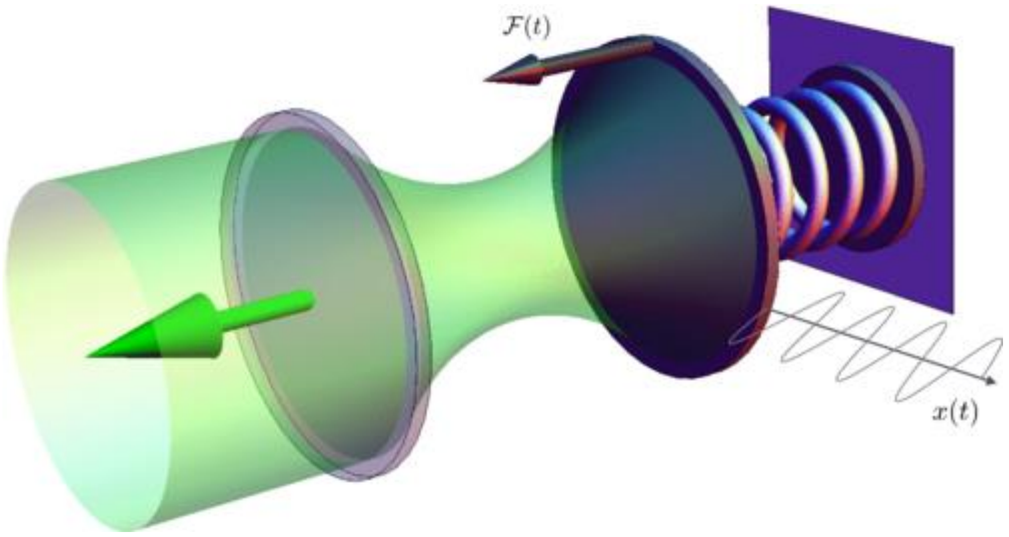
Non-separability of phonon pairs in a time modulated Bose-Einstein Condensate

Acoustic Analog of the Dynamical Casimir Effect

Victor Gondret, Charlie Leprince, Quentin Marolleau, Clothilde Lamirault, Denis Boiron & Chris Westbrook

The Dynamical Casimir Effect (DCE)



Macrì, V. *et al*. PRX (2018).

Dynamical Casimir effect : fast oscillation of the mirror creates photons out of vacuum.

$$N_{photons} \sim \omega\tau \left(\frac{v}{c}\right)^2 F$$

Lambrecht *et al* PRL (1996)

Experimental plateforms to observe DCE

Move the mirror

Change the refraction index

Change the sound speed (in BEC)

Outline of the talk

① Parametric creation of phonon in a cigar shape Bose-Einstein condensate

② Experimental procedure and phonon creation

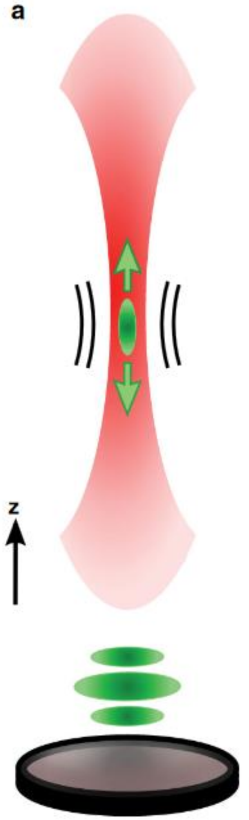
③ Correlation analysis of the phonons

$x(t) = x_0(1 + \epsilon \sin \omega t)$

$n(t) = n_0(1 + \epsilon \sin \omega t)$

Creation of pairs of phonon with frequency

$\omega_1 + \omega_2 = \omega$



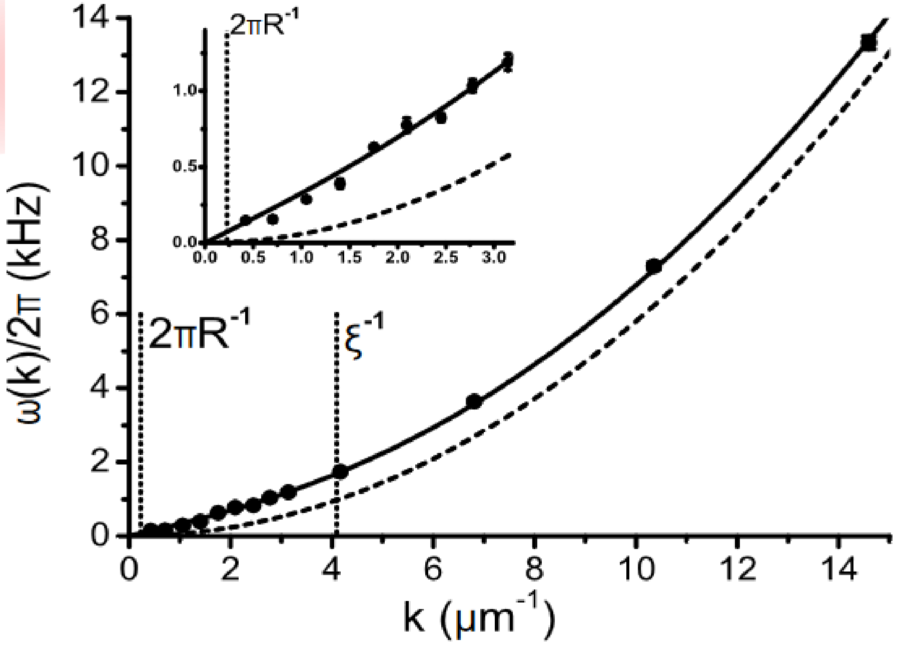
$c_s(t) = c_{s,0}(1 + \epsilon \sin \omega t)$

Wilson *et al.* Nature (2011).
Lähteenmäki *et al.* PNAS (2013)
Vezzoli, S. *et al.* Com Phys (2019).

Jaskula *et al.* PRL (2012).

Theoretical description

Cigar shape BEC (10s)
- $\omega_{\parallel} = 70$ Hz (~100 µm)
- $\omega_{\perp} = 1{,}3$ kHz (~5 µm)



Steinhauer *et al*, PRL (2002)

Describe the system as

small perturbation of the field

$$\hat{\Psi} = \Phi_0(r,t)(1 + \hat{\phi}(z,t))$$

Gaussian ansatz
(σ, L, N)

At lowest order, collective excitations :

$$i\partial_t \hat{a}_k = \Omega_k \hat{a}_k + \frac{-i\dot{\Omega}_k}{2\Omega_k} \hat{a}^{\dagger}_{-k}$$

with $\Omega_k = \sqrt{c_s^2 k^2 + \left(\frac{\hbar k^2}{2m}\right)^2}$

- recover Bogoliubov dispersion relation with sound speed

$$c_s = \left(\frac{2\, a_s N}{L\, \sigma^2}\right)^{1/2}$$

- modes k and –k evolve independently when c_s= cst
- k/–k mixing when the sound speed varies !

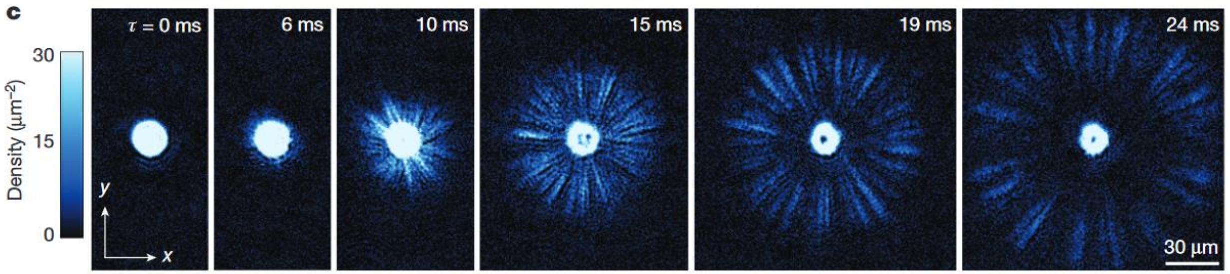
Busch *et al.* PRA (2013) ; Robertson *et al. PRD (2018)*

How to create collective excitations ?

How to change the sound speed ? $c_s = \sqrt{g_1 n_1/m} = \left(\frac{2\, a_s N}{L\, \sigma^2}\right)^{1/2}$

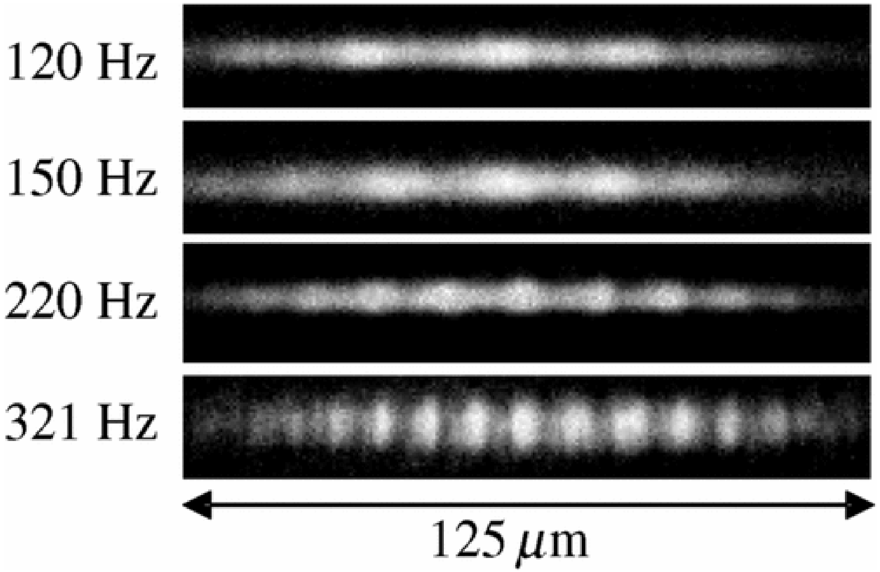
- a_s atomic scattering length
- $L\,, \sigma$ length and width of the BEC

Change the scattering length
Chicago, Heidelberg



Bose fireworks

Change the BEC width
Trento, Palaiseau



In situ imaging of Faraday waves

Clark *et al*. Nature (2017)

Engels *et al*. PRL (2007)

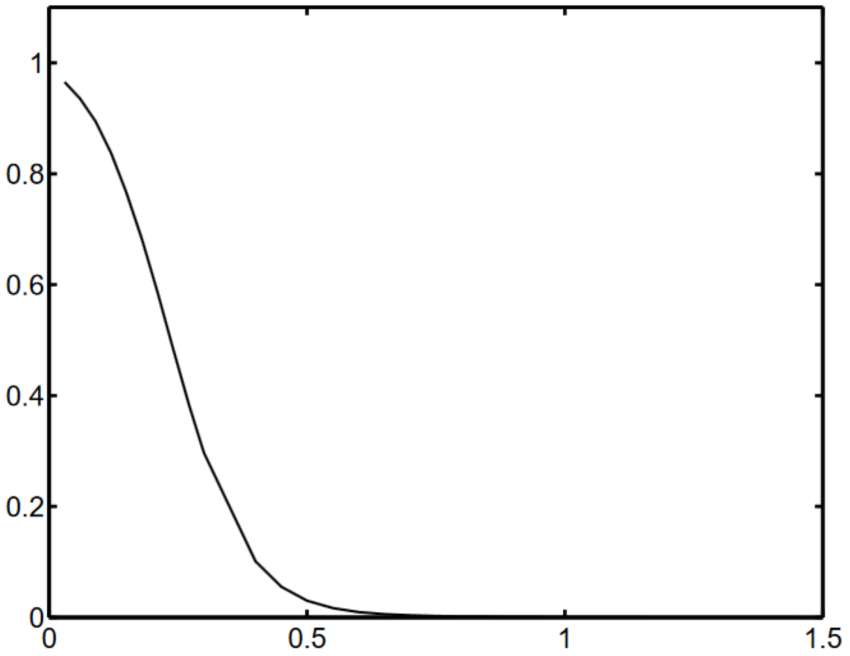
How to measure collective excitations ?

How to measure a_k ?

How collective excitation evolves when one release the BEC ?

Particles $$\begin{pmatrix} \hat{\phi}_k \\ \hat{\phi}^\dagger_{-k} \end{pmatrix} = \begin{pmatrix} u_k & v_k \\ v_k & u_k \end{pmatrix} \begin{pmatrix} \hat{a}_k \\ \hat{a}^\dagger_{-k} \end{pmatrix}$$ Quasi-particles

Ratio $\left.\frac{v_k}{u_k}\right|_{t\to\infty}$



Momentum k [a_ρ]

For high enough momentum, the collective excitation is mapped to a single atom.

(For us : k x a_ρ ~ 0.8)

Tozzo & Dalfovo, PRA (2004)

Outline of the talk

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① Parametric creation of phonon in a cigar shape Bose-Einstein condensate

② Creation of quasi-particles in a Bose-Einstein condensate of He^*

③ Correlation analysis of the phonon

Summary of ①

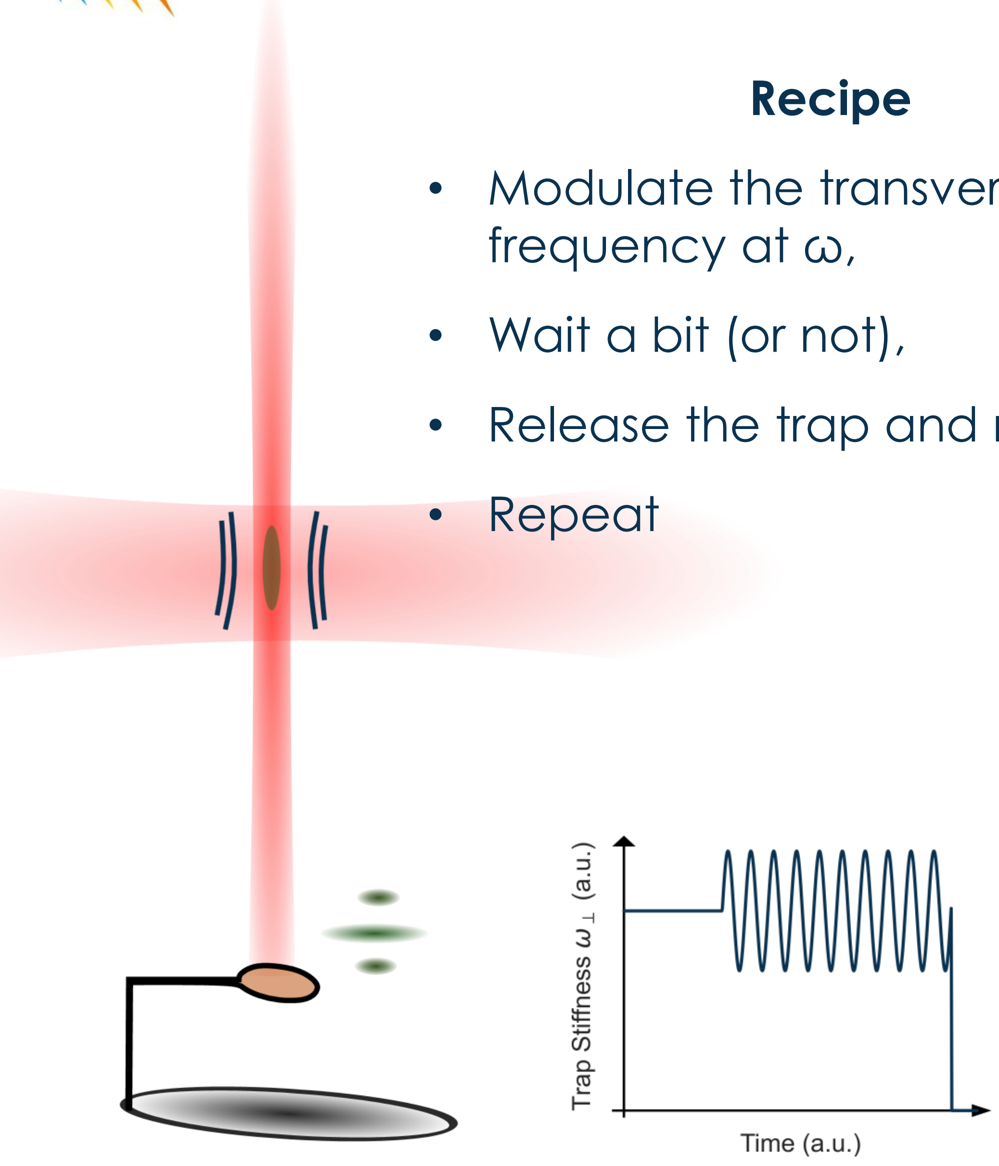
- Detecting the momentum of individual atoms
- Create entangled pairs of phonons when varies density
- Phonons adiabatically transform to atoms when release the trap

