Agencia Nacional de Investigación y Desarrollo
Ministerio de Ciencia, Tecnología, Conocimiento e Innovación
Gobierno de Chile

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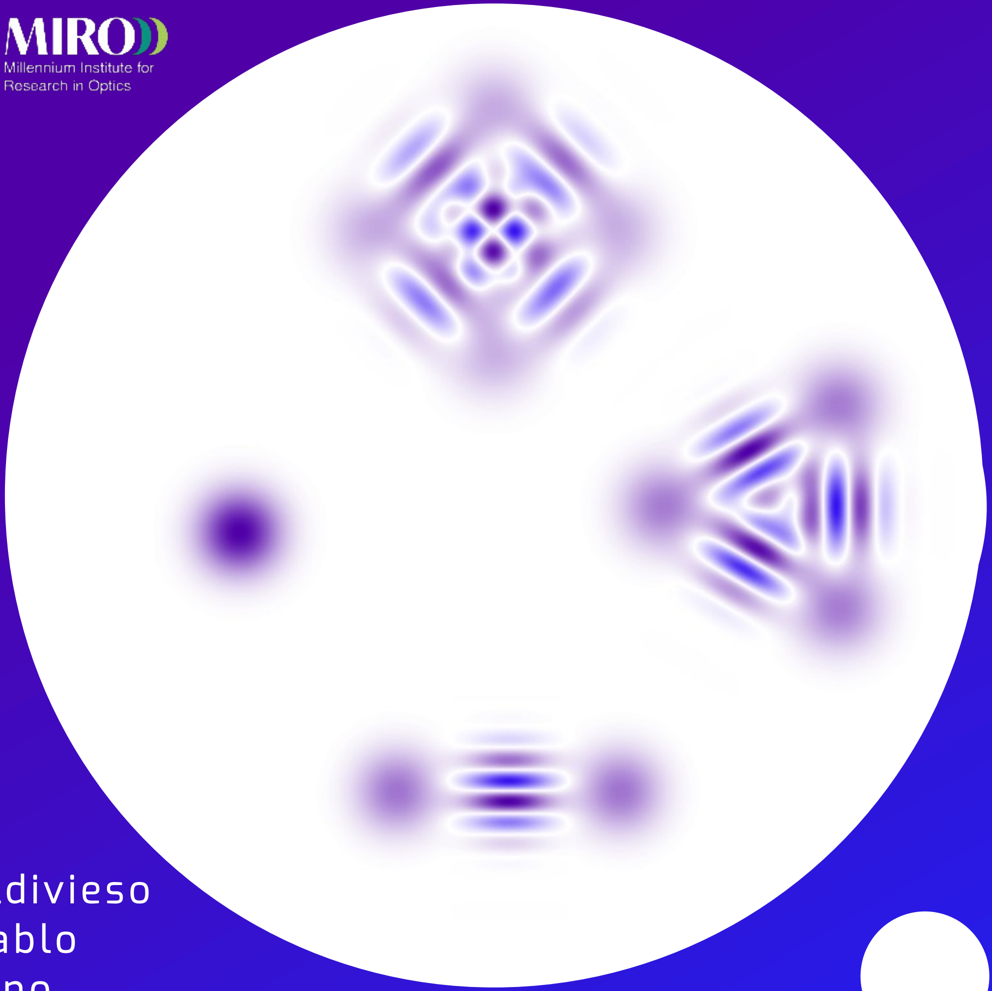
AMAZING QUANTUM

MIRO
Millennium Institute for Research in Optics

Characterizing the quantumness of Generalized Coherent States with intensity-field correlation



Slides

Victor Gondret, Ignacio Salinas Valdivieso
Gerd Hartmann S., Mariano Uria, Pablo
Solano, and Carla Hermann-Avigliano

OUTLINE

④ SO DOES IT WORK?

② WHAT IS THAT?

Characterizing the quantumness of Generalized Coherent States with intensity-field correlation

③ WHAT DOES THIS MEAN?

① OK... BUT WHY?

I. Context

II. Generalized Coherent States

III. Non-classicality of a field with $G^{(3/2)}$

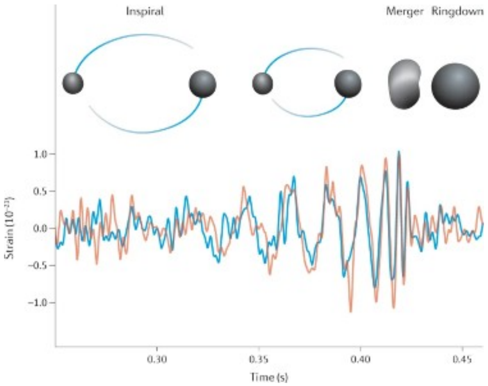
IV. Results

SQUEEZING TO ENHANCE PRECISION IN METROLOGY

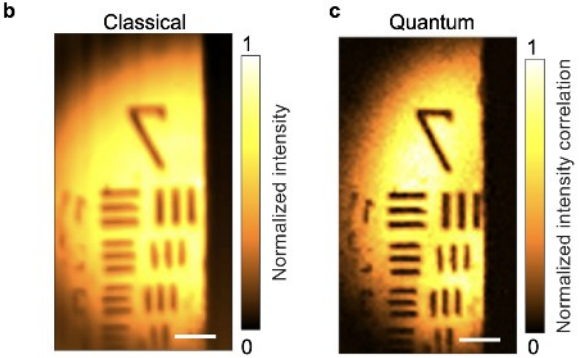
WHY?

