



# Creation and non-separability of phonon pairs in a modulated BEC

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Optique Nice 2022

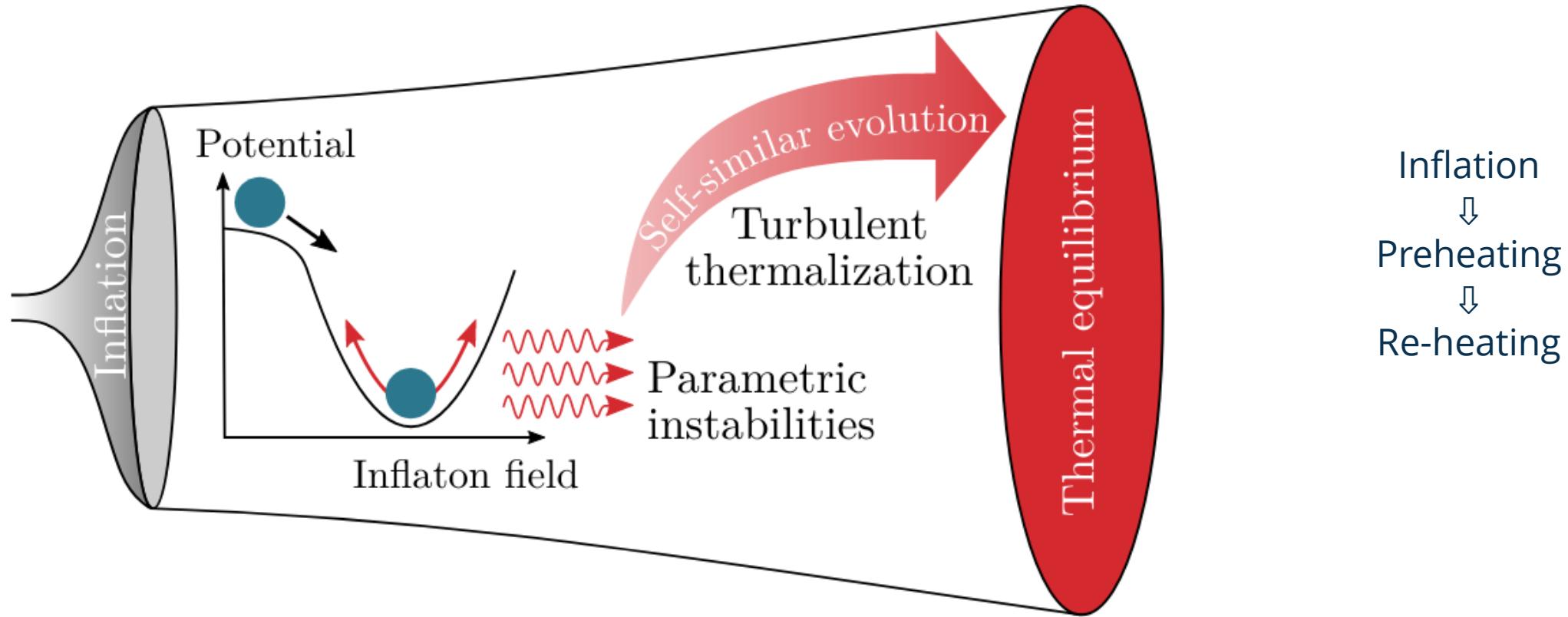


# Summary

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- ① Context
- ② Experimental setup
- ③ Phonon pair creation and non-separability of the phonon pair
- ④ First experimental result