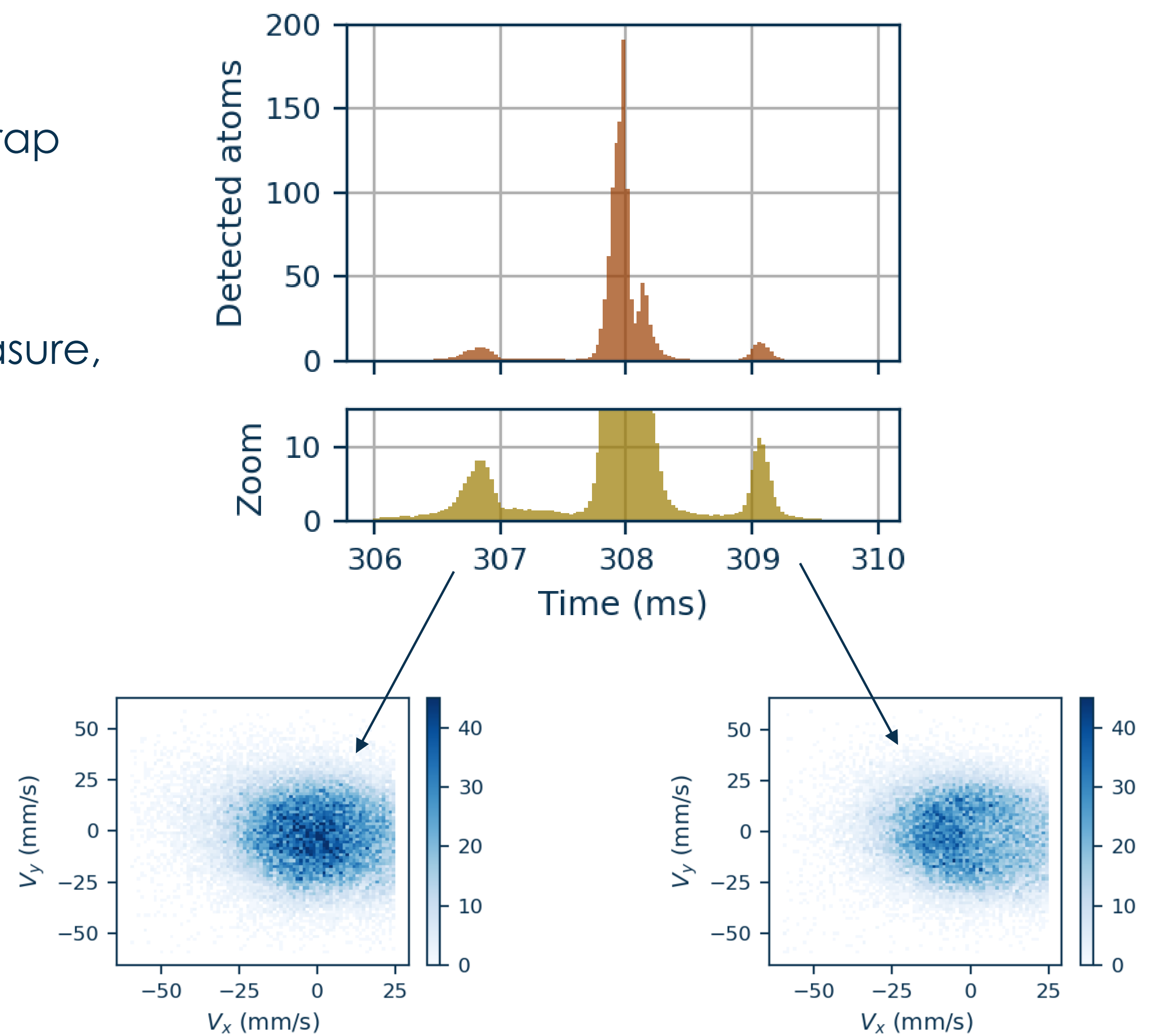
Let's do that !

Experimental procedure

Recipe

- Modulate the transverse trap frequency at ω,
- Wait a bit (or not),
- Release the trap and measure,
- Repeat

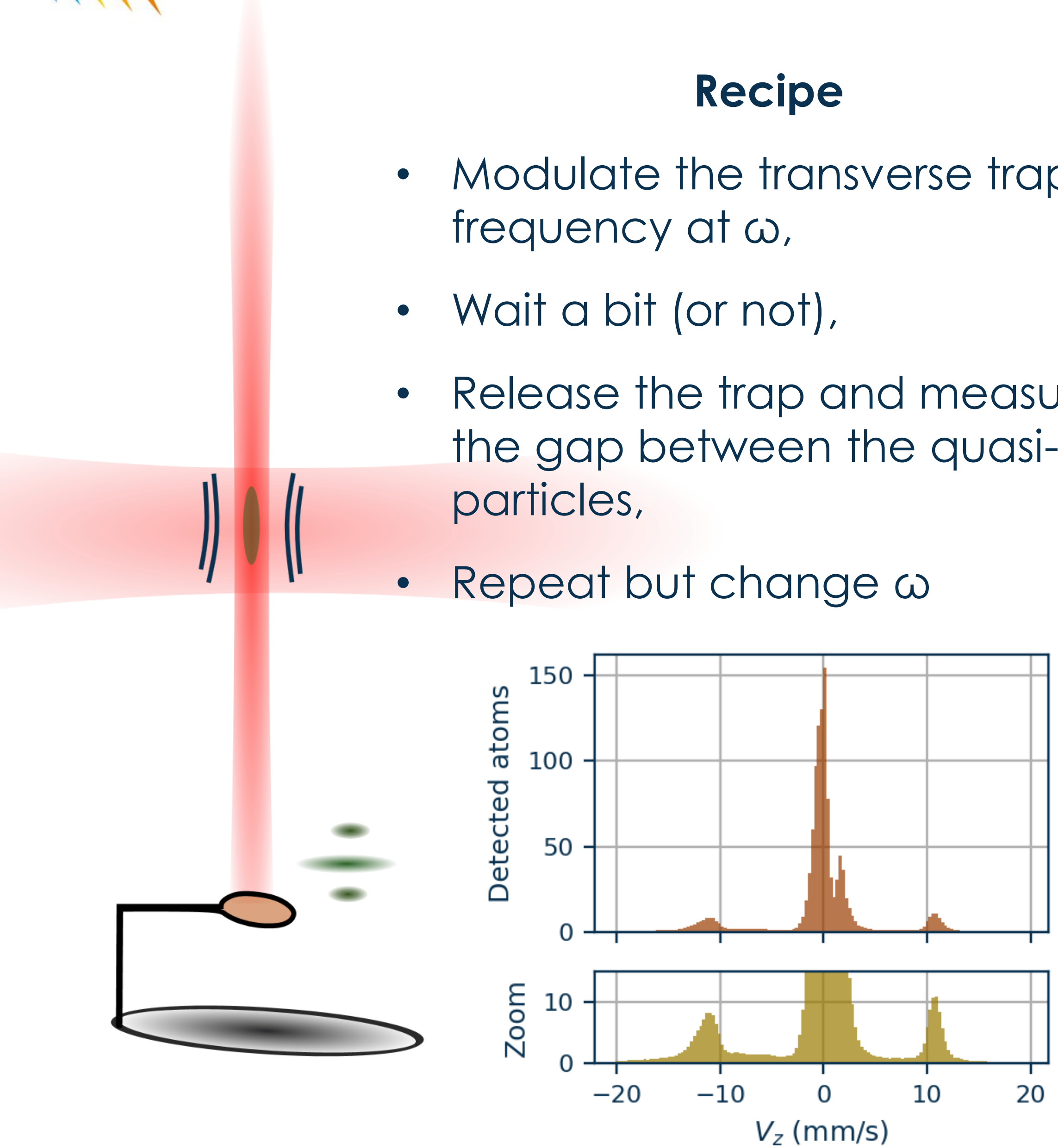


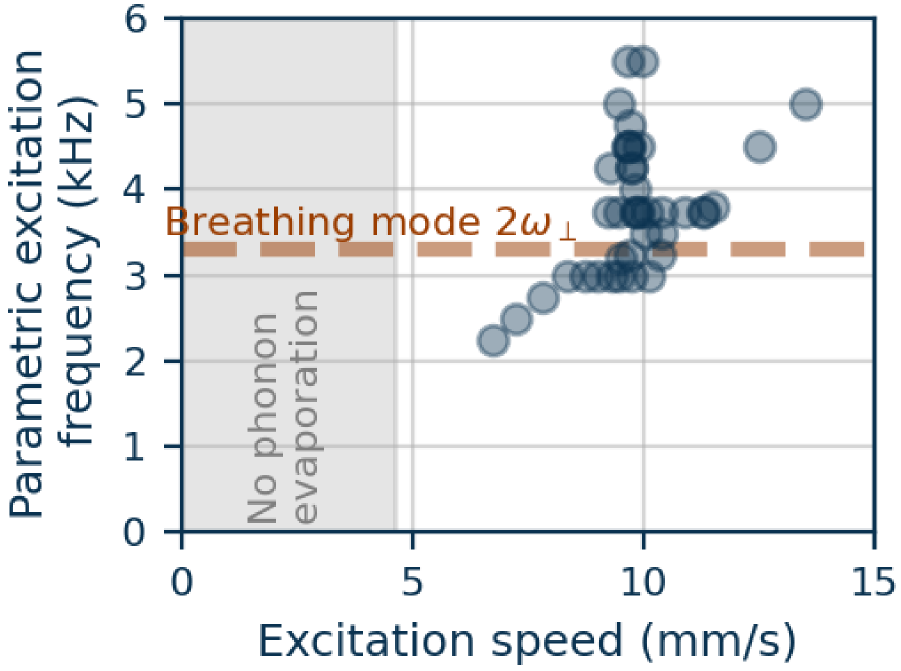


Measuring the dispersion relation

Recipe

- Modulate the transverse trap frequency at ω,
- Wait a bit (or not),
- Release the trap and measure the gap between the quasi-particles,
- Repeat but change ω

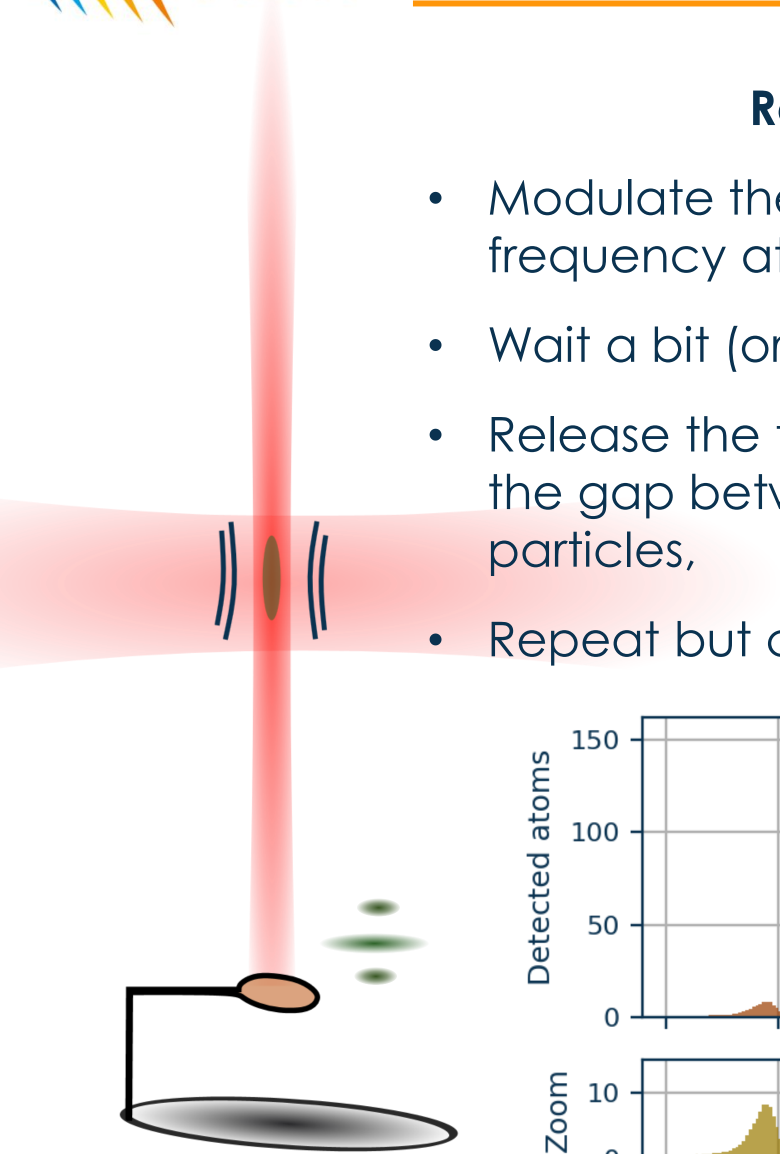




Parametric excitation near the breathing mode excites the BEC at resonance.

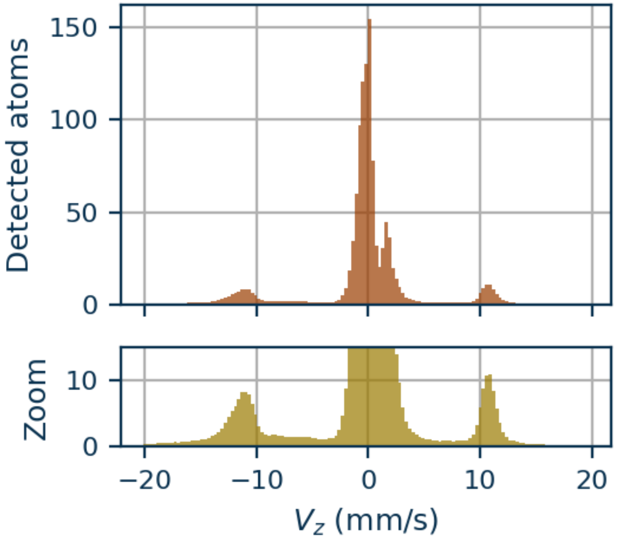
Tozzo & Dalfovo, PRA (2004)

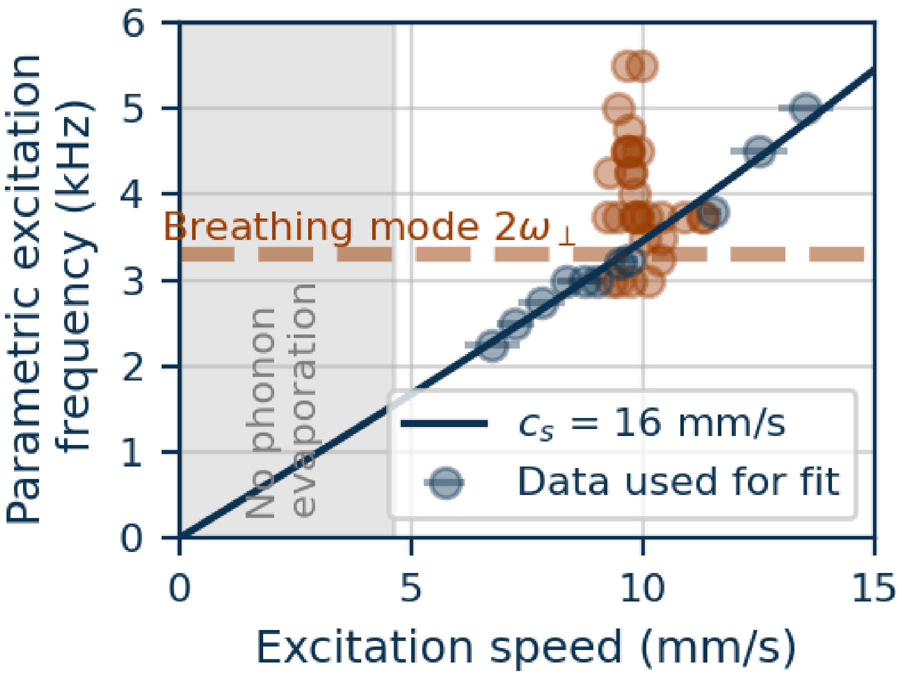
Measuring the dispersion relation



Recipe

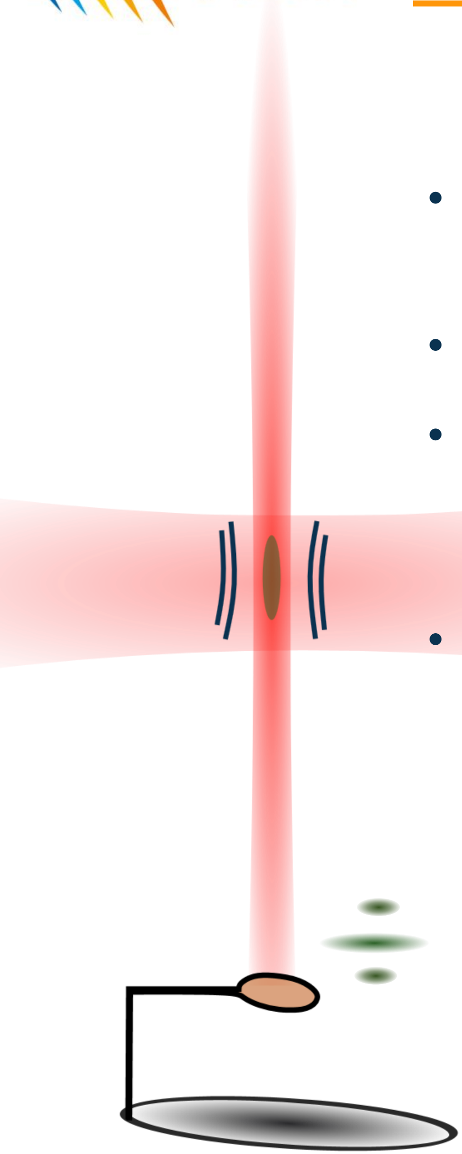
- Modulate the transverse trap frequency at ω,
- Wait a bit (or not),
- Release the trap and measure the gap between the quasi-particles,
- Repeat but change ω





