no more information, this is still the most reasonable hypothesis. Thanks to this assumption, we can use the following property: $< e^{\pm i \int_0^\tau \Delta\nu(t)dt} >= e^{-\frac{1}{2}<\left[\int_0^\tau \Delta\nu(t)dt\right]^2>}$.

Once this approximation is done, we can compute:

$$S_E(f) = E_0^2 \int_{-\infty}^{+\infty} e^{2\pi i(\nu_0 - f)\tau} e^{-2\int_{-\infty}^{+\infty} S_{\Delta\nu}(g) \frac{\sin^2(\pi g \tau)}{g^2} dg} d\tau \quad (2)$$

This equation is exactly what we were looking for: a direct expression of the laser spectrum in function of the spectrum of frequency noises. Let's see what comes out for a specific case: when the spectrum of frequency power density is constant.

C. Case $S_{\Delta\nu}(f)$ constant

We write $h_0 = S_{\Delta\nu}(f)$ (independent from f). With this notation,

$$\int_{-\infty}^{+\infty} h_0 \frac{\sin^2(\pi g \tau)}{\pi^2 g^2} dg = h_0 |\tau|$$

This leads to

$$S_E(f) = E_0^2 \int_{-\infty}^{+\infty} e^{2\pi i(\nu_0 - f)\tau} e^{-2\pi^2 h_0 |\tau|} d\tau = \frac{4E_0^2 \frac{1}{h_0}}{1 + (\frac{f-\nu_0}{\frac{2\pi h_0}{2}})^2}$$

At the end of those computations, we have found that with a constant spectrum of frequency noise power h_0 the line-shape of the laser is a Lorentzian with a full width at half maximum of $2\pi h_0$. This π seems quite strange because we worked only with frequency, without taking the π out of the exponentials. It actually comes out from a Dirichlet integral ($\int_{-\infty}^{+\infty} \frac{\sin(\omega)}{\omega} d\omega = \pi$).

IV. EXPERIMENTAL

We now know that the idea of analysing the frequency noise allows to compute The spectrum of the laser. Furthermore as we can expect to be able to reduce low frequency noises with a feedback control, knowing the frequency noise power spectrum help estimating the improvement we can bring to the laser.

A. Set up

To perform the measurement, one wants to convert the frequency fluctuation into intensity fluctuation in a known way. To achieve this goal it is essential to keep the conversion factor from frequency to optical power almost constant. This happens only if we work in an area in which the slope of the transmission curve is almost constant. This implies that the linewidth of the laser should be smaller than the linewidth of the cavity. This is important for the choice of cavity: we have to make sure the linewidth is large enough. Another danger is the slow drift. A solution to solve it is to lock the piezoelectric crystal within the tunable cavity to a given voltage on the photo-diode.

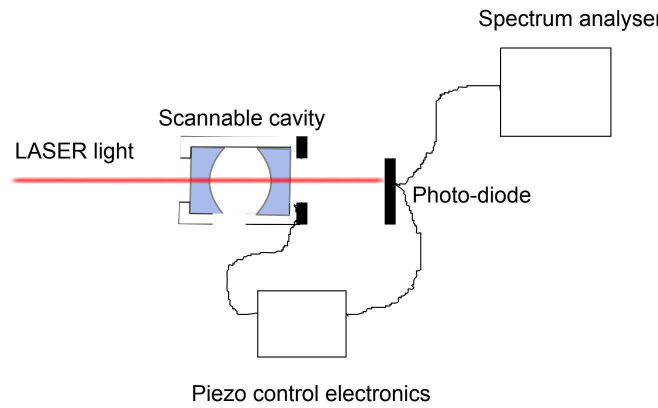

FIG. 3. Experimental set-up for the characterisation of the laser. the cavity is locked at half resonance.

The definitive set-up was the following: The laser light is taken from where we can access it to an optical fibre (with a half-wave plate and a beam polarisation splitter to be able to reduce the power).

Then it is directed (after a mirror) to a tunable Fabry-Pérot cavity. Ideally the linewidth of the cavity is between 10 times and 100 times the linewidth of the laser. Indeed the narrower the cavity, the higher the frequency intensity conversion coefficient will be. But we also want this coefficient to be almost constant: the laser linewidth have to be small in comparison with the cavity linewidth.

At the output of the cavity the light is converted into voltage by a fast avalanche photo-diode.

The signal is then split in two: some goes to a lock-box to slowly lock the cavity at half resonance of the laser. Most of the signal goes to a spectrum analyser.

Of course all the optics elements need to be precisely aligned.

B. Locking the cavity

After some unfortunate attempts to use an old lock-box built some decades ago by an unknown undergraduate that leads me only into some smoke, and after some sophisticated attempts to lock the laser at the cavity, I decided to build my own lock-box.