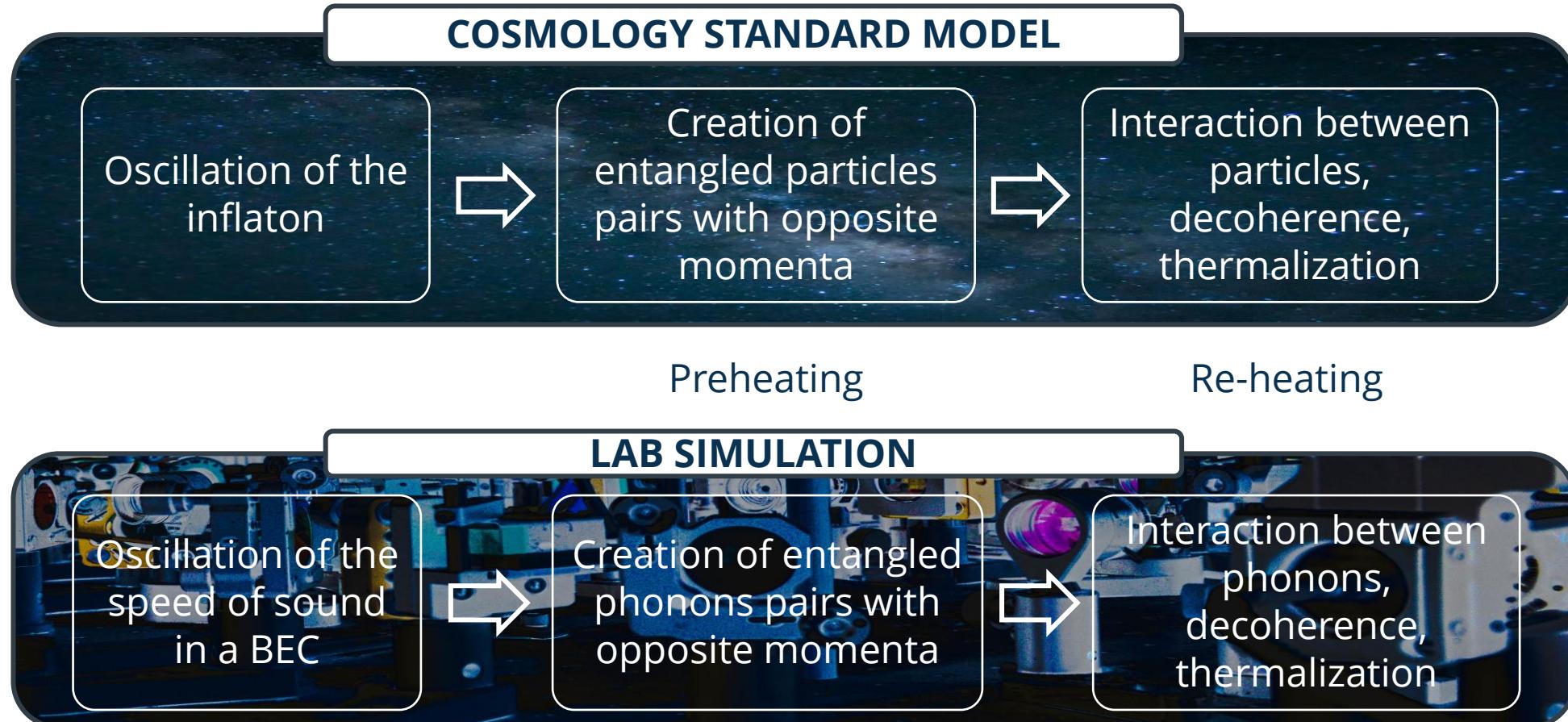
# The story of the universe in a nutshell