- Parametric excitation near the breathing mode excites the BEC at resonance.
- Excitations on the phonon branch

Measuring the production rate of phonon

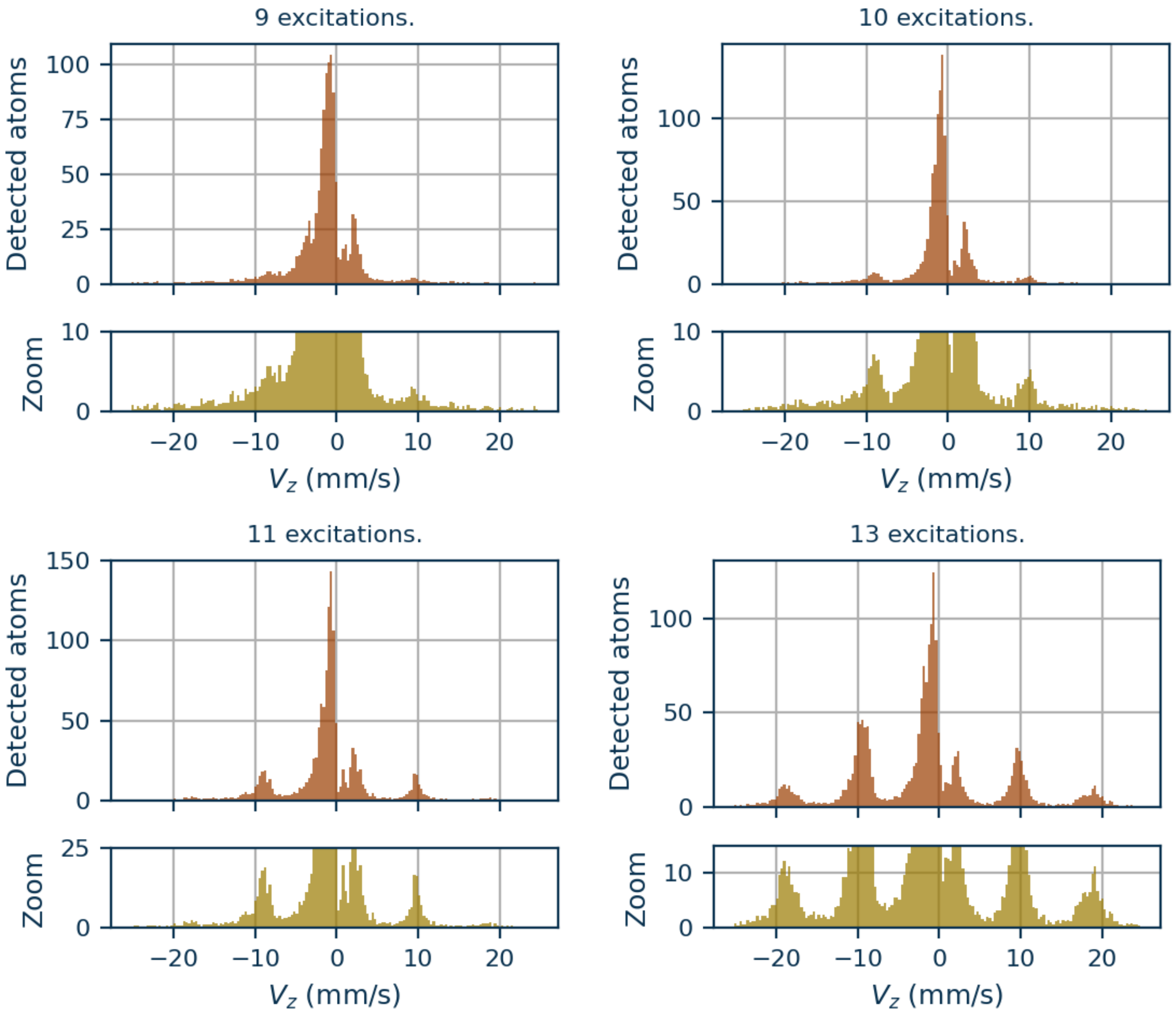


Recipe

- Modulate the transverse trap frequency at ω,
- Wait a bit (or not),
- Release the trap and measur the gap between the quasi-particles,
- Repeat but change the number of excitations

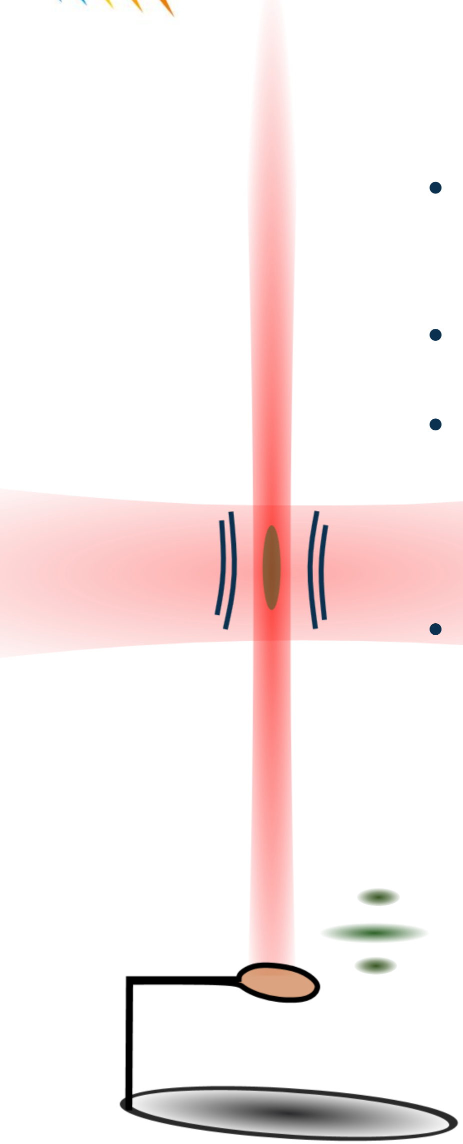
One expects[(1)] :

- exponential creation of phonons,
- higher orders phonons



(1) Robertson *et al*. PRD (2017)

Measuring the production rate of phonon



Recipe

- Modulate the transverse trap frequency at ω,
- Wait a bit (or not),
- Release the trap and measure the gap between the quasi-particles,
- Repeat but change the number of excitations

One expects[1] :

- exponential creation of phonons,
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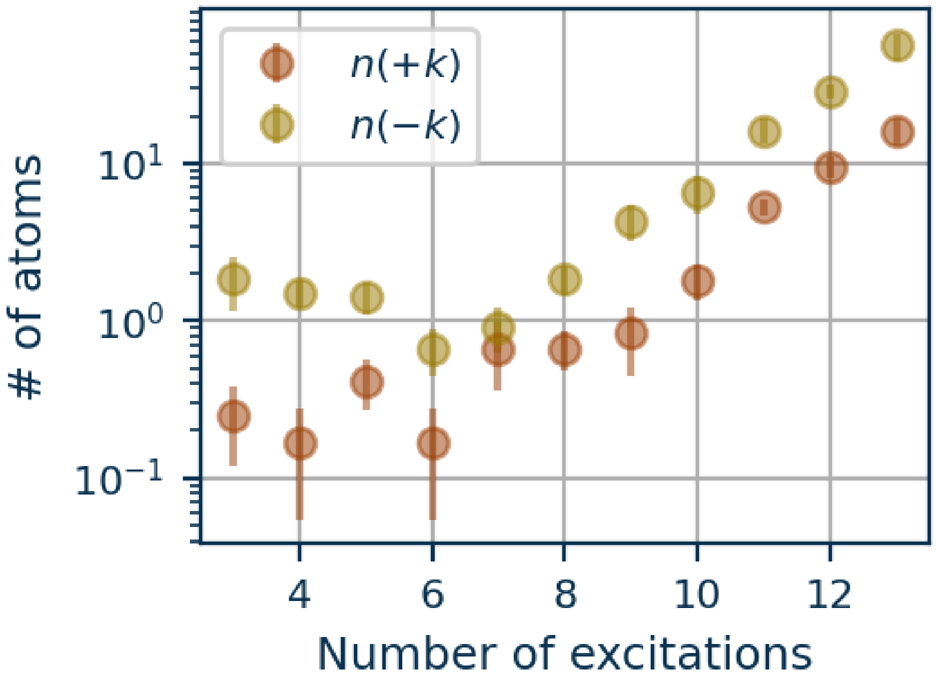


➢ Check the decay of the production rate[2]

(1) Robertson *et al*. PRD (2017)
(2) Micheli & Robertson, PRB (2022)

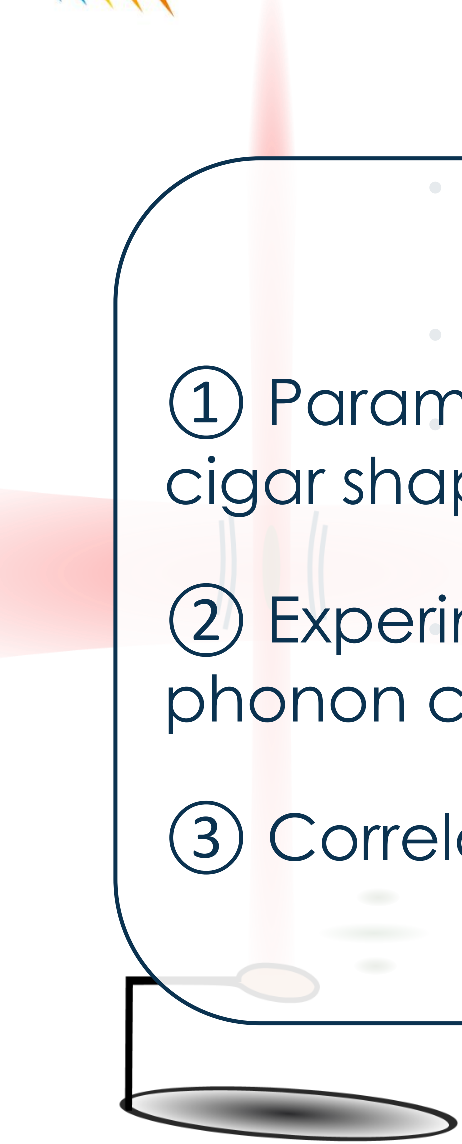
Measuring the production rate of phonon

Recipe

- Modulate the transverse trap
- Wait a bit (or not),
- particles,
- of excitations

One expects[1] :

- exponential creation of phonons,
- higher orders phonons



Outline of the talk

① Parametric creation of phonon in a cigar shape Bose-Einstein condensate

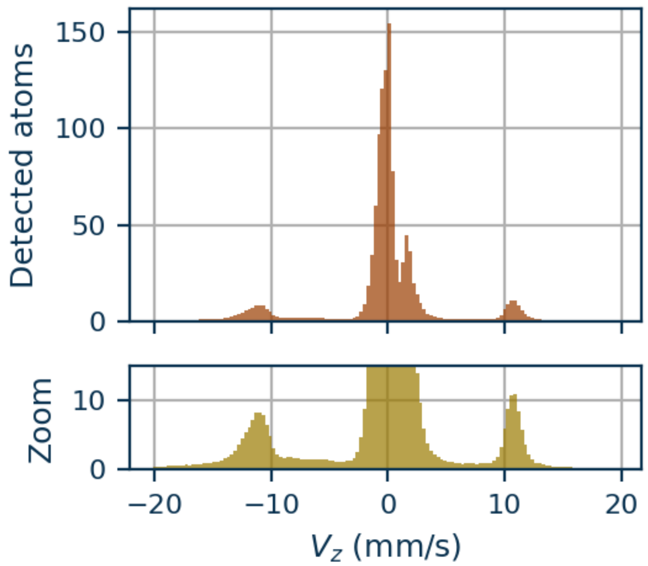
② Experimental procedure and phonon creation

③ Correlation analysis of the phonon

Summary of ②

- Creation and detection of quasi-particles in a BEC
- Population grows exponentially with excitation parameters
- Quasi-particles are phonons, in a BEC with c_s = 16 mm/s

Two mode squeezed state



We expect to create :

$$|TMS\rangle = \frac{1}{\cosh r}\sum_n \left(e^{-i\phi}\tanh r\right)^n |n\rangle_k \otimes |n\rangle_{-k}$$

Second order correlation function

$$g^{(2)}(q,k) \stackrel{\text{def}}{=} \frac{\langle :\hat{n}_q \hat{n}_k: \rangle}{\langle \hat{n}_q \rangle \langle \hat{n}_k \rangle}$$

$$g^{(2)}(k,k) = 2$$

1. Bosonic bunching

$$\langle n \rangle = \sinh r$$

$$P(n) = \frac{\tanh^n r}{\cosh r}$$

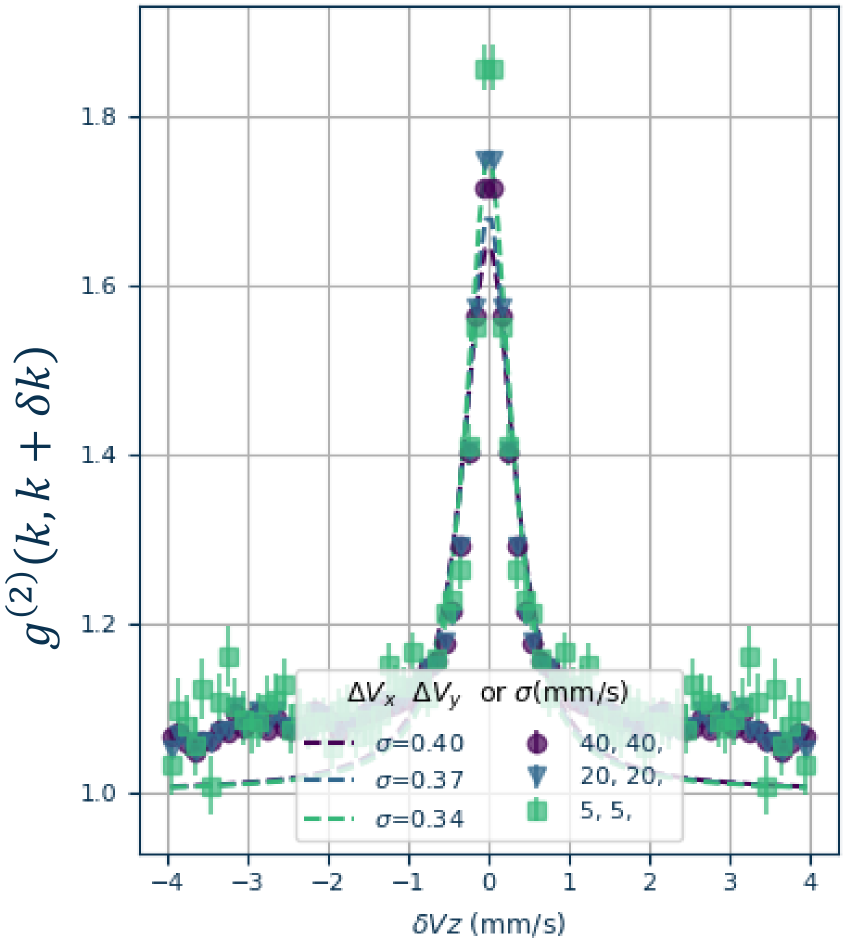
2. Counting statistics

$$g^{(2)}(k,-k) = 2 + \frac{1}{\langle n \rangle}$$

3. Entanglement

Tracing over one of the modes leaves the remaining mode in a thermal state

Probing (local) correlations



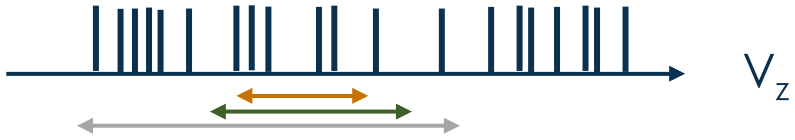
Local correlations (normal)

$g^{(2)}(k,k) = 2$ in the limit where the integration volume goes to 0.