Light can be squeezed to improve sensitivity to a measurement

Gravitational wave detection



Improved microscopy

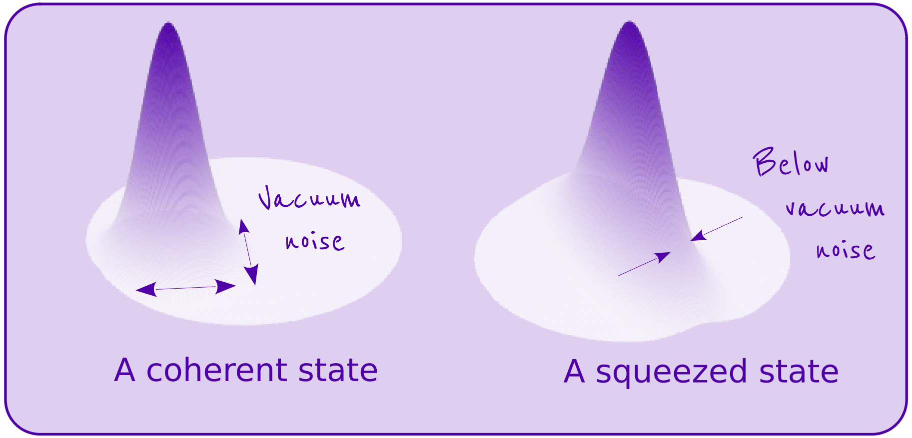


HOW MUCH?

In an ideal experiment, only remains noise due to vacuum fluctuations. Coherent states set the **standard quantum limit** in metrology

$$\Delta\phi_{SQL} \sim \frac{1}{\sqrt{N}}$$ (Or shot noise)

Wigner description of a quantum state



The LIGO Sci. Coll. *et al.* Nature Phys **7**, 962–965 (2011).
Z. He *et al.* Nat Commun **14**, 2441 (2023).
M. Bailes *et al.* Nat Rev Phys **3**, 344–366 (2021).

NON-GAUSSIAN STATES TO FURTHER ENHANCE PRECISION

Standard Quantum Limit

$$\Delta\phi_{\mathrm{SQL}} \sim \frac{1}{\sqrt{N}}$$

Gaussian squeezed states

Some other non-Gaussian states

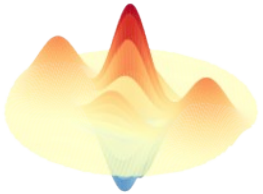
Heisenberg Limit

$$\Delta\phi_{\mathrm{Heis}} \sim \frac{1}{N}$$

Negative Quasiprobabilities Enhance Phase Estimation in Quantum-Optics Experiment [1]

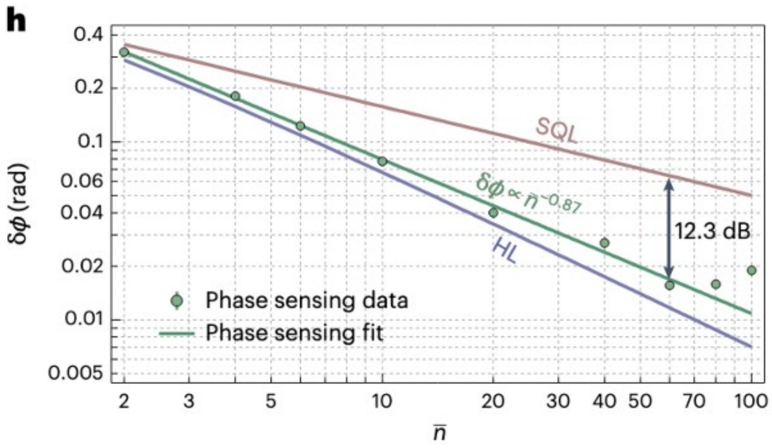
Pure quantum states which are not Gaussian have a non-positive Wigner function

Schrödinger cat states



A superposition of coherent states allows displacement measurements at the Heisenberg limit [3]

Fock states for metrology



Quantum metrology with large Fock states [2]

[1] N. Lupu-Gladstein *et al*. Phys. Rev. Lett. **128**, 220504 (2022)
[2] X. Deng *et al.* Nat. Phys. **20**, 1874–1880 (2024).
[3] A. Gilchrist *et al.* J. Opt. B: Quantum Semiclass. Opt. **6**, S828–S833 (2004).

NON-GAUSSIAN STATES FOR QUANTUM COMPUTATION

Metrology

Standard Quantum Limit

$$\Delta\phi_{\mathrm{SQL}} \sim \frac{1}{\sqrt{N}}$$

Gaussian states

Some other non-Gaussian states

Heisenberg Limit

$$\Delta\phi_{\mathrm{Heis}} \sim \frac{1}{N}$$

Computation

GKP states for error correction

Gottesman, Kitaev & Preskill, Phys. Rev. A **64**, 012310 (2001).

The negativity of the Wigner function is a requirement for building a universal quantum computer with continuous variables [1,2]

(negativity of the state or the detector)

Generation and characterization of non-Gaussian quantum states is an active field of research

[1] S. Lloyd and S. L. Braunstein, Phys. Rev. Lett. **82**, 1784 (1999).
[2] Walschaers, PRX Quantum **2**, 030204 (2021)

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HOW TO GENERATE NON-GAUSSIAN STATES?

Non-deterministic approach

Herald a photon in one mode of an entangled state.

Start from a Gaussian state

Welcome to the non-Gaussian world!