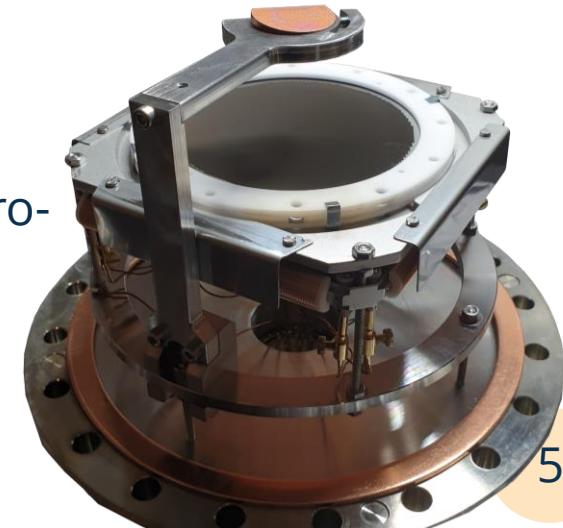
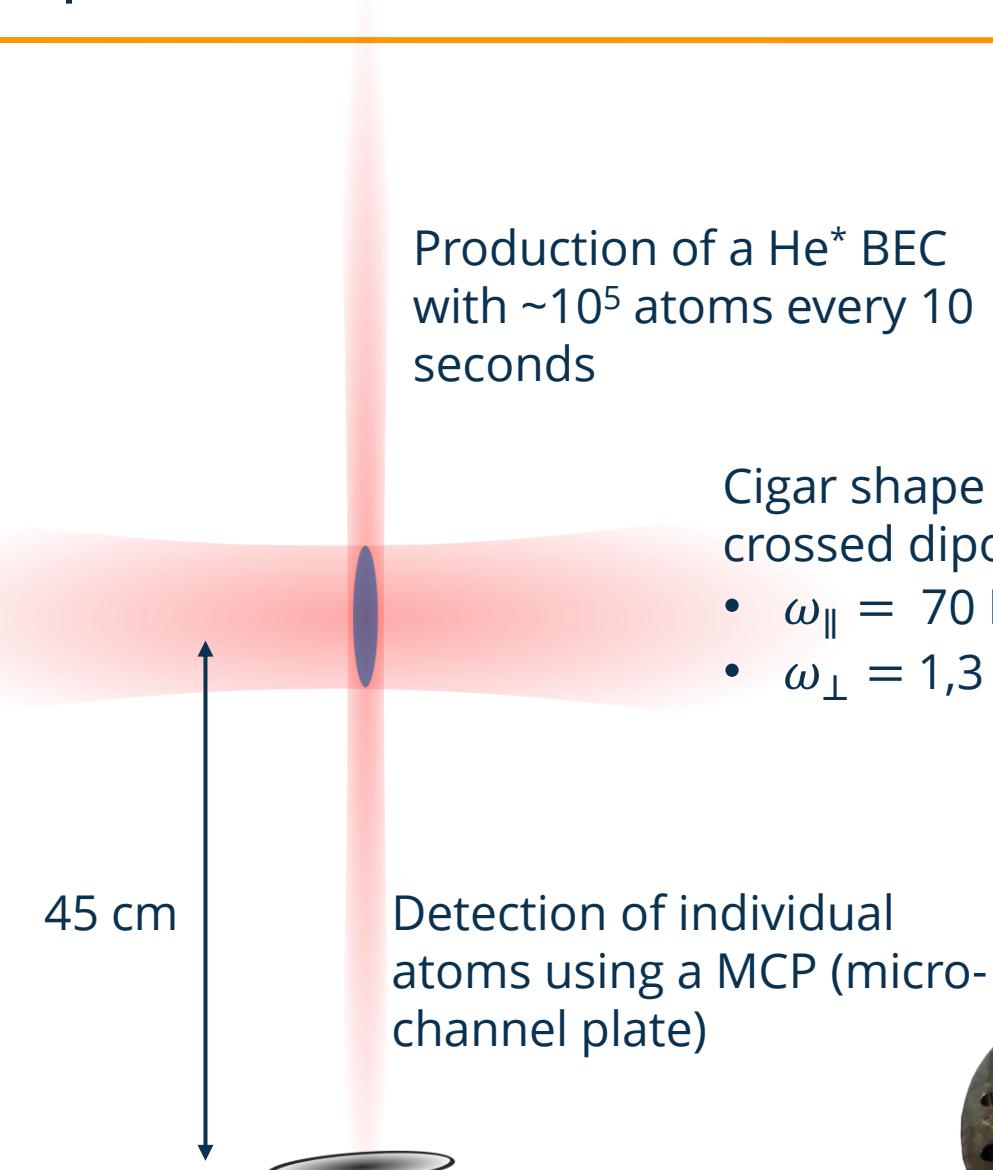
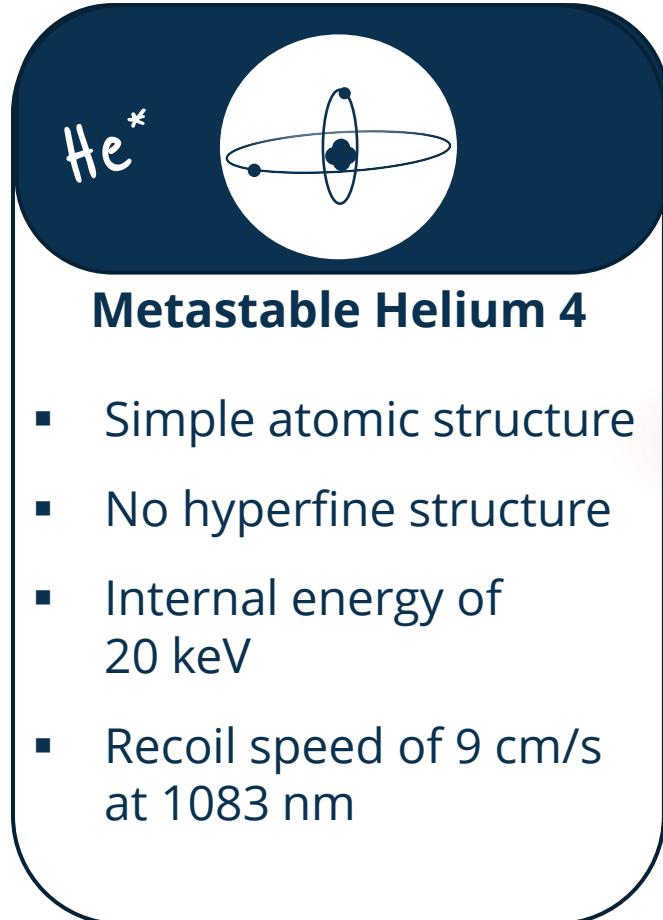
Chatrchyan *et al.*, (2021) Analog cosmological reheating in an ultracold Bose gas, Phys. Rev. A 104, 023302.