(In the limit where pixels size goes to zero)

How to measure the number of particles in mode ? *(for counting statistics)*

How to define a mode size ? $(2\pi/L)$



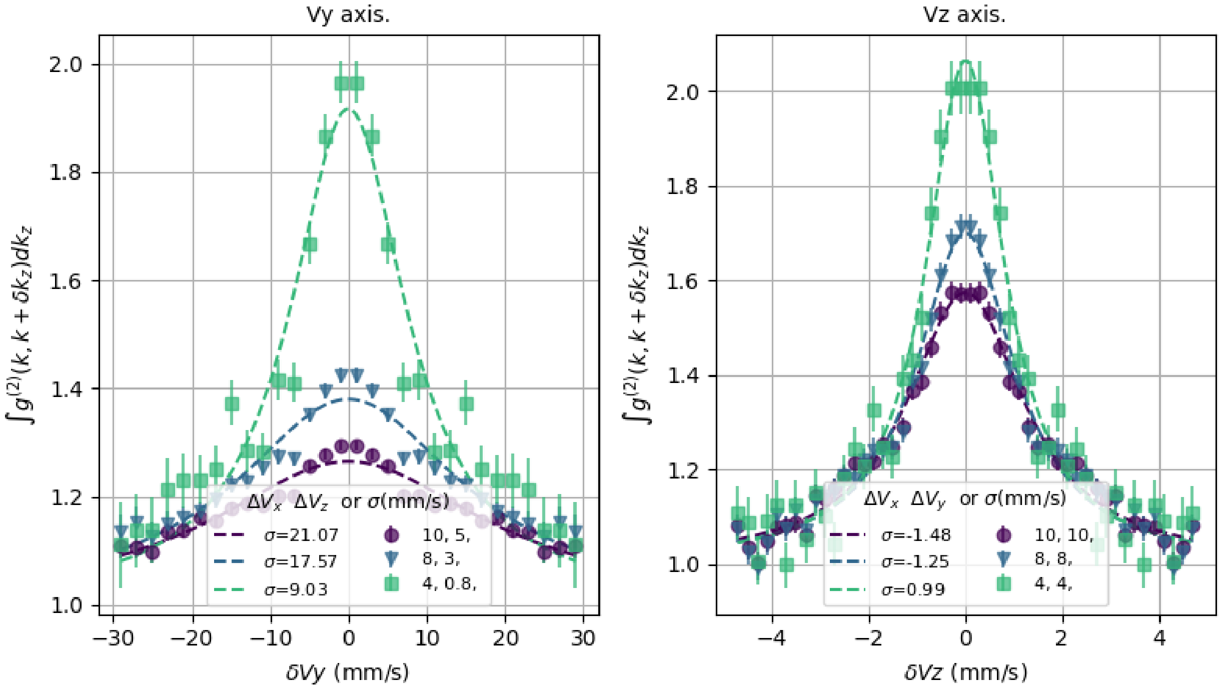
Use the HBT effect !

Bosonic bunching

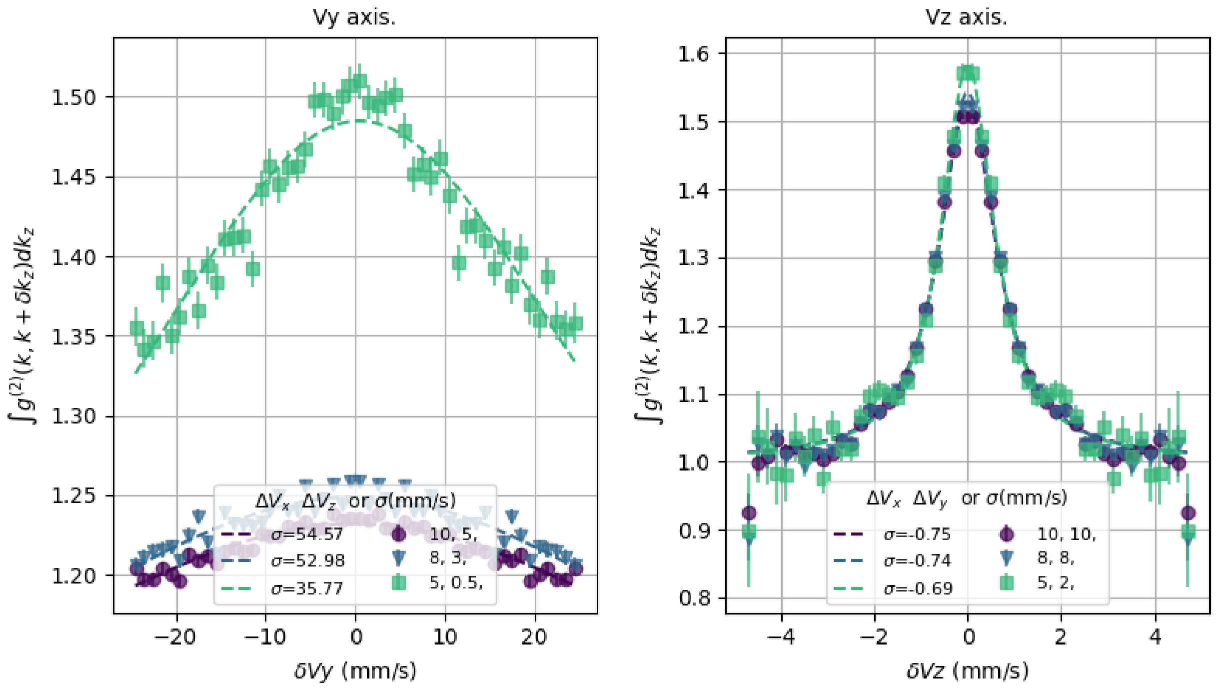
The width of the auto-correlation gives the mode size.

Is it really the size of the source ?

Parametric amplification in a moving lattice

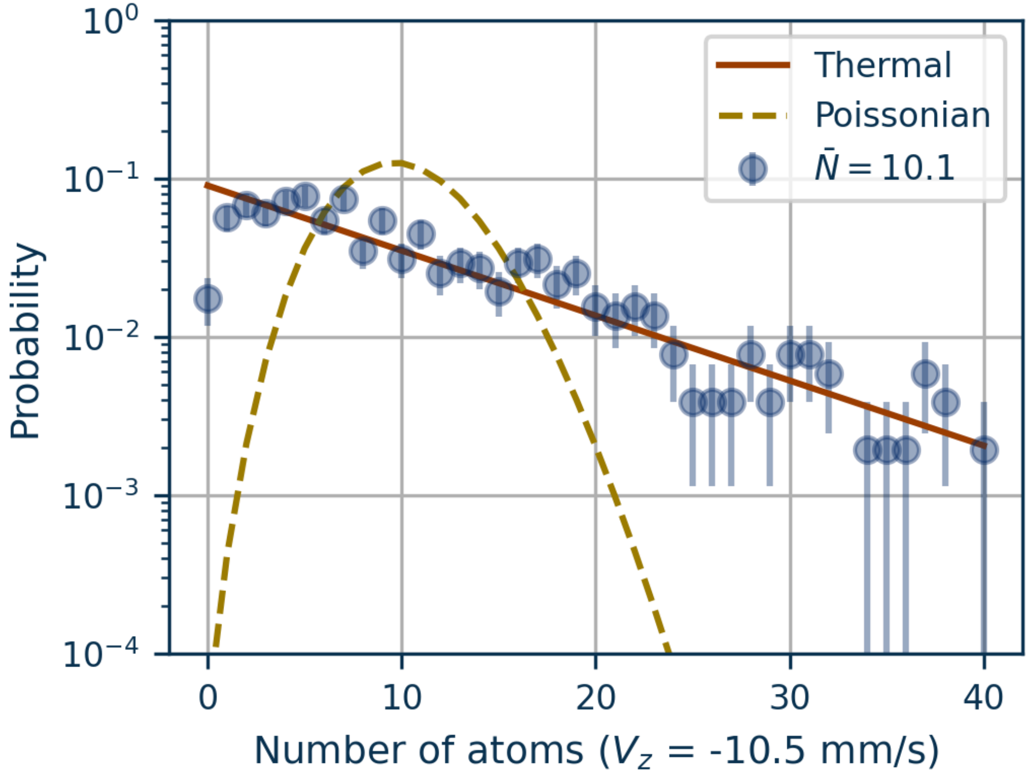


This presentation (DCE)



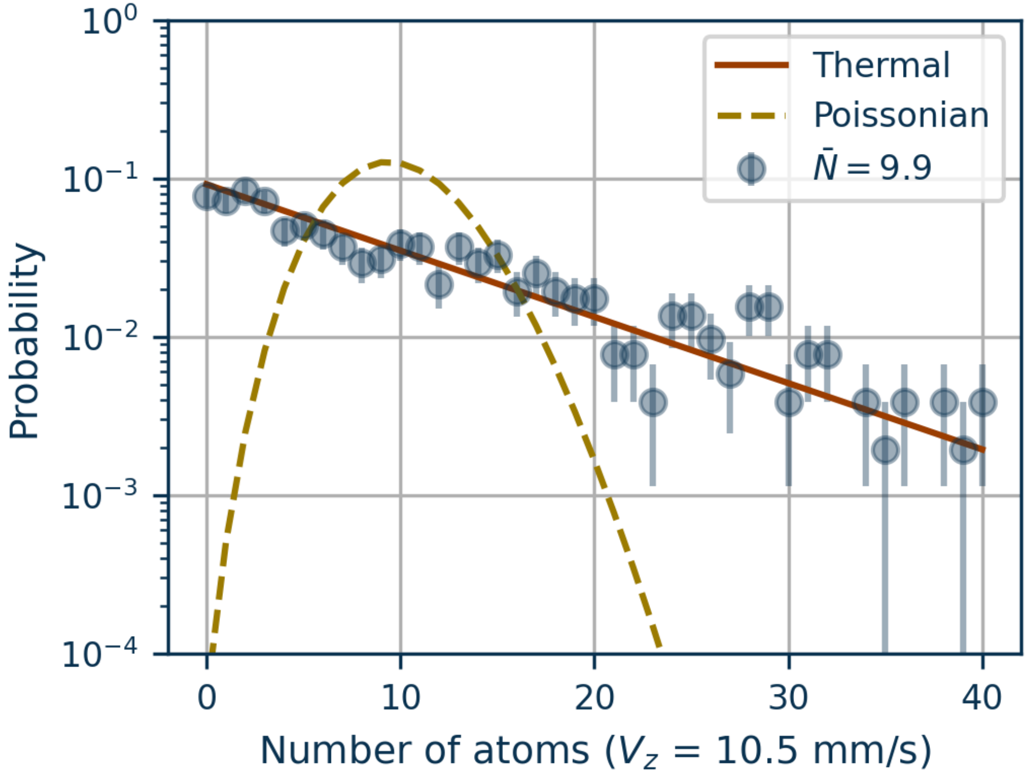
Counting statistics

For a thermal state, the mean number of particles determines the distribution statistics.