Make it evolve under a higher than quadratic Hamiltonian

Deterministic approach

A. I. Lvovsky et al. arXiv.2006.16985 (2020)

I. Context

II. Generalized Coherent States

III. Non-classicality of a field with $G^{(3/2)}$

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AMAZING QUANTUM

A GENERAL MODEL FOR NONLINEAR INTERACTIONS

Generic Hamiltonian nonlinear in the photon number operator

Atomic operator

$|\alpha\rangle$

$$\hat{H}_{int} = \hbar g \hat{n}^{\epsilon} \hat{O}_a$$

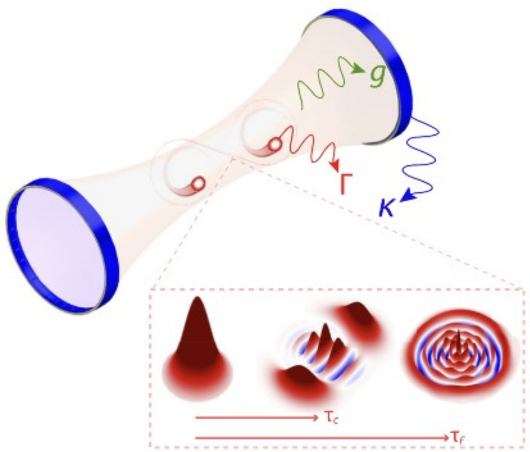
$|\alpha_{\varepsilon,t}\rangle$

Nonlinear parameter ε

τ: effective interaction time

Dimensionless time $t = g\tau$

$\epsilon=1/2$



Includes the Jaynes-Cummings Hamiltonian in the resonant intense field limit.

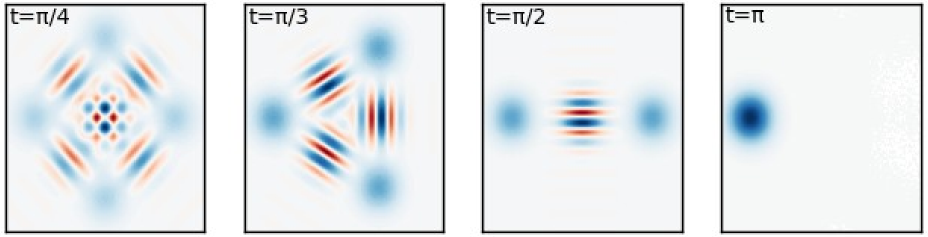
Deterministic generation of large (displaced) Fock states.

Up to 100 photons Fock state with current capability!

$\epsilon=2$

Kerr Hamiltonian

Deterministic generation of large superpositions of coherent states.



M. Uria *et al*. Phys. Rev. Research **5**, 013165 (2023).
M. Uria *et al*. Phys. Rev. Lett. **125**, 093603 (2020).

GENERALIZED COHERENT STATES

Generic Hamiltonian nonlinear in the photon number operator

Atomic operator

$|\alpha\rangle$

$$\hat{H}_{int} = \hbar g \hat{n}^{\epsilon} \hat{O}_a \longrightarrow |\alpha_{\varepsilon,t}\rangle$$

Nonlinear parameter ε

τ: effective interaction time

Dimensionless time $t = g\tau$

$|\alpha_{\varepsilon,t}\rangle$ is a Generalized Coherent State (GCS)

$$|\alpha_{\varepsilon,t}\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} e^{-itn^{\varepsilon}} |n\rangle_F$$

Nonlinear parameter

Effective interaction time

Fock basis

Properties

Coherent in the sense of Glauber [1]

(see their poissonian statistics)

$$\forall i,\ g^{(i)} = \frac{\langle (\hat{a}^{\dagger})^i \hat{a}^i \rangle}{\langle \hat{a}^{\dagger}\hat{a}\rangle^i} = 1$$

They can have a large negativity in their Wigner function [2]

Provide a large metrological quantum advantage [2]

[1] D. Stoler. Phys. Rev. D **4**, 2309–2312 (1971).
[2] M. Uria *et al*. Phys. Rev. Research **5**, 013165 (2023).

PROBING THE QUANTUMNESS OF GCSS

$|\alpha_{\varepsilon,t}\rangle$ is a Generalized Coherent State (GCS)

$$|\alpha_{\varepsilon,t}\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} e^{-itn^{\varepsilon}} |n\rangle$$

Nonlinear parameter

Effective interaction time

Properties

Coherent in the sense of Glauber

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They can have a large negativity in their Wigner function

Provide a large metrological quantum advantage

HOW TO PROBE EXPERIMENTALLY THE QUANTUMNESS OF THESE STATES?

Quantum state tomography

Too long! Can we find other ways??

→ Use (low order) correlation functions

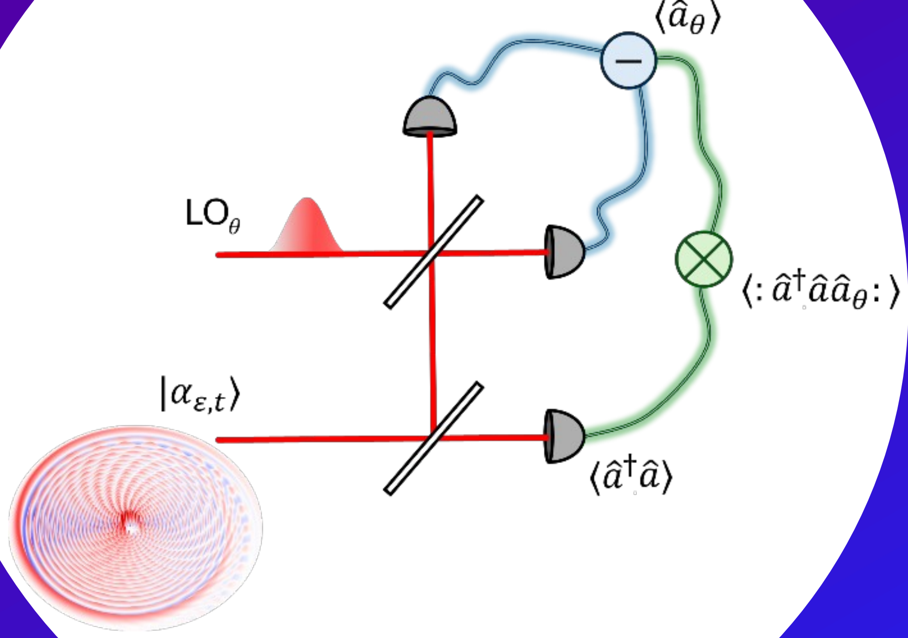
Intensity correlation functions are useless.