García-Bellido, J. (1999). The origin of matter and structure in the universe. *Philosophical Transactions of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences* 357.1763 (1999): 3237-3257.

# Analogy : simulating the early universe in our lab



# Experimental setup



# Phonon pair creation

Laser Dipole Trap

BEC

The physics is 1D  $\hat{\Phi} = \Phi_0(r, t) \times (1 + \hat{\phi}(z, t))$ :

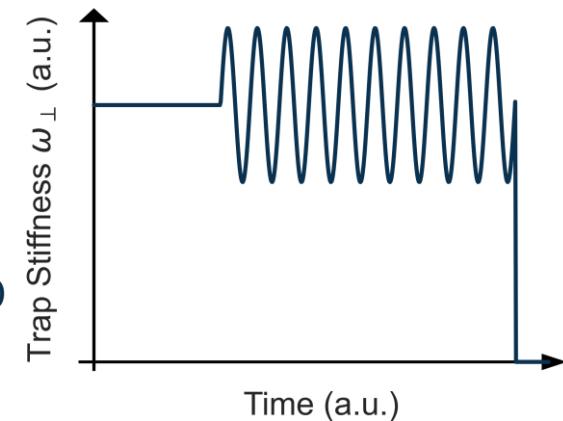
$$i\partial_t \hat{\phi} = -\frac{1}{2m} \partial_{zz} \hat{\phi} + g_1(t) n_1 (\hat{\phi} + \hat{\phi}^\dagger)$$