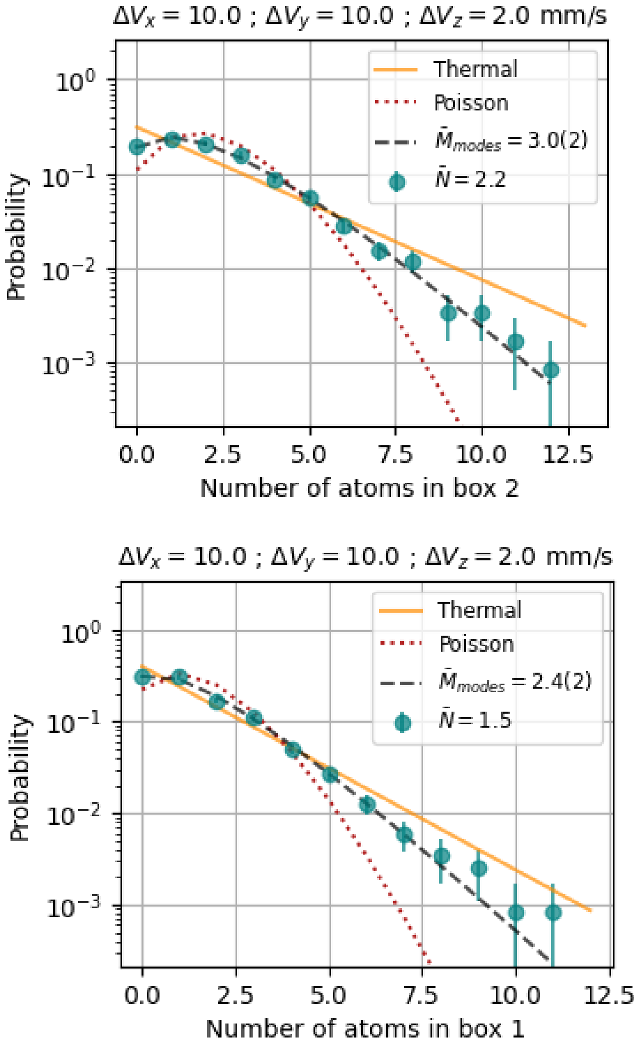


$$\langle n \rangle = \sinh r$$

$$P(n) = \frac{\tanh^n r}{\cosh r}$$

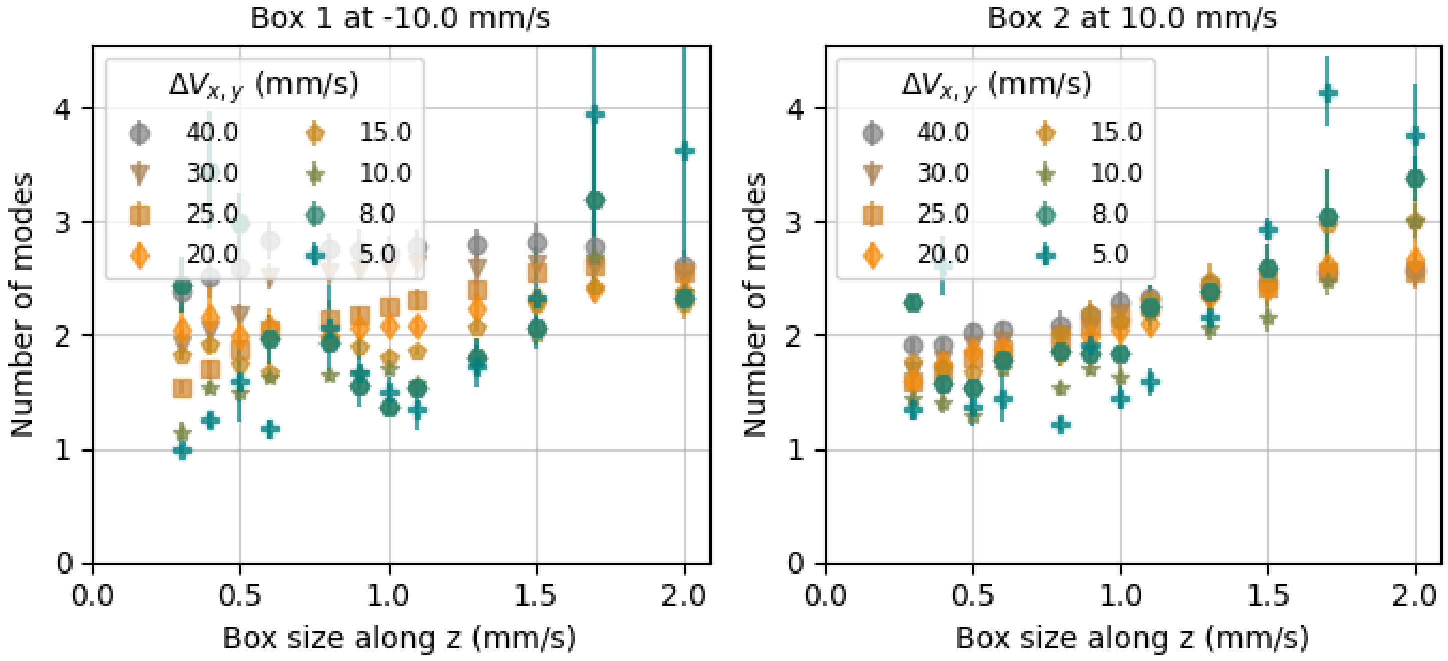


Counting statistics



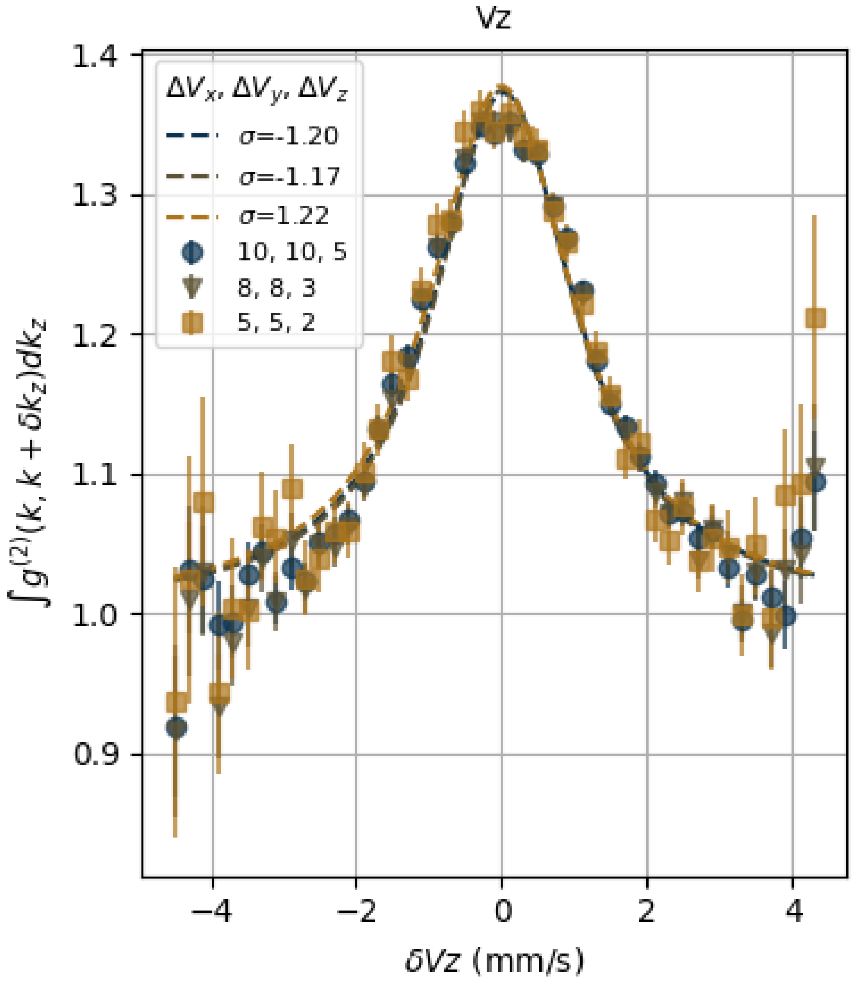
When the size of the box is too large, the distribution is no longer thermal.

Assuming equal population per mode, the number of modes can be fitted.



Probing (cross) correlations

$$g^{(2)}(-k, k + \delta k)$$



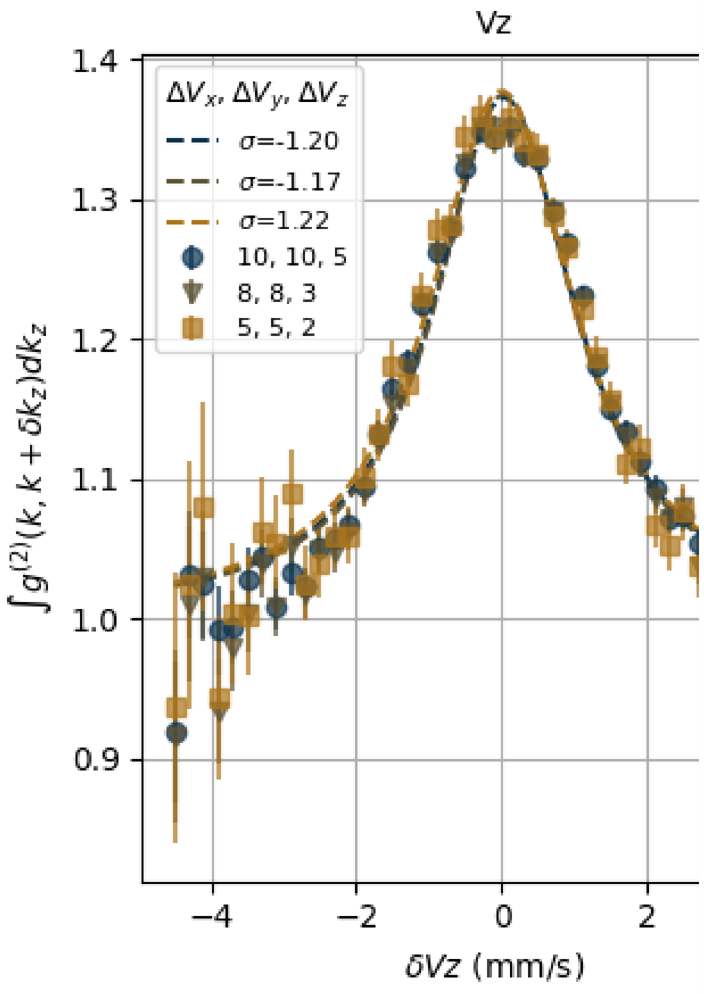
- $g^{(2)}(k, -k) < 2$
- Pic value of ~1.3 (< 2 of local correlations)

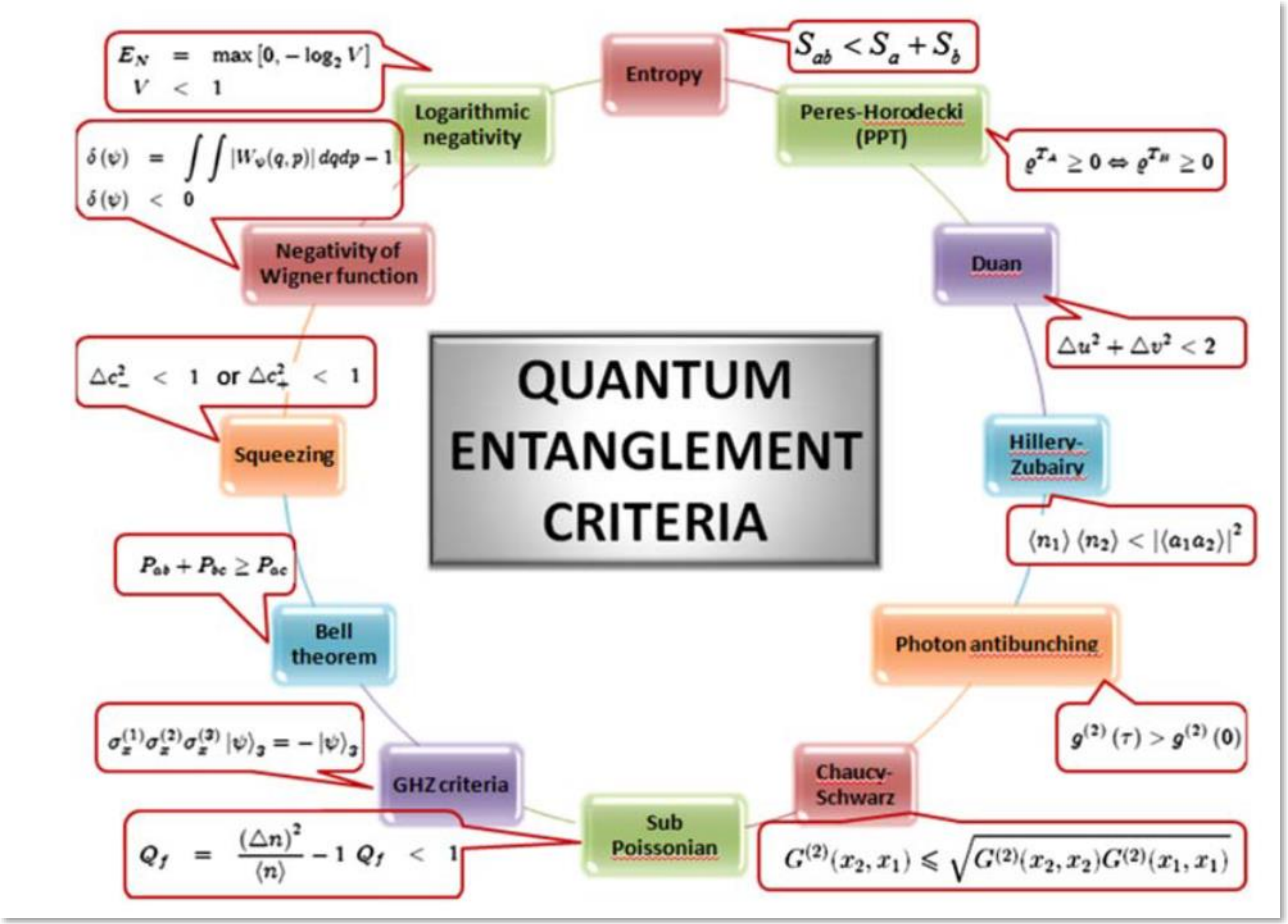
WHY ???? ☹☹☹

Shot-to-shot fluctuations of the BEC initial speed kills cross-correlations

Probing (cross) correlations

$g^{(2)}(-k, k$

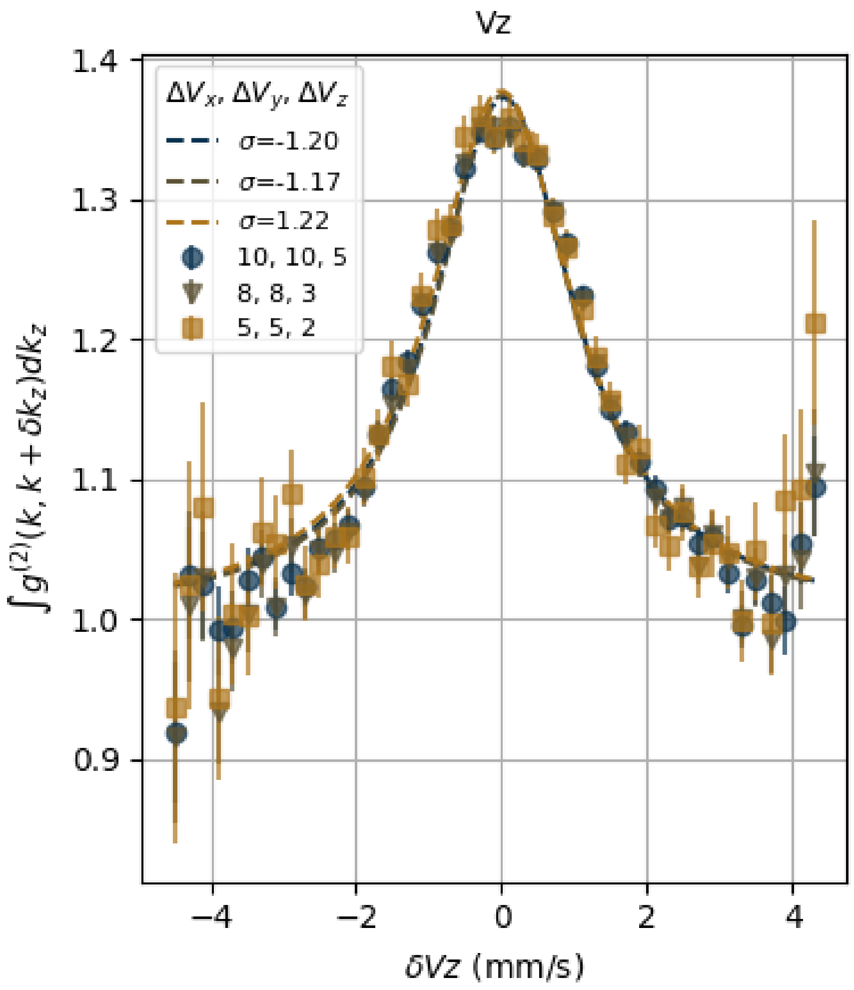




Sumairi *et al.* Journal of Mod. Op (2013)

Probing (cross) correlations

$$g^{(2)}(-k, k + \delta k)$$



- $g^{(2)}(k, -k) < 2$
- Pic value of ~1.3 (< 2 of local correlations)

WHY ???? ☹☹☹

Shot-to-shot fluctuations of the BEC initial speed kills cross-correlations

Use other entanglement witnesses

Cauchy-Schwarz inequality

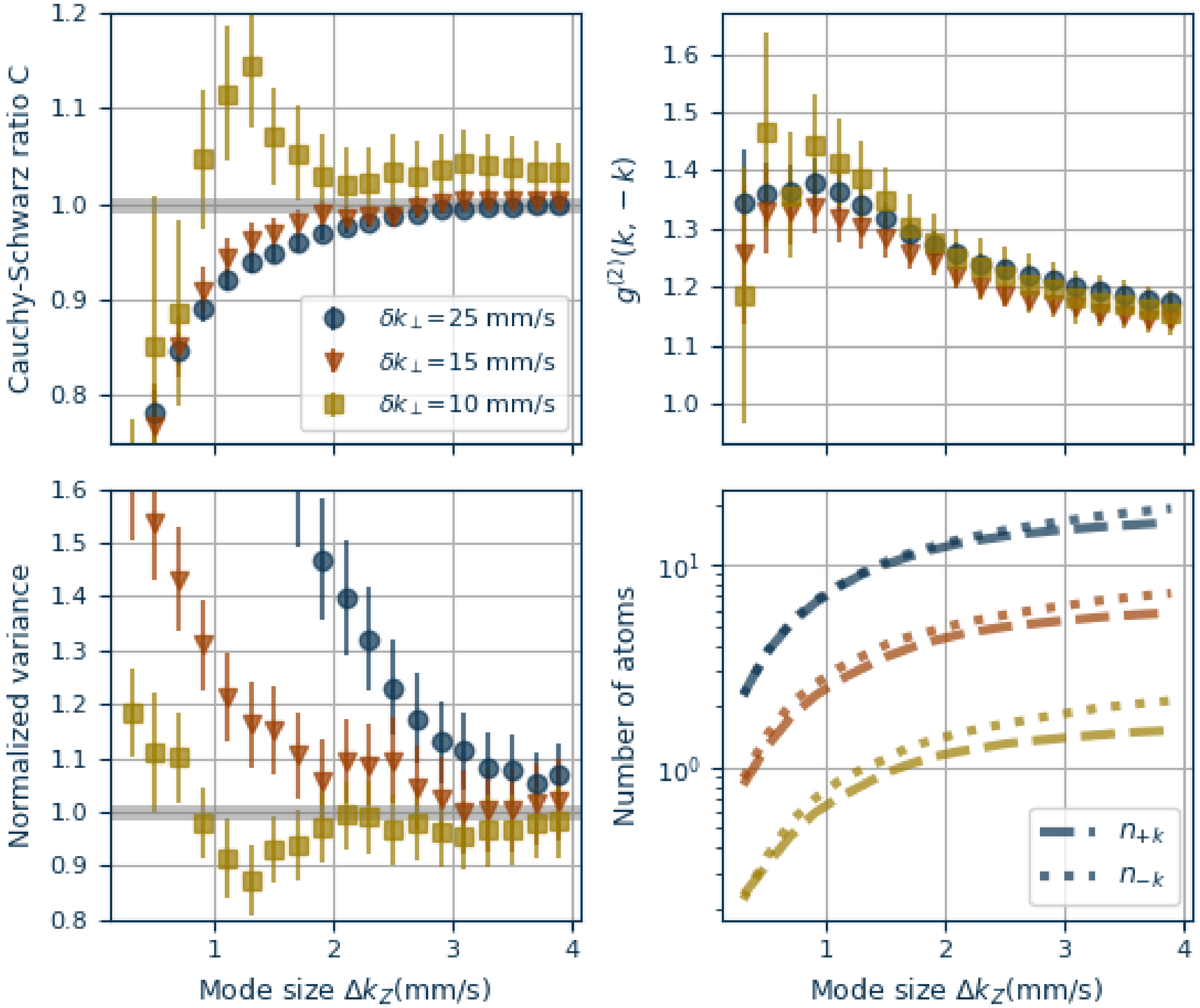
$$C = \frac{G^{(2)}(k, -k)}{\sqrt{G^{(2)}(-k, -k) \times G^{(2)}(k, k)}} < 1$$

Normalized variance

$$V = \frac{Var(n_k - n_{-k})}{n_k + n_{-k}} > 1$$

Kheruntsyan *et al.* PRL (2012)