I started from the model used in the lab and modified it a little. Basically it takes the signal from the photo-diode, add a tunable offset, integer it and add a new offset. There is also the possibility to switch to a signal produced by an external function generator. The switch is designed so that when you switch from the external source to the locked position, the output voltage stay continuous. If there is any voltage discontinuity when you try to lock the cavity, it will fail. For instance the (bad quality) function generator I used had an offset (even when it was supposed to be 0) and the locking process failed. I had to add a cut-low filter before my lock-box. Once this problem solved, the lock was quite efficient.

C. Calibrations

In the theoretical part, we have seen how to compute the spectrum of a laser from the spectral frequency noise power density. But the spectrum analyser measures an electrical power. To make the conversion, one needs to know the slope of the transmission curve at half resonance (where the cavity is locked). In order to have an absolute value of this slope, two quantities have to be determined experimentally.

The first one is the voltage emitted by the photo-diode when the transmission of the cavity is maximum. As the input impedance of the spectrum analyser depends on the frequency (DC block), we can't just measure the value when we are at half resonance and multiply by 2. The easiest way I've found is to scan the cavity and look at the photo-diode output voltage with an oscilloscope equipped with a 50 Ω input.

The second one is the linewidth of the cavity. Due to the general set-up of the lab, it is possible to shift the lasers by an arbitrary frequency. A side-band (created by modulation) is locked so as long as the lock follows, changing the modulation frequency changes the frequency of the laser. This is controlled by a computer. With a good function generator I was able to send ramps to the piezo of the cavity when ordered by the computer. With this sophisticated material it was possible to send ramps at the piezo so that at each one the frequency of the laser has been shifted (one time up, one time down). By averaging the photo-diode signal on an oscilloscope and recording it on a computer, I was able to extract the linewidth of the cavity by using the absolute frequency value given by the shift between the maxima (shift in the modulation frequency).

With the exact value of the coefficient to convert frequency into voltage fluctuations, we are ready to analyse the data from the spectrum analyser.

D. Analysis

We have seen that we can reconstruct the spectrum of a laser from the analysis of the measurement data. As we hope to be able to make low-frequency noise decreasing significantly with a feedback control, looking at high frequency noises gives us an indication of the best spectrum we will be able to obtain with an electronic control that can reduce low frequency noises.

Indeed when we look at laser spectra, we notice that they seem to finish at a flat stage. We have seen that the corresponding spectrum is a pure Lorentzian. So it behaves like if the lasers had a Lorentzian spectrum polluted at low frequency by "technical" noises.

To implement numerically the formula 2 we have infinite integrals to evaluate whereas we can only access experimentally to a finite window. A solution is to assume $S_{\Delta\nu}$ to be constant outside of the experimental window.

With this hypothesis we can use the linearity of the integral within the exponential to have two terms: one have non zero values only in the experimental window (and a numerical integration is possible without special problem) and the other one correspond to a Lorentzian whose spectrum characteristics (the linewidth) can be determined by just knowing the level of the stage at one point.

V. CONCLUSION

As we have seen, laser quality is essential in atomic physics to reach high levels of precision. The presented technique and theories gives an opportunity to evaluate the characteristics of lasers.

In this study what I call "intrinsic Lorentzian spectrum" seems to be the best quality we can reach with a given laser. In fact it exist techniques to lower the linewidth of this Lorentzian. One possible technique is to use a long external cavity[5]. Perhaps with this technique it will be possible to improve the lasers of the main experiment enough to create a very efficient optical quantum transistor.

Appendix A: figures

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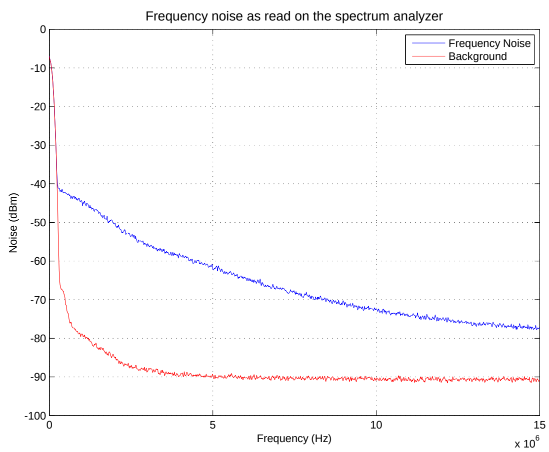

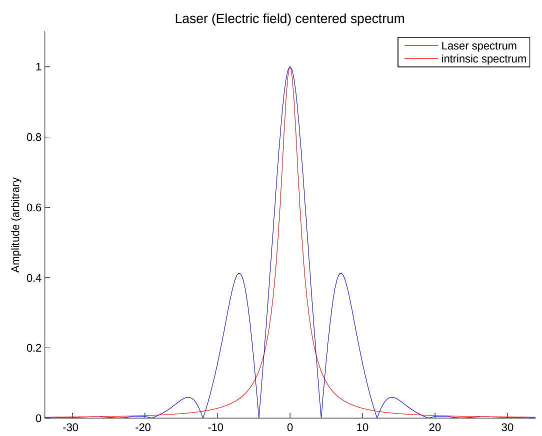

FIG. 4. Experimental measurement on the spectrum analyser and laser spectrum after processing