1D density of the BEC  
# of atoms / length

effective 1D interaction strength

Oscillation of the transverse trap frequency

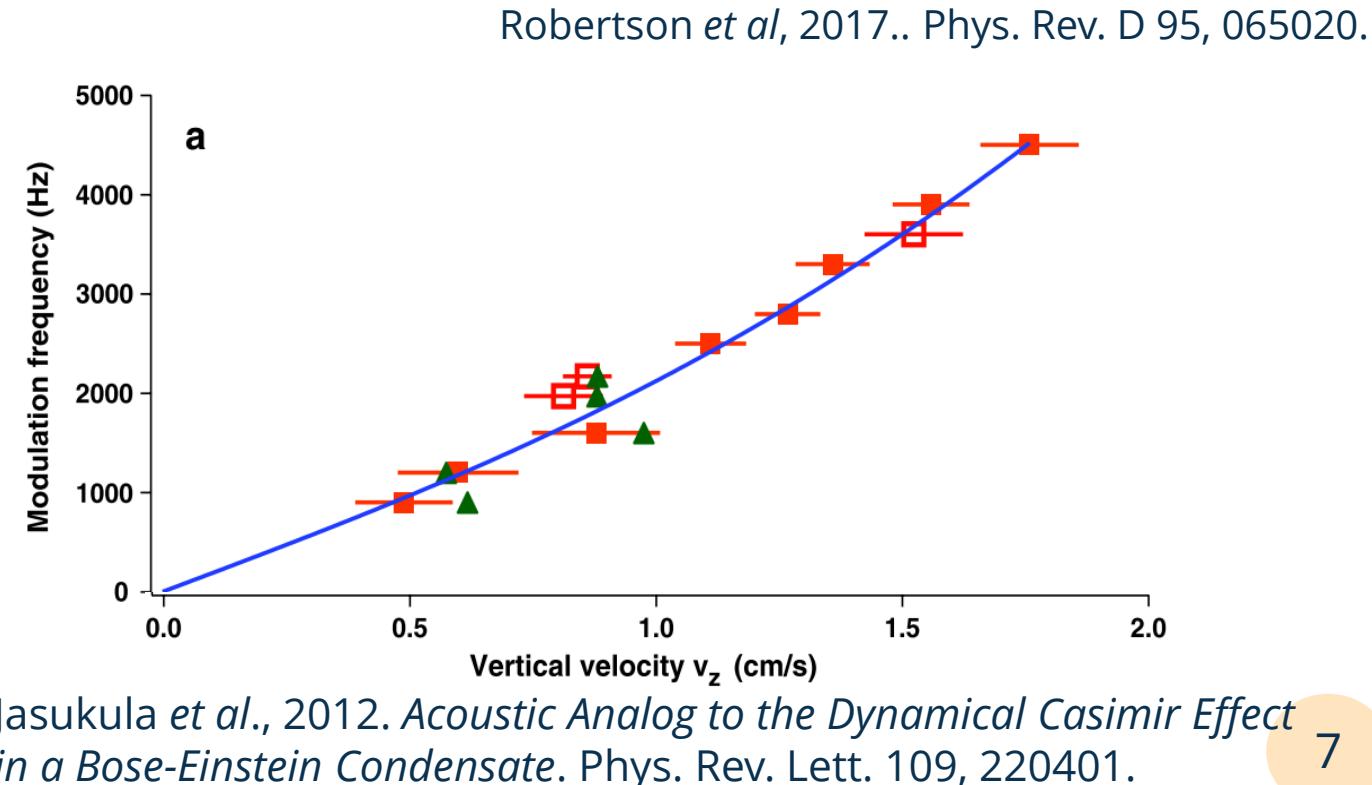
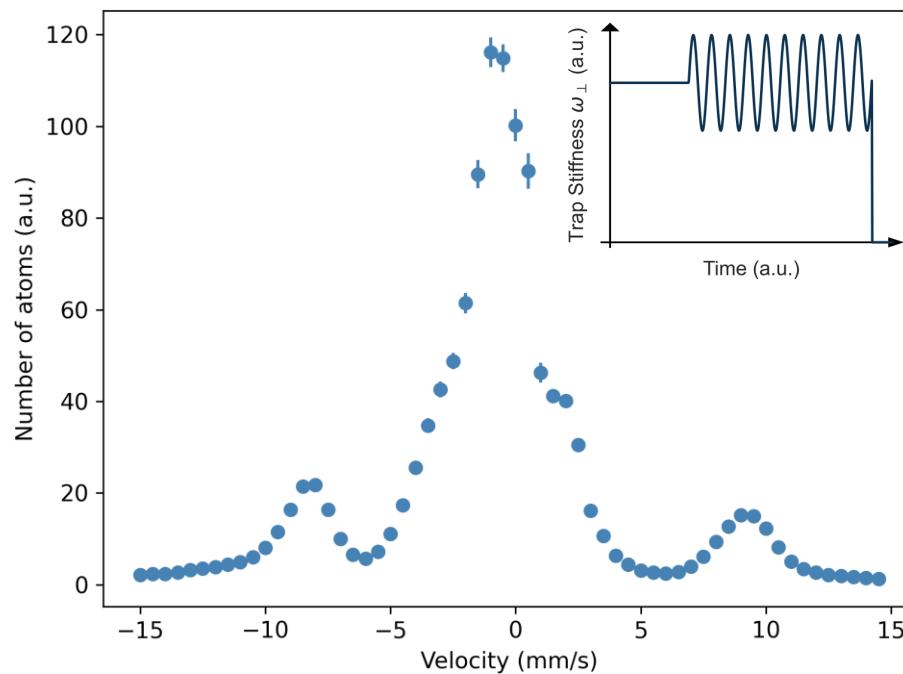
Oscillation of the effective 1D interaction strength



# Phonon pair creation

$$i\partial_t \begin{pmatrix} \hat{a}_k \\ \hat{a}_{-k}^\dagger \end{pmatrix} = \begin{pmatrix} \Omega_k & -i\partial_t\Omega_k/2\Omega_k \\ -i\partial_t\Omega_k/2\Omega_k & -\Omega_k \end{pmatrix} \begin{pmatrix} \hat{a}_k \\ \hat{a}_{-k}^\dagger \end{pmatrix} \quad \text{with } \Omega_k^2 = \frac{g_1 n_1 k^2}{m} + \left(\frac{k^2}{2m}\right)^2$$

- $\hat{a}_k$  is the annihilation operator for collective excitations (phonons or quasi-particles)
- When  $\partial_t\Omega_k = 0$ :  $k$  and  $-k$  modes evolve independently from each other
- When  $\partial_t\Omega_k \neq 0$ : mixing between  $k$  and  $-k$  modes