Sub-shot noise variance and CSI



Cauchy-Schwarz inequality

$$C = \frac{G^{(2)}(k,-k)}{\sqrt{G^{(2)}(-k,-k) \times G^{(2)}(k,k)}} < 1$$

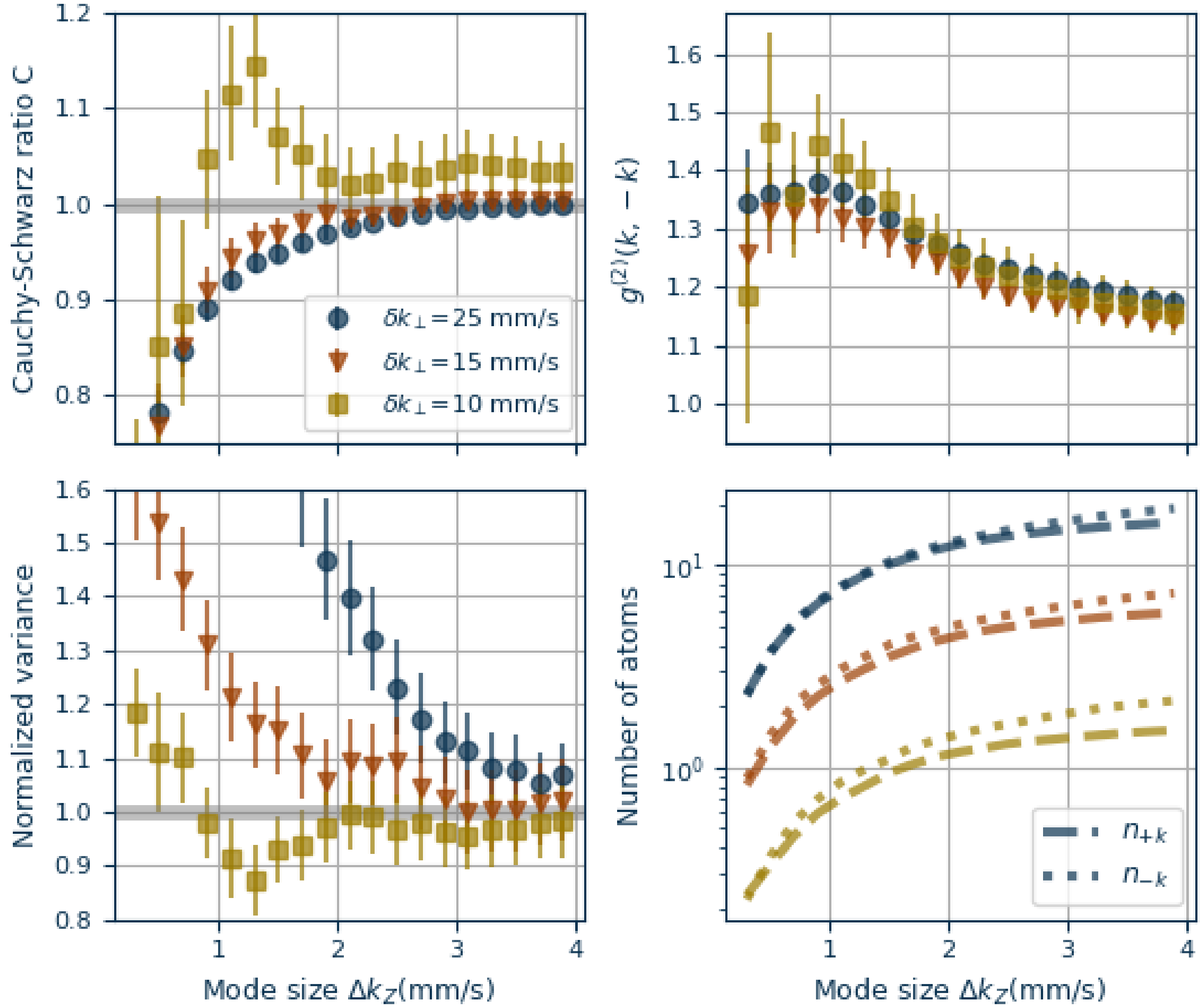
Normalized variance

$$V = \frac{Var(n_k - n_{-k})}{n_k + n_{-k}} > 1$$

We observe for the « *right* » pinhole :

- Violation of the Cauchy-Schwarz inequality,
- Sub-shot noise variance

Sub-shot noise variance and CSI



> *The observation of [the CSI] proves the presence of particle entanglement in a many-body system of bosons. This applies with either a fixed or a fluctuating number of particles, provided that there is no coherence between different number states.*
>
> Wasak et al., « Cauchy-Schwarz Inequality and Particle Entanglement » PRA (2014)

We observe for the « *right* » pinhole :

- Violation of the Cauchy-Schwarz inequality,
- Sub-shot noise variance

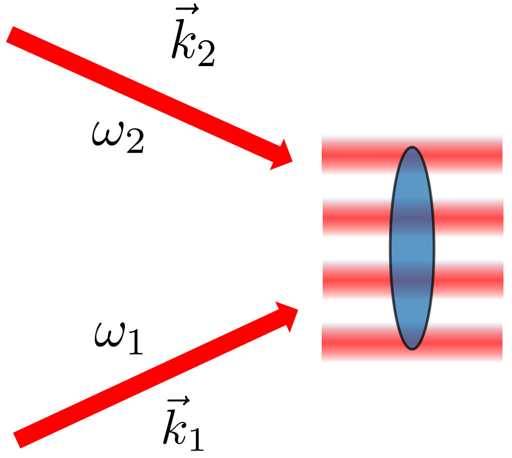
Check the $g^{(2)} > 2$ criteria

$$\langle \hat{n}_k \hat{n}_{-k} \rangle = \langle \hat{b}_k^\dagger \hat{b}_{-k}^\dagger \hat{b}_k \hat{b}_{-k} \rangle = n_k n_{-k} + \underbrace{\left|\langle \hat{b}_k \hat{b}_{-k} \rangle\right|^2}_{\leq n_k n_{-k}} + \underbrace{\left|\langle \hat{b}_k^\dagger \hat{b}_{-k} \rangle\right|^2}_{0\ ????}$$

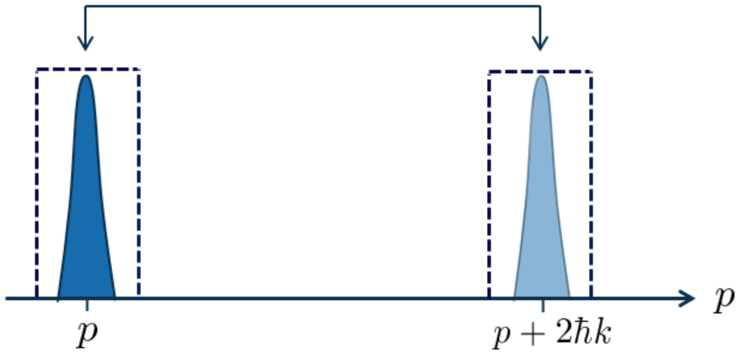
if the state is separable :

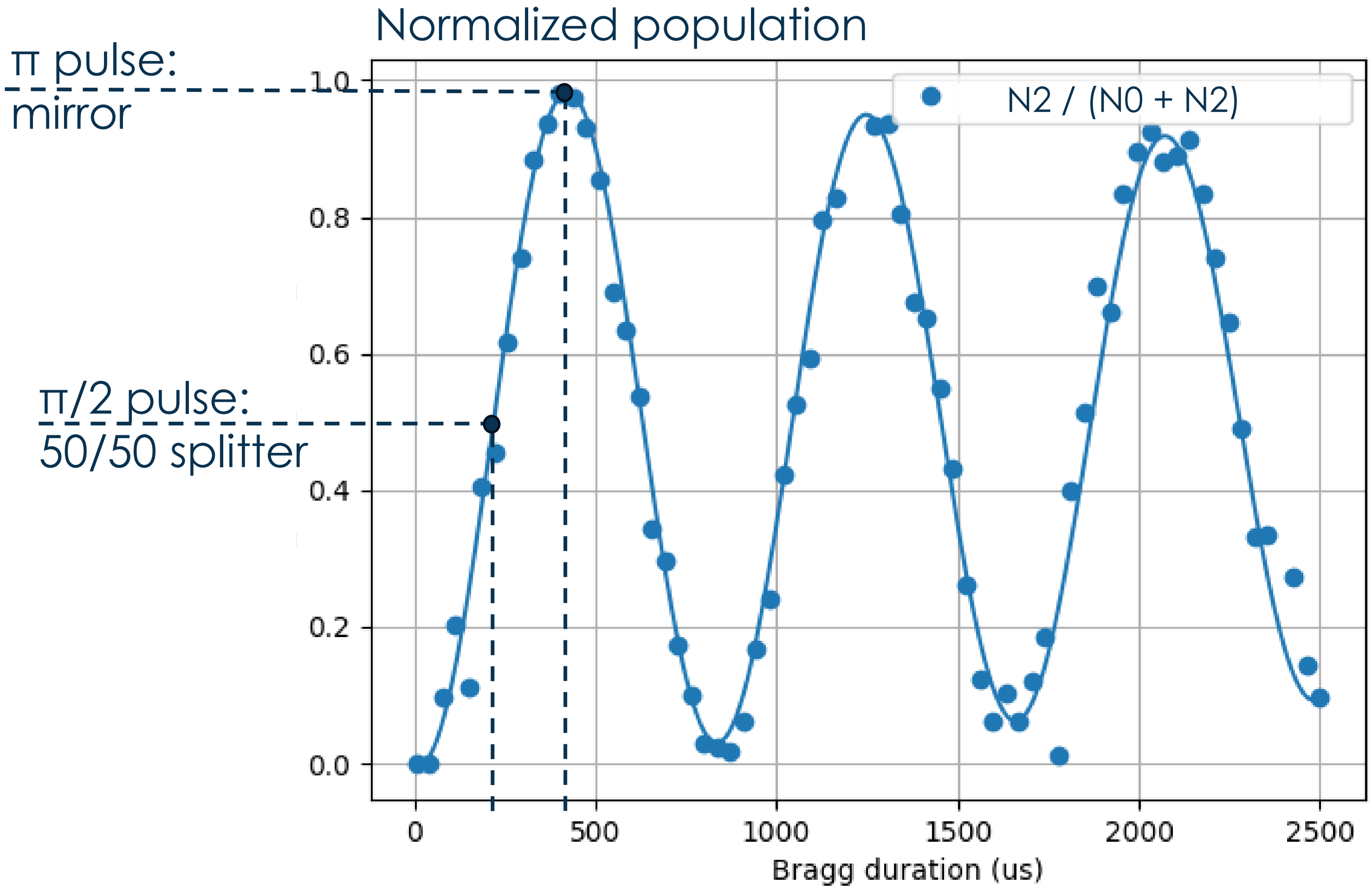
g2 >2 <-> entanglement witness

Momentum transfer: Bragg diffraction



Two photons momentum transfer between p and p + 2ħk





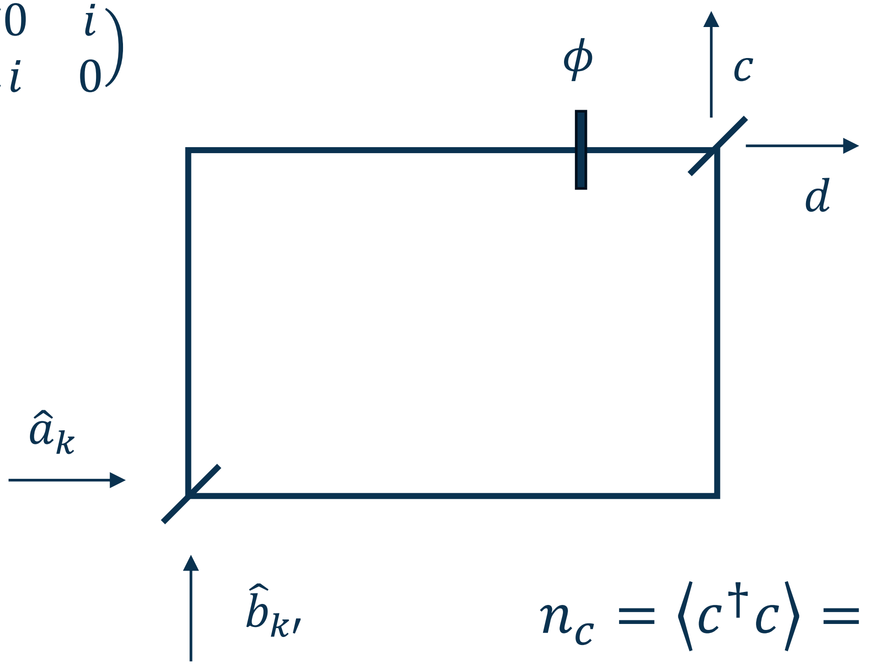
Check the $g^{(2)} > 2$ criteria

$$\langle \hat{n}_k \hat{n}_{-k} \rangle = \langle \hat{a}_k^\dagger \hat{a}_{-k}^\dagger \hat{a}_k \hat{a}_{-k} \rangle = n_k n_{-k} + \underbrace{|\langle \hat{a}_k \hat{a}_{-k} \rangle|^2}_{\le n_k n_{-k}} + \underbrace{|\langle \hat{a}_k^\dagger \hat{a}_{-k} \rangle|^2}_{0\ ????}$$

?

if the state is separable :

$$\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$



$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & ie^{2i\phi} \\ ie^{-2i\phi} & 1 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

$$c = \cos(\phi)\, a + \sin(\phi)\, b$$

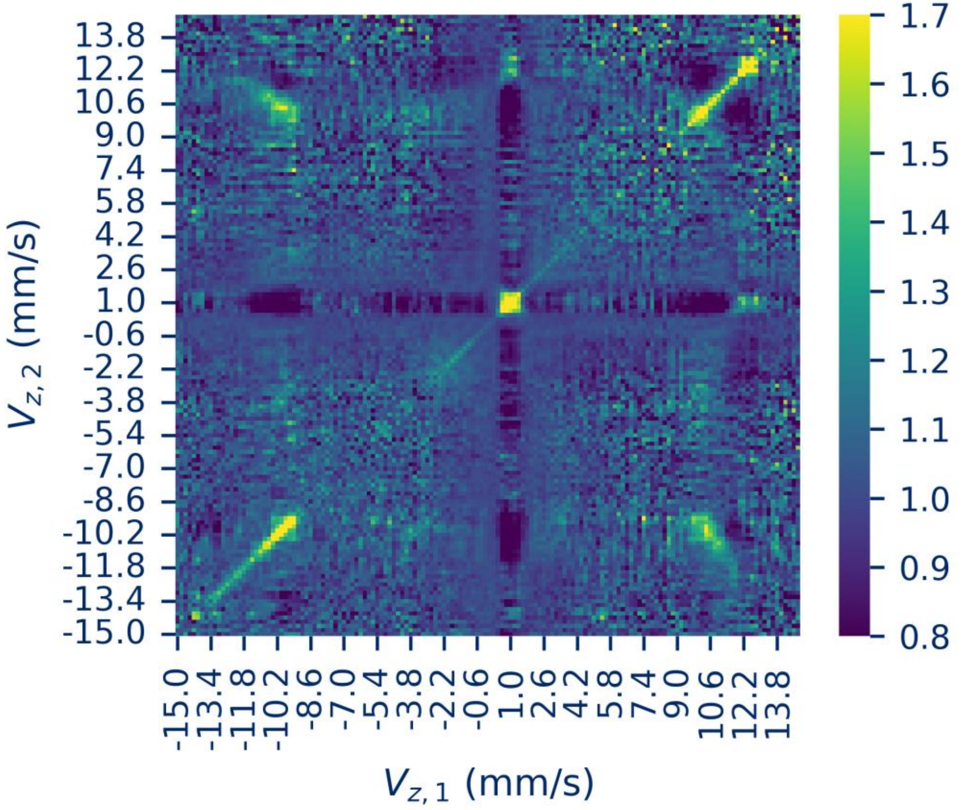
$$n_c = \langle c^\dagger c \rangle = \cos(\phi)^2\, a^\dagger a + \sin(\phi)^2\, b^\dagger b + \cos\phi \sin\phi\, (a^\dagger b + b^\dagger a)$$

Conclusion

What we saw

- Creation and detection of (evaporated) phonons in a Bose-Einstein Condensate,
- Bogoliubov dispersion relation and exponential creation of phonons,
- Counting statistics compatible with a two-mode squeezed state,
- Preliminary results on non-separability
- Setup of the interferometer to check mode coherence.