$$\langle (\hat{a}^{\dagger})^k \hat{a}^k \rangle = \langle \hat{a}^{\dagger}\hat{a}\rangle^k$$

We must consider correlation function with a different number of â and â†.

$$\langle \hat{a}^{\dagger}\hat{a}^2 \rangle$$

AMAZING QUANTUM



THE INTENSITY-FIELD CORRELATION FUNCTION

The intensity field correlation function

$$g_\theta^{(3/2)} = \frac{\langle : \hat{n}\hat{a}_\theta : \rangle}{\langle \hat{n} \rangle \langle \hat{a}_\theta \rangle}$$

$$\hat{a}_\theta = \hat{a}e^{-i\theta} + \hat{a}^\dagger e^{i\theta}$$

Carmichael *et al.* have shown that any classical Gaussian state must satisfy the inequality*

$$0 \leq |g^{(3/2)}_{\theta=\arg\langle\hat{a}\rangle} - 1| \leq \frac{2}{1 + |\hat{a}|^2/\langle\Delta\hat{a}^\dagger\Delta\hat{a}\rangle}$$

$$\Delta\hat{a} = \hat{a} - \langle\hat{a}\rangle$$

*under the LO phase conditions θ=arg(<â>).

Only applies for Gaussian states...

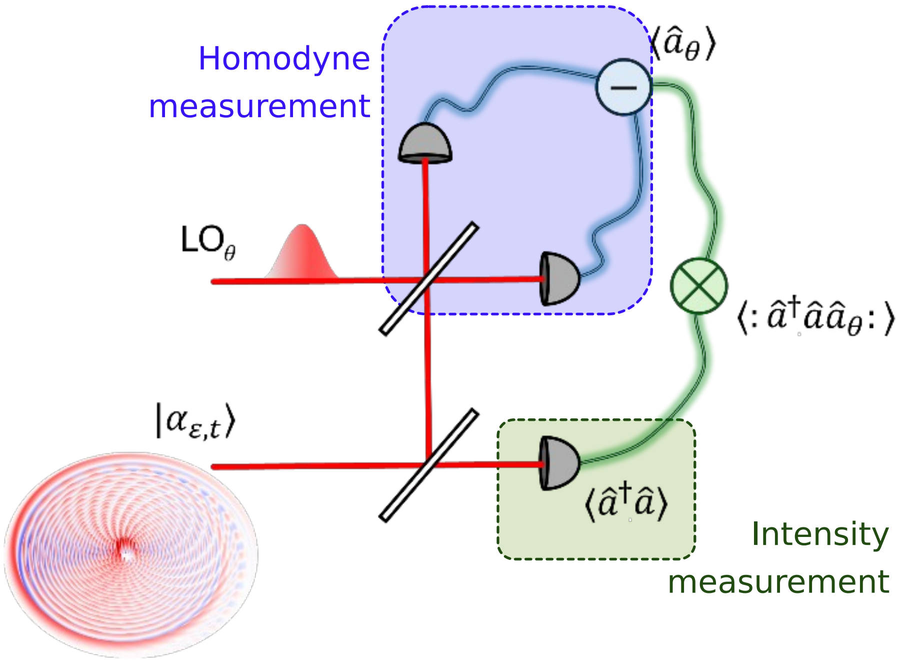


Fig: Measurement principle.

H. J. Carmichael *et al.* Phys. Rev. Lett. **85**, 1855–1858 (2000).
H. J. Carmichael *et al.* in Progress in Optics **46** 355–404 (2004).

WHAT MEANS NON-CLASSICAL?

In [1], non-classicality relies on

$$\langle : \hat{O}^2 : \rangle - \langle \hat{O} \rangle^2 \geq 0$$

for any classical state.

Normal ordering does not matter for classical fields

Ex: antibunching is a non-classical signature: $\langle : \hat{n}^2 : \rangle - \langle \hat{n} \rangle^2 < 0$

Non-classicality definition

The state of an electromagnetic field is nonclassical if:

a) $\langle \hat{n} \rangle < 1$

b) The P-distribution $\hat{\rho} = \int_{\mathbb{C}} d^2\alpha\, P(\alpha) |\alpha\rangle\langle\alpha|$

is not a probability density i.e. it is more singular than a delta function.

L. Mandel. Phys. Scr. **T12**, 34–42 (1986).

Glauber-Sudarshan P-function

The Wigner function is a "smoothed" P–function

$$W(\alpha) = \frac{2}{\pi} \int_{\mathbb{C}} d^2\beta \, \exp\left(-2|\alpha - \beta|^2\right) P(\beta).$$

We cannot measure in general the P–function.

The P–function is directly related to normal ordered correlation functions

$$\langle (\hat{a}^\dagger)^m \hat{a}^n \rangle = \int_{\mathbb{C}} d^2\alpha \, P(\alpha) \, (\alpha^*)^m \, \alpha^n.$$

P(α) is a probability distribution $\xrightarrow{[4]}$

$$D_\theta = \begin{vmatrix} 1 & \langle \hat{a}_\theta \rangle & \langle \hat{n} \rangle \\ \langle \hat{a}_\theta \rangle & \langle : \hat{a}_\theta^2 : \rangle & \langle : \hat{a}_\theta \hat{n} : \rangle \\ \langle \hat{n} \rangle & \langle : \hat{a}_\theta \hat{n} : \rangle & \langle : \hat{n}^2 : \rangle \end{vmatrix} \geq 0$$

[1] H. J. Carmichael *et al.* Phys. Rev. Lett. **85**, 1855–1858 (2000).
[2] R. J. Glauber. Phys. Rev. **131**, 2766–2788 (1963).
[3] E. C. G. Sudarshan. Phys. Rev. Lett. **10**, 277–279 (1963).
[4] E. V. Shchukin & W. Vogel. Phys. Rev. A 72, 043808 (2005).