# Non-separability of the phonon pairs

In the experiment, we count the number of atoms arriving of the detector and compute

$$g^{(2)}(k, -k) = \langle : \hat{n}_k \hat{n}_{-k} : \rangle / \langle \hat{n}_k \rangle \langle \hat{n}_{-k} \rangle$$

Noting that  $\hat{n}_k = \hat{a}_k^\dagger \hat{a}_k$  and using Wick contraction

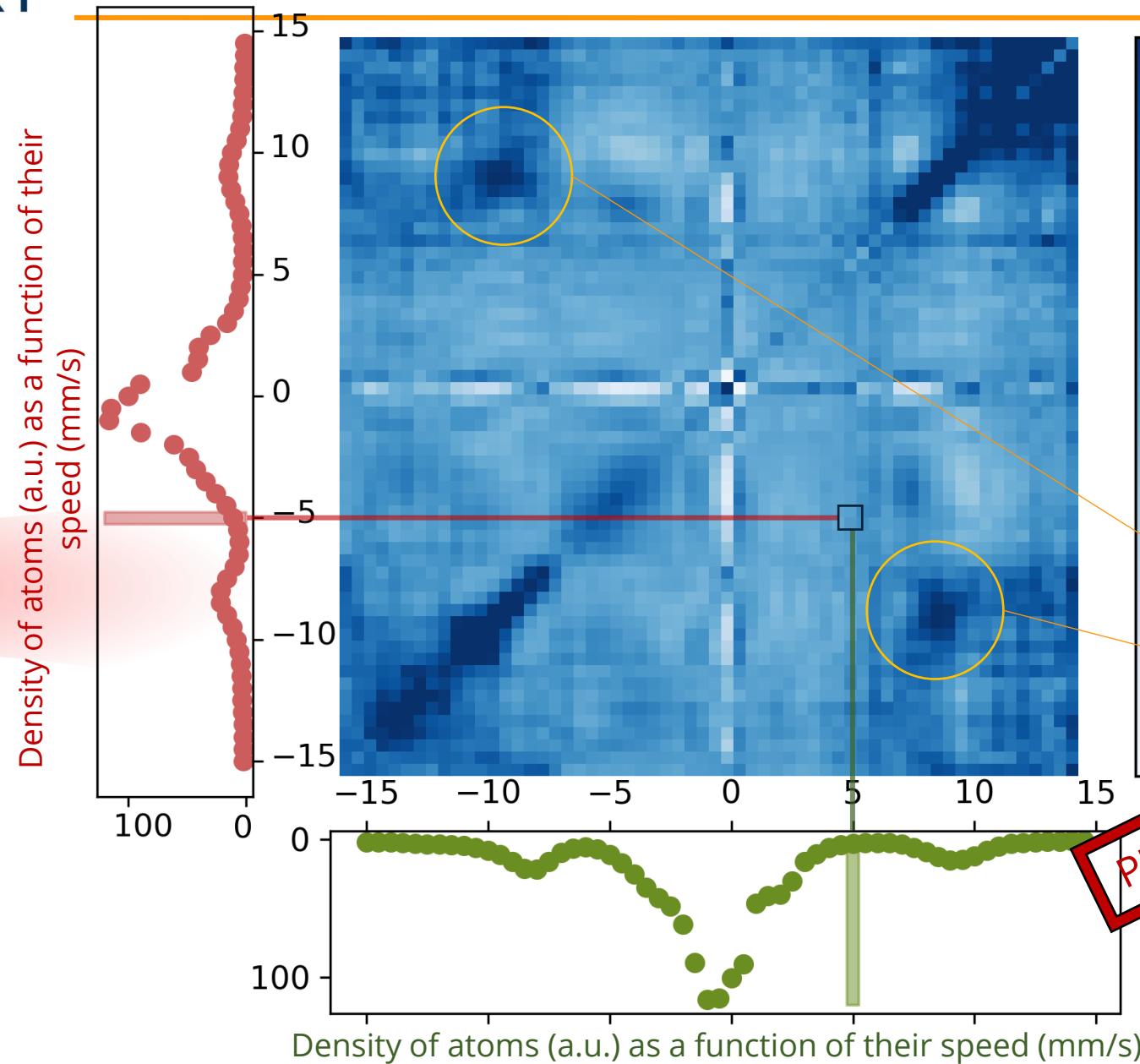
$$\langle : \hat{n}_k \hat{n}_{-k} : \rangle = \langle : \hat{a}_k^\dagger \hat{a}_{-k}^\dagger \hat{a}_k \hat{a}_{-k} : \rangle = n_k n_{-k} + |\