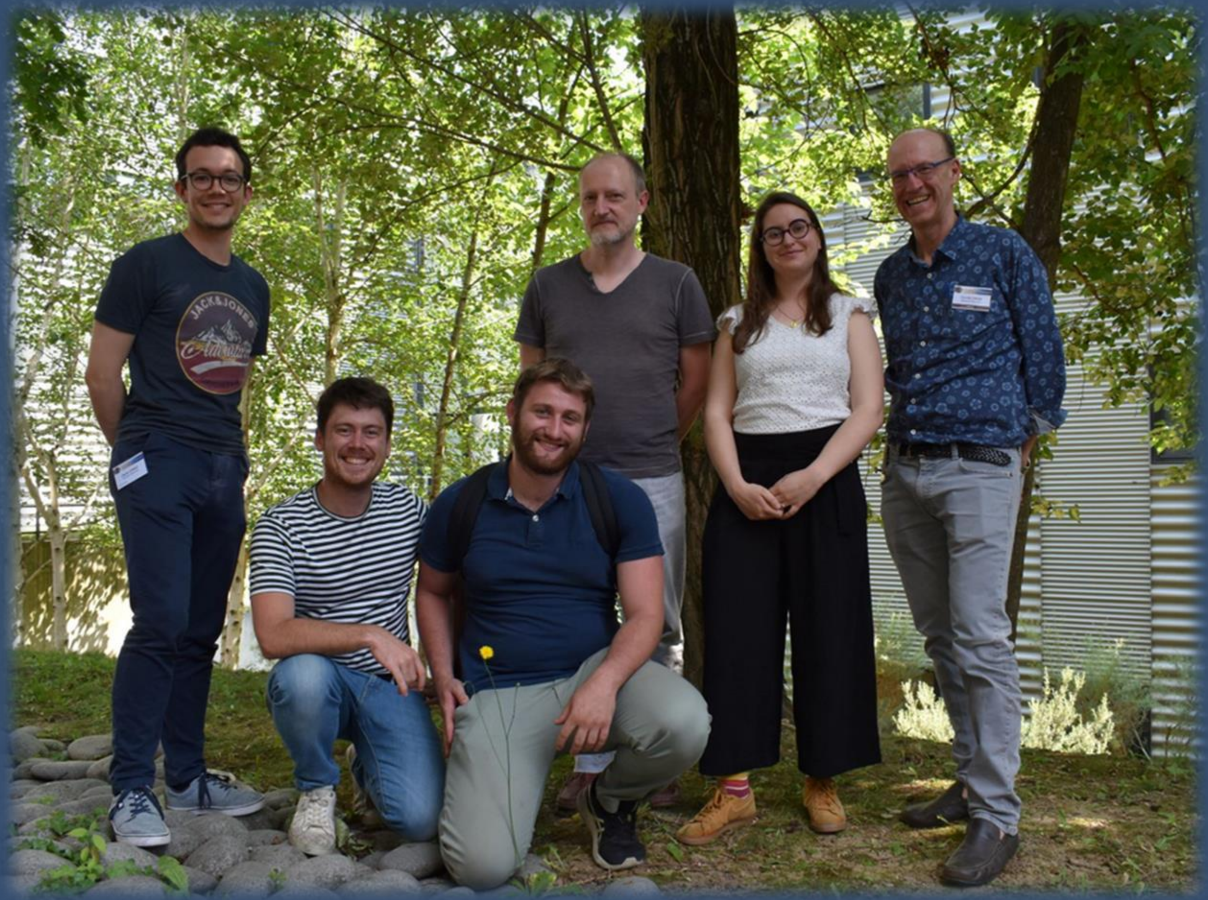
2D correlation map of $g^{(2)}(V_{z,1}, V_{z,2})$



Thank you for your time !

Some lecture

- J.-C. Jaskula *et al.*, Phys. Rev. Lett. **109**, 220401 (2012).
- S. Robertson, F. Michel, and R. Parentani, Phys. Rev. D **95**, 065020 (2017).
- A. Micheli and S. Robertson, Phys. Rev. B **106**, 214528 (2022).



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From the BdG Eq to collective excitations

Describe the system as

$$\hat{\Psi} = \Phi_0(r,t)(1+\hat{\phi}(z,t))$$

Φ_0 : gaussian ansatz for the transverse profile with width σ

$\hat{\phi}$: small perturbation of the field, depending only on z and time.

The perturbation obeys the Bogoliubov - de Gennes equation

$$i\hbar\partial_t\hat{\phi} = \frac{-1}{2m}\partial_{zz}\hat{\phi} + g_1 n_1(\hat{\phi}+\hat{\phi}^\dagger)$$

where $g_1 n_1$ depends on transverse profile Φ_0 (and time !). Fourier transforming this eq :

$$i\hbar\partial_t\hat{\phi}_k = \frac{k^2+g_1 n_1}{2m}\widehat{\phi_k} + \hat{\phi}_k^\dagger$$

$$\begin{pmatrix}\hat{\phi}_k \\ \hat{\phi}_{-k}^\dagger\end{pmatrix} = \begin{pmatrix}u_k & v_k \\ v_k & u_k\end{pmatrix}\begin{pmatrix}\hat{b}_k \\ \hat{b}_{-k}^\dagger\end{pmatrix}$$

with

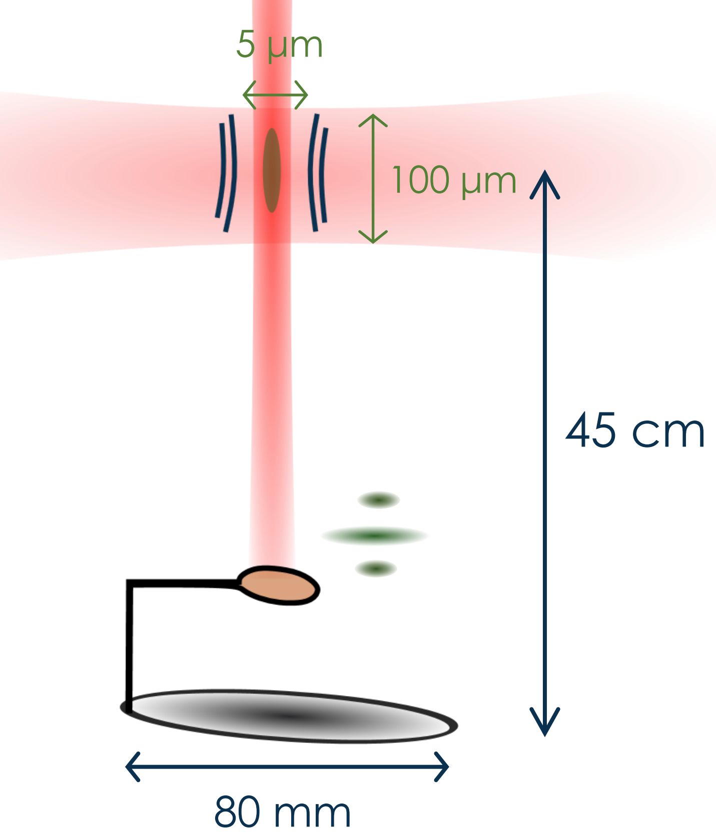
$$\begin{matrix}u_k \\ v_k\end{matrix} = \frac{\sqrt{\Omega_k+mc^2}\pm\sqrt{\Omega_k-mc^2}}{norm} \quad \text{and } \Omega_k^2 = \frac{g_1 n_1 k^2}{m} + \left(\frac{k^2}{2m}\right)^2$$

$$i\partial_t\begin{pmatrix}\hat{b}_k \\ \hat{b}_{-k}^\dagger\end{pmatrix} = \begin{pmatrix}\Omega_k & -i\partial_t\Omega_k/2\Omega_k \\ -i\partial_t\Omega_k/2\Omega_k & -\Omega_k\end{pmatrix}\begin{pmatrix}\hat{b}_k \\ \hat{b}_{-k}^\dagger\end{pmatrix}$$

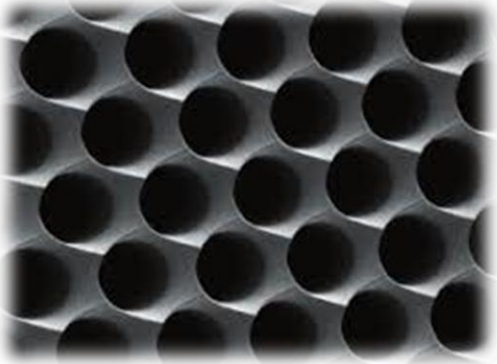
- $\hat{b}_k$ is the annihilation operator for collective excitations (phonons)
- When $\partial_t\Omega_k = 0$: k and $-k$ modes evolve independently from each other
- When $\partial_t\Omega_k \neq 0$: mixing between k and $-k$ modes

Detecting single atoms

Cigar shape BEC (10s)
- $\omega_{\parallel} = 10$ Hz
- $\omega_{\perp} = 1{,}6$ kHz

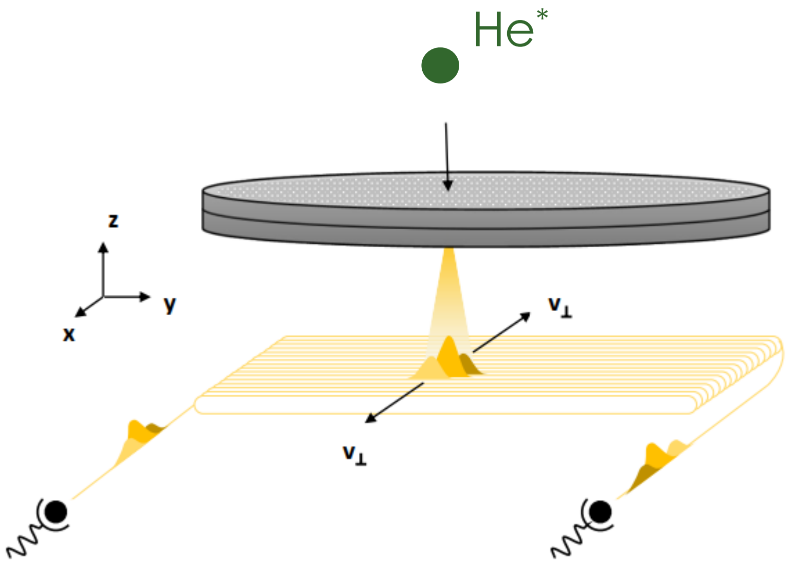


Metastable (20 eV , 2h) helium atom



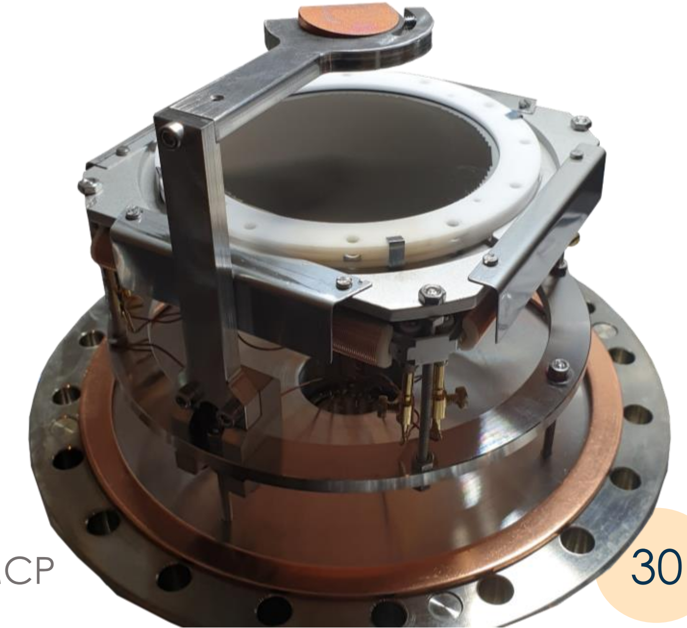
80 µm

© *Hamamatsu Photonics*



A metastable helium atom hitting the detector triggers an electronic fountains :

→ Measure : X, Y, T.



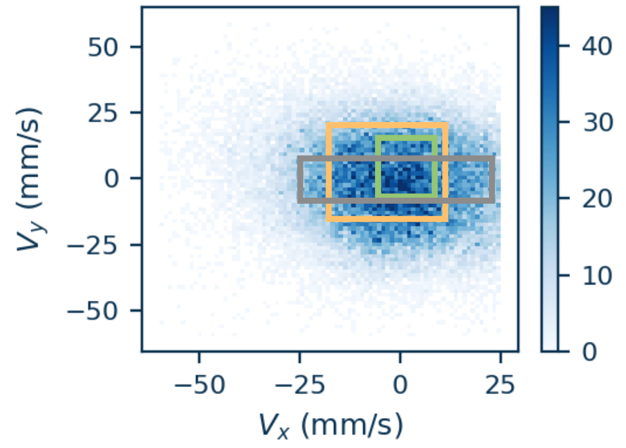
Picture of the MCP

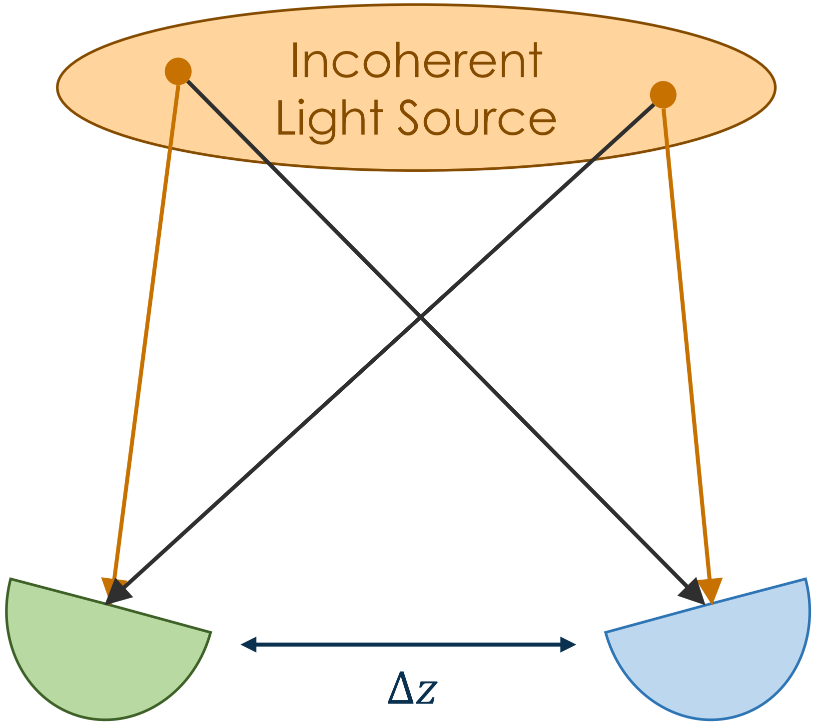
Probing (local) correlations

How to measure

$$g^{(2)}(k,q) = \langle : \hat{n}_q \hat{n}_k : \rangle / \langle \hat{n}_k \rangle \langle \hat{n}_q \rangle$$

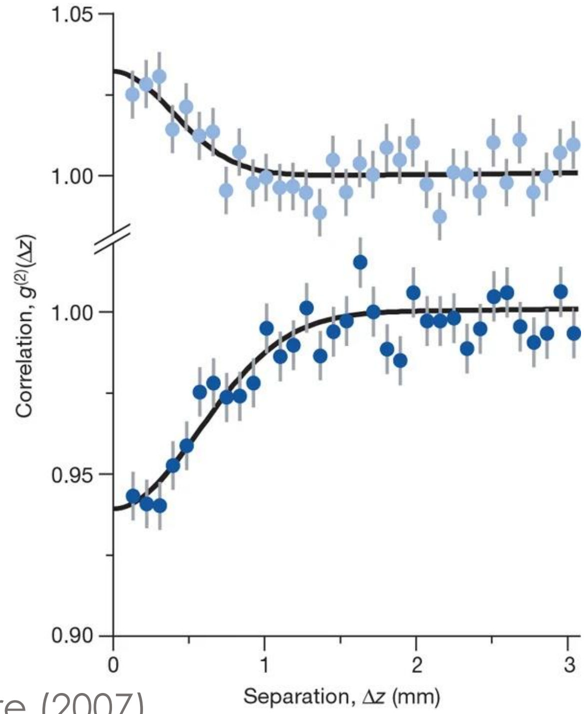
How to define a mode size ? ($2\pi/L$)