NON CLASSICALITY BASED ON THE MOMENTS

P(α) is a probability distribution

$$D_\theta = \begin{vmatrix} 1 & \langle \hat{a}_\theta \rangle & \langle \hat{n} \rangle \\ \langle \hat{a}_\theta \rangle & \langle : \hat{a}_\theta^2 : \rangle & \langle : \hat{a}_\theta \hat{n} : \rangle \\ \langle \hat{n} \rangle & \langle : \hat{a}_\theta \hat{n} : \rangle & \langle : \hat{n}^2 : \rangle \end{vmatrix} \geq 0$$

If $$D_\theta = \boxed{\left(\langle : \hat{a}_\theta^2 : \rangle - \langle \hat{a}_\theta \rangle^2 \right)} \times \boxed{\left(\langle : \hat{n}^2 : \rangle - \langle \hat{n} \rangle^2 \right)} - \boxed{\langle \hat{a}_\theta \rangle^2 \langle \hat{n} \rangle^2 \left(g_\theta^{(3/2)} - 1 \right)^2} < 0$$

The state is non-classical.

A **x** **B** **–** **C²**

This can be negative or positive

This is negative or null

NON CLASSICALITY FOR GCSs

P(α) is a probability distribution → $D_\theta = \begin{vmatrix} 1 & \langle \hat{a}_\theta \rangle & \langle \hat{n} \rangle \\ \langle \hat{a}_\theta \rangle & \langle : \hat{a}_\theta^2 : \rangle & \langle : \hat{a}_\theta \hat{n} : \rangle \\ \langle \hat{n} \rangle & \langle : \hat{a}_\theta \hat{n} : \rangle & \langle : \hat{n}^2 : \rangle \end{vmatrix} \geq 0$

If $D_\theta = \cancel{\left(\langle : \hat{a}_\theta^2 : \rangle - \langle \hat{a}_\theta \rangle^2 \right) \left(\langle : \hat{n}^2 : \rangle - \langle \hat{n} \rangle^2 \right)} - \langle \hat{a}_\theta \rangle^2 \langle \hat{n} \rangle^2 \left(g_\theta^{(3/2)} - 1 \right)^2 < 0$ → The state is non-classical.

GCSs are Glauber coherent $\langle : \hat{n}^2 : \rangle = \langle \hat{n} \rangle^2$

For GCSs, if $g_\theta^{(3/2)} = \dfrac{\langle : \hat{n} \hat{a}_\theta : \rangle}{\langle \hat{n} \rangle \langle \hat{a}_\theta \rangle} \neq 1$, the state is nonclassical.

- Easy to measure
- No data analysis
- Real-time measurement

I. Context

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INTENSITY-FIELD CORRELATION FUNCTION WITNESS

$$g_\theta^{(3/2)} = \frac{\langle : \hat{n}\hat{a}_\theta : \rangle}{\langle \hat{n} \rangle \langle \hat{a}_\theta \rangle} \neq 1 \;\Rightarrow\; \text{nonclassicality}$$