Use the HBT effect !
Bosonic bunching

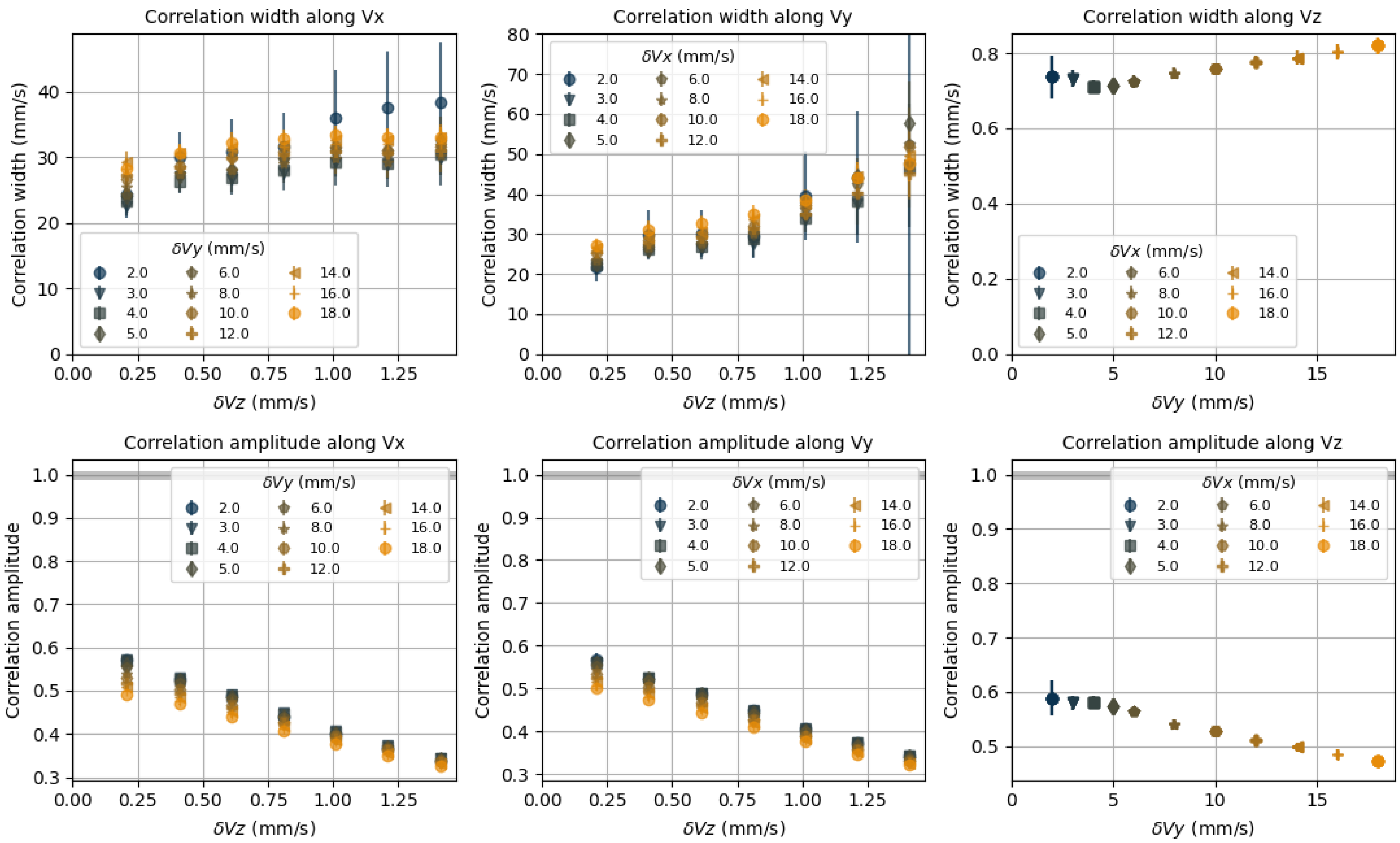
- Constructive interferences for bosons,
- Destructive interference for fermions



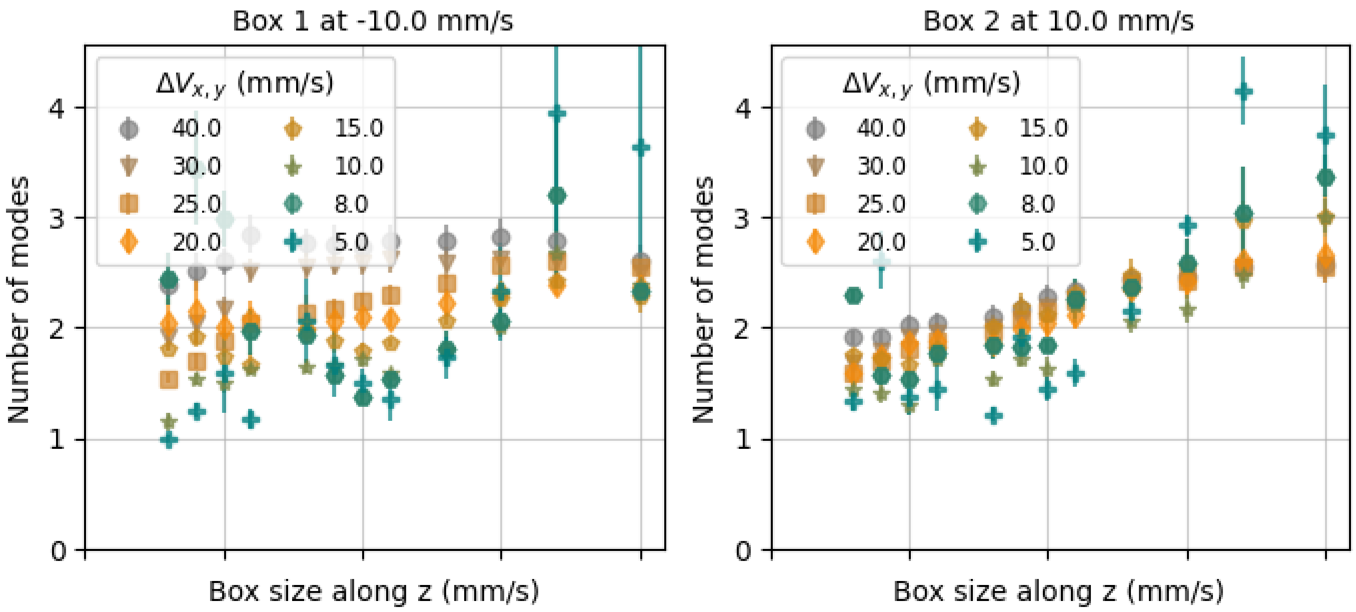
Jeltes *et al*, Nature (2007)

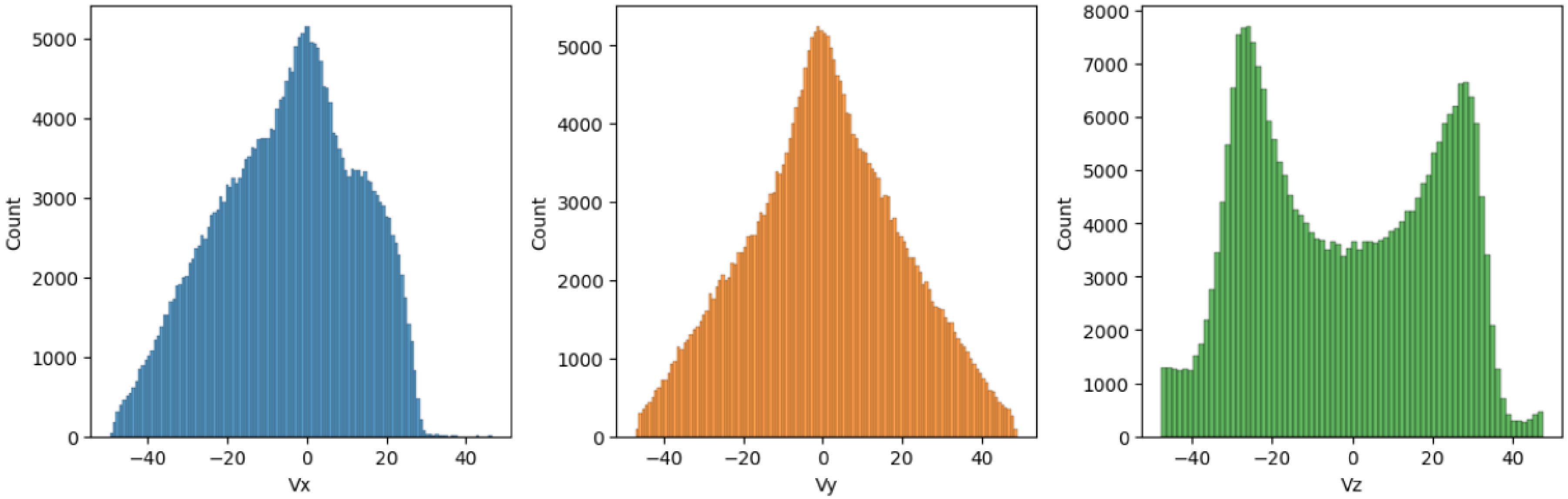
HBT : Correlation width (2)

Correlations width (up) and amplitude (down) of beam a as a function of voxel size. Fit function is lorentzian



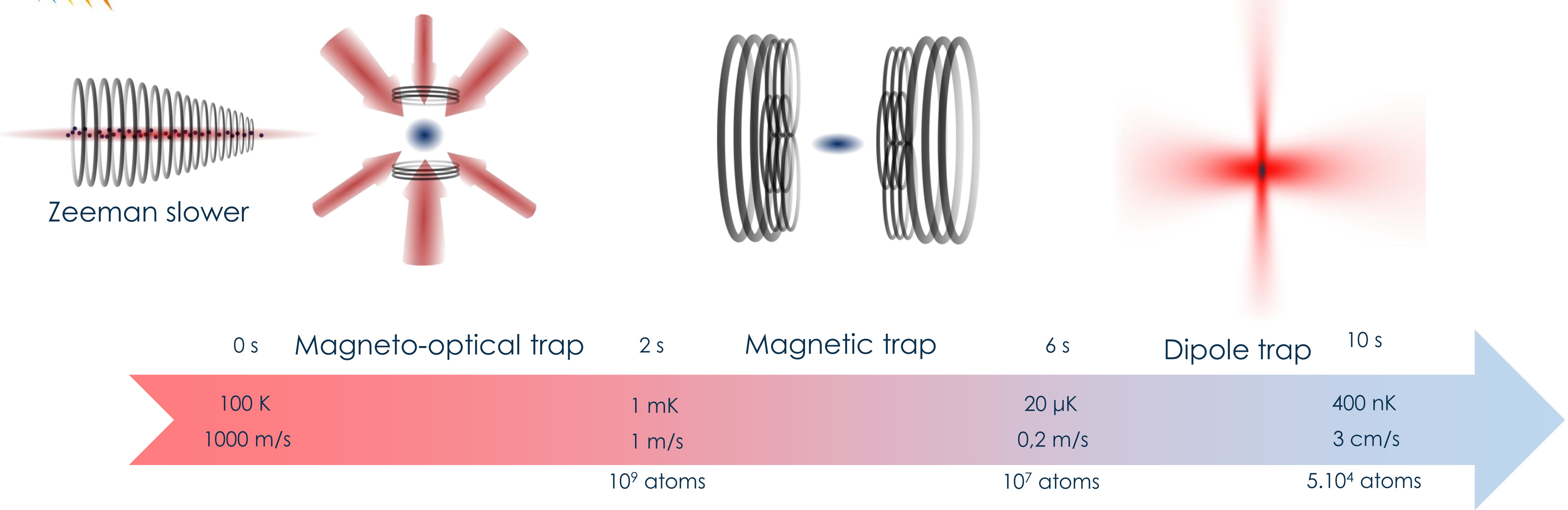
Nombre de modes et nombre d'atomes par modes pour différentes tailles de boîte.





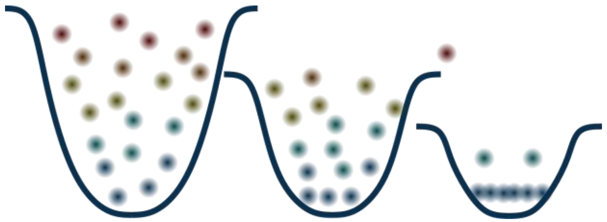
Count
Vx
Count
Vy
Count
Vz

(Not that fast) Production of He^* BEC



Plasma to excite He to metastable state : 20 eV, 2h.

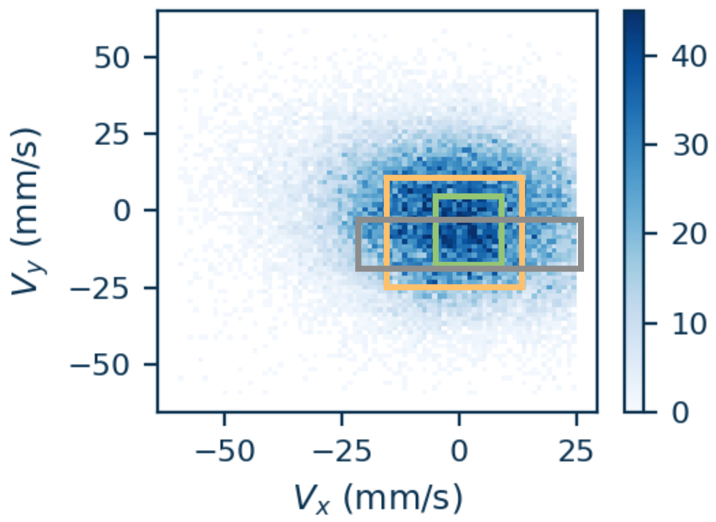
Evaporative cooling to obtain a BEC

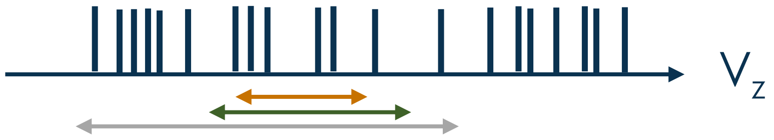


Probing (local) correlations

How to measure the number of particles in mode ? *(for counting statistics)*

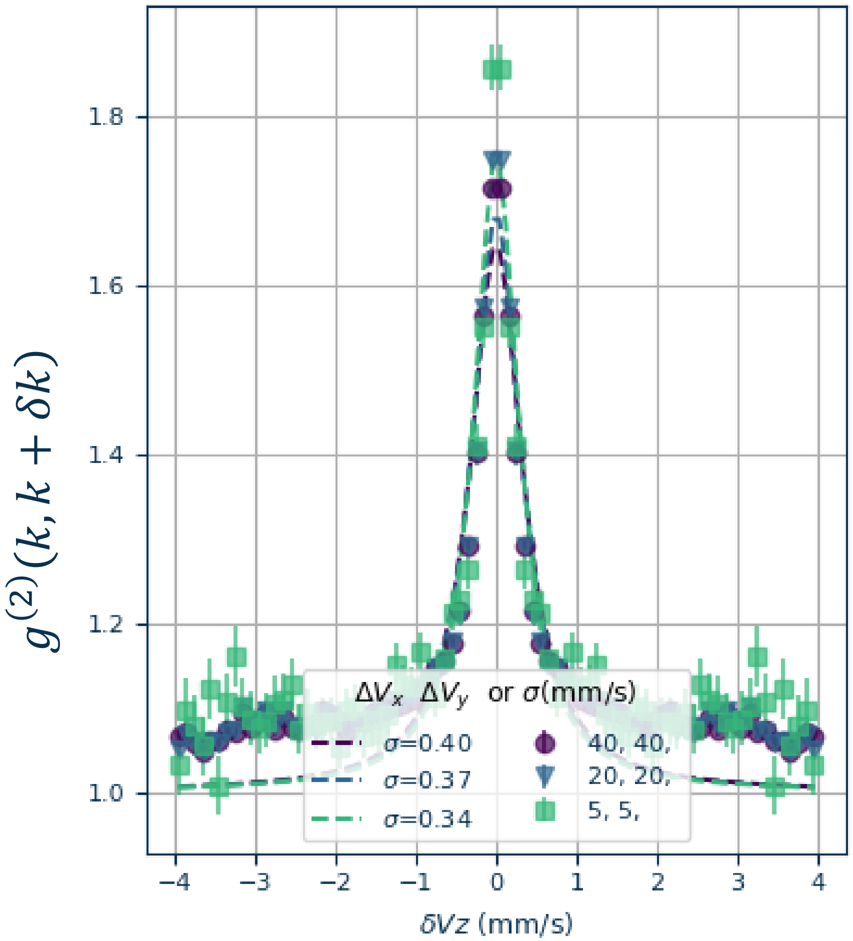
How to define a mode size ? ($2\pi/L$)





Use the HBT effect !

Bosonic bunching



Local correlations (normal)

$g^{(2)}(k,k) = 2$ in the limit where the integration volume goes to 0.

The width of the auto-correlation gives the mode size.