Double check

4 parameters : $|\alpha_{\varepsilon,t}\rangle$

n population,

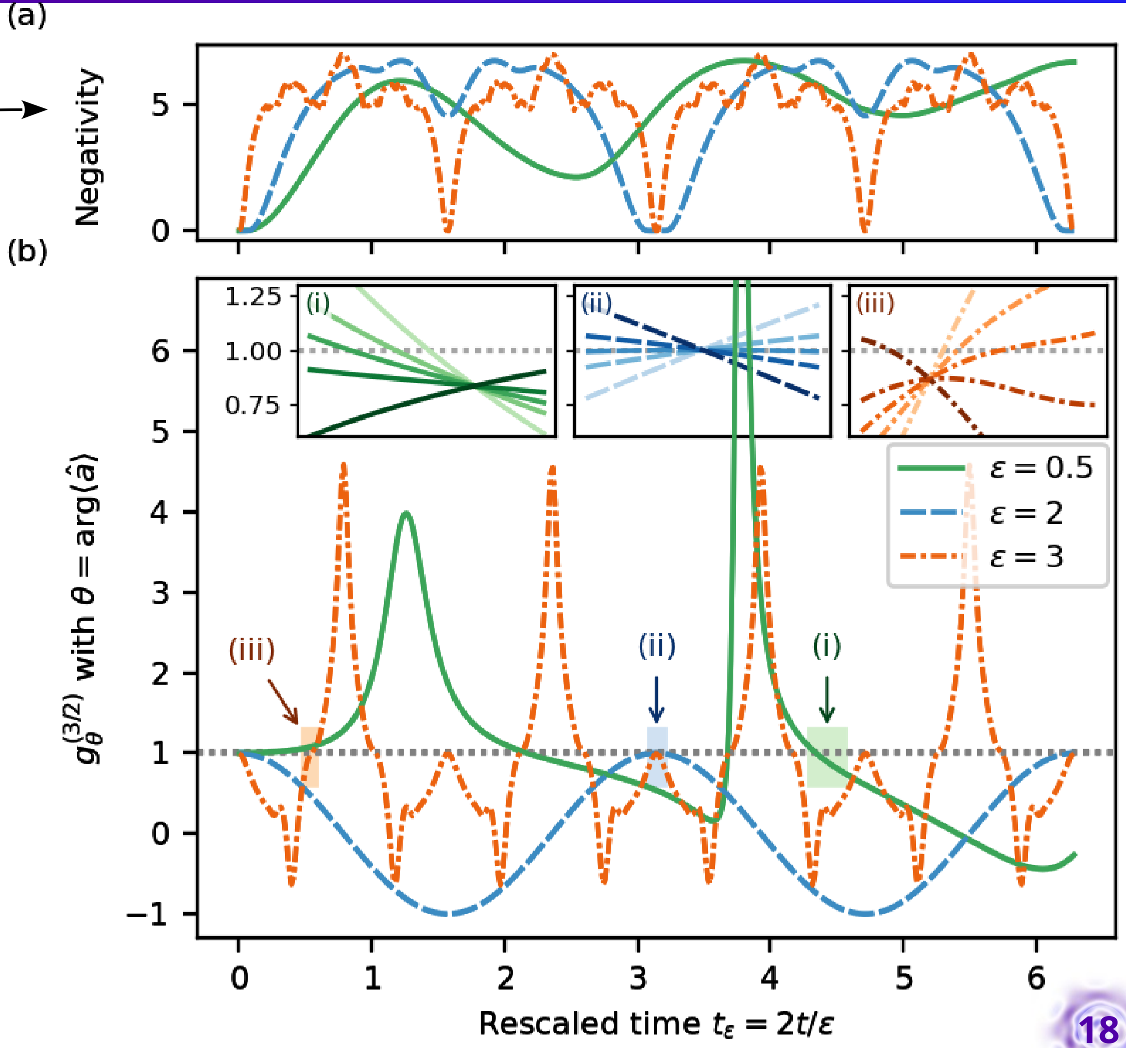
ϵ nonlinear parameter,

t time,

θ local oscillator phase

$\theta = \arg(\langle \hat{a} \rangle)$

The intensity-field function captures well the nonclassicality of the state



[1] Salinas *et al.* Characterization of Generalized Coherent States through Intensity-Field Correlation, ArXiv, 2512.15655 (2025)

A SIMPLE ANALYTIC FORMULA FOR KERR STATES

$$g_{\theta}^{(3/2)} = \frac{\langle : \hat{n}\hat{a}_\theta : \rangle}{\langle \hat{n} \rangle \langle \hat{a}_\theta \rangle} \neq 1 \;\Rightarrow\; \text{nonclassicality}$$

4 parameters : $|\alpha_{\varepsilon,t}\rangle$

n population,

ϵ nonlinear parameter,

t time,

θ local oscillator phase

$\theta = \arg(\langle \hat{a} \rangle)$

For Kerr states:

$$g^{(3/2)}_{\theta=\arg\langle \hat{a}\rangle} = \cos 2t$$

It does not depend on n !!

Very different for $g^{(2)}$ of Fock states

$$g^{(2)}_{|n\rangle} = 1 - 1/n$$

but its “measurement” does...

$$\langle \hat{a}_\theta \rangle \propto \sqrt{n} e^{-n(1-\cos(2t))}$$

$$\langle : \hat{n}\hat{a}_\theta : \rangle \propto n^{3/2} e^{-n(1-\cos(2t))}$$

Define a connected correlation function

$$G^{(3/2)}_{c,\,\theta} = \langle : \hat{n}\hat{a}_\theta : \rangle - \langle \hat{n} \rangle \langle \hat{a}_\theta \rangle .$$

$G^{(3/2)}_{c,\,\theta} \neq 0 \Rightarrow$ non-classicality

PROPERTIES OF THE WITNESS

Define a connected correlation function

$$G^{(3/2)}_{c,\theta} = \langle : \hat{n}\hat{a}_\theta : \rangle - \langle \hat{n} \rangle \langle \hat{a}_\theta \rangle .$$

$G^{(3/2)}_{c,\theta} \neq 0 \Rightarrow$ non-classicality

$\epsilon=1/2$

$\epsilon=2$

Hard to detect non-classicality here...

$\epsilon=3$

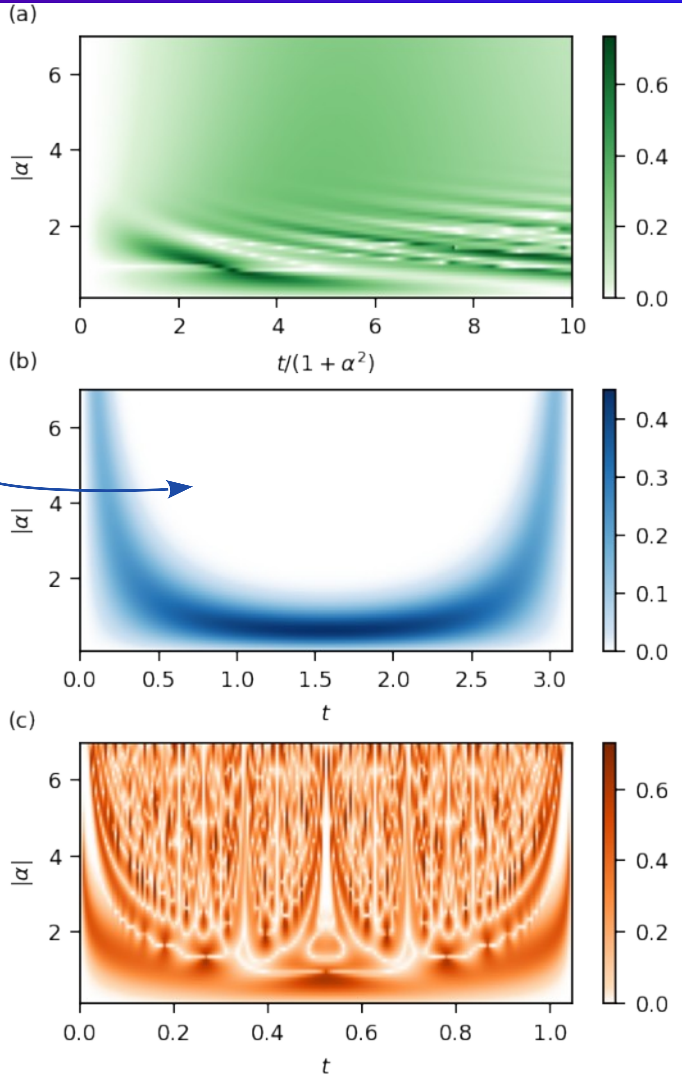


Fig: value of $|G^{(3/2)}_{c,\theta}/\alpha^{3/2}|$

[1] Salinas *et al.* Characterization of Generalized Coherent States through Intensity-Field Correlation, ArXiv, 2512.15655 (2025)
[2] M. Uria *et al.* Phys. Rev. Lett. **125**, 093603 (2020).

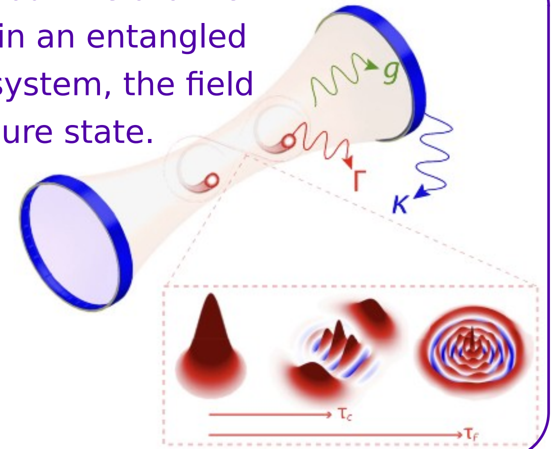
MIXED STATES

$$g_\theta^{(3/2)} = \frac{\langle : \hat{n}\hat{a}_\theta : \rangle}{\langle \hat{n} \rangle \langle \hat{a}_\theta \rangle}$$

$$G_{c,\theta}^{(3/2)} = \langle : \hat{n}\hat{a}_\theta : \rangle - \langle \hat{n} \rangle \langle \hat{a}_\theta \rangle .$$

What About Mixed States?

If you trace out the atomic subsystem in an entangled field-atom system, the field is not in a pure state.



$$\hat{\rho} = \sum_i p_i |\alpha_{i,\varepsilon_i,t_i}\rangle\langle\alpha_{i,\varepsilon_i,t_i}|$$

positive

If $|\alpha_i|=|\alpha_j|$ $\forall$ i,j, the state remains Glauber coherent and $G_\theta^{(3/2)}$ is still a nonclassical witness.

In a general scenario, we expect such a mixture [3].

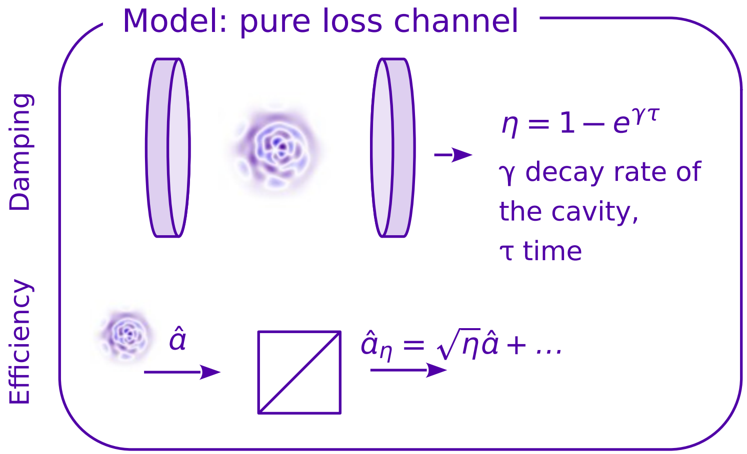
[1] Salinas *et al.* Characterization of Generalized Coherent States through the Intensity-Field Correlation Function, Submitted?
[2] M. Uria *et al.* Phys. Rev. Lett. **125**, 093603 (2020).
[3] M. Uria *et al.* Phys. Rev. Research **5**, 013165 (2023).

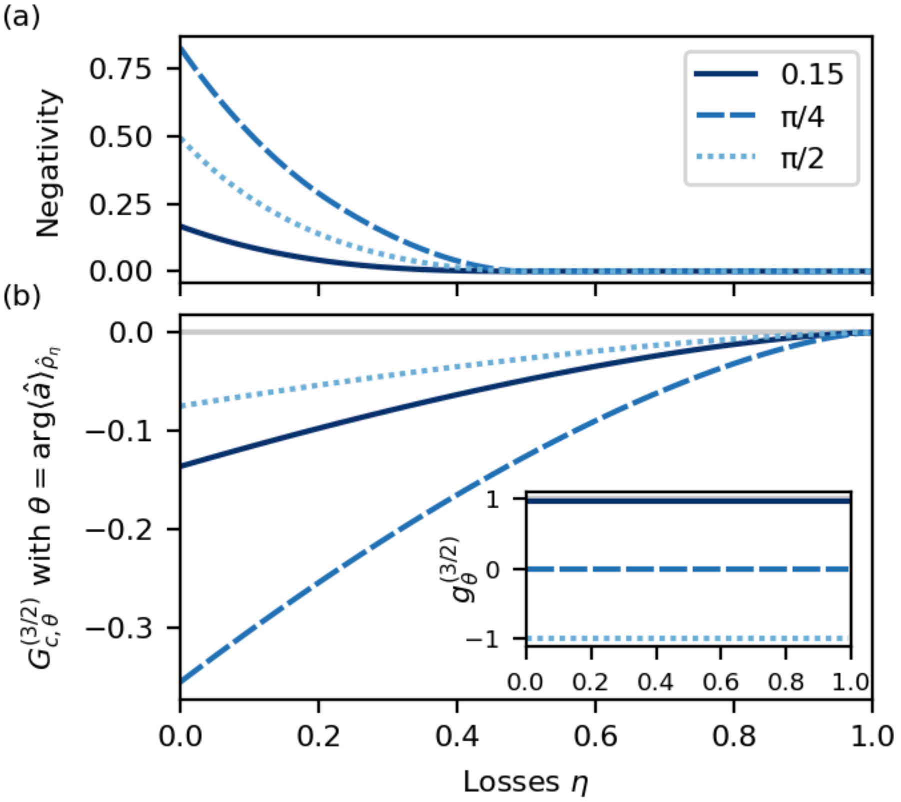
EFFECT OF LOSSES

$$g_\theta^{(3/2)} = \frac{\langle : \hat{n}\hat{a}_\theta : \rangle}{\langle \hat{n} \rangle \langle \hat{a}_\theta \rangle}$$

$$G_{c,\theta}^{(3/2)} = \langle : \hat{n}\hat{a}_\theta : \rangle - \langle \hat{n} \rangle \langle \hat{a}_\theta \rangle.$$

- Pure loss channel: Glauber coherence is preserved.
- Normalized correlation function: does not depend on the detection efficiency.





[1] Salinas *et al.* Characterization of Generalized Coherent States through the Intensity-Field Correlation Function, Submitted?

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¡Thank you!

- Non-Gaussian states are bakan in quantum optics
- Generalized Coherent States arise in many nonlinear situations
- We can probe their quantumness using the intensity field correlation function

Our paper available since this morning!!


Carla Hermann-Avigliano


Pablo Solano


Mariano Uria

Ignacio Salinas Valdivieso


Victor Gondret


Gerd Hartmann S.

Fondecyt grant no 123089

NON CLASSICALITY BASED ON THE MOMENTS

P(α) is a probability distribution ⟶

$$D_\theta = \begin{vmatrix} 1 & \langle \hat{a}_\theta \rangle & \langle \hat{n} \rangle \\ \langle \hat{a}_\theta \rangle & \langle : \hat{a}_\theta^2 : \rangle & \langle : \hat{a}_\theta \hat{n} : \rangle \\ \langle \hat{n} \rangle & \langle : \hat{a}_\theta \hat{n} : \rangle & \langle : \hat{n}^2 : \rangle \end{vmatrix} \geq 0$$

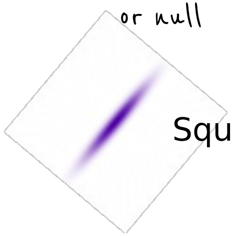
If $D_\theta = \boxed{\left(\langle : \hat{a}_\theta^2 : \rangle - \langle \hat{a}_\theta \rangle^2\right)} \times \boxed{\left(\langle : \hat{n}^2 : \rangle - \langle \hat{n} \rangle^2\right)} - \boxed{\langle \hat{a}_\theta \rangle^2 \langle \hat{n} \rangle^2 \left(g_\theta^{(3/2)} - 1\right)^2} < 0$ ⟶ The state is non-classical.

A x **B** – **C²**

This can be negative or positive

This is negative or null

Coherent state: $D_\theta = 0$



Superbunching

Squeezed state: $D_\theta = \boxed{(-)} \times \boxed{(+)} - \boxed{(+)} < 0$

Depends on θ

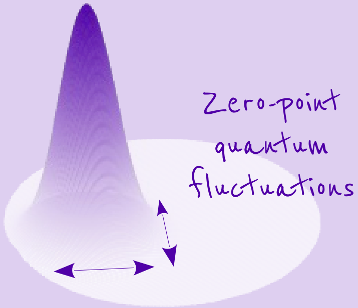
antibunching

Fock state: $D_\theta = \boxed{(+)} \times \boxed{(-)} - \boxed{(+)} < 0$

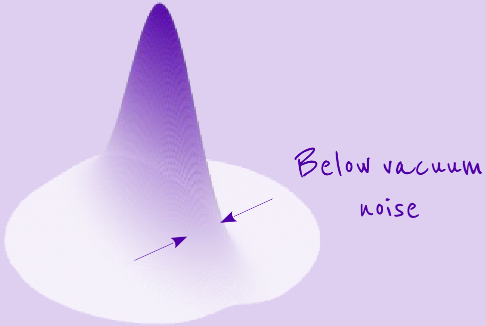
GCS state?????

SQUEEZING TO ENHANCE PRECISION IN METROLOGY

Wigner description of a quantum state



A coherent state in phase space



A coherent state in phase space

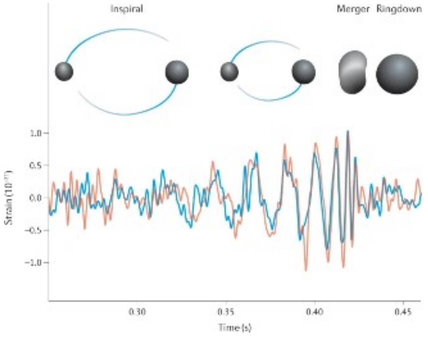
Zero-point vacuum fluctuations (thus coherent states) set the **standard quantum limit** in metrology

$$\Delta\phi_{SQL} \sim \frac{1}{\sqrt{N}}$$ (Or shot noise)

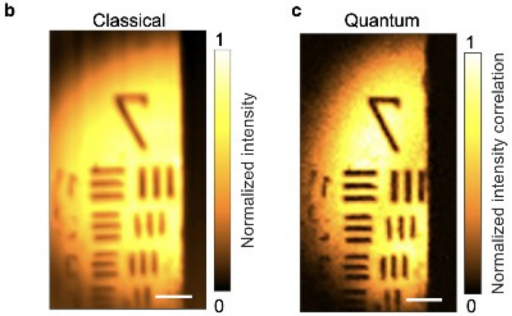
But you can beat the standard quantum limit (in one direction) by squeezing one quadrature.

APPLICATIONS

Gravitational wave detection



Improved microscopy



The LIGO Sci. Coll. *et al.* Nature Phys **7**, 962–965 (2011).
Z. He *et al.* Nat Commun **14**, 2441 (2023).
M. Bailes *et al.* Nat Rev Phys **3**, 344–366 (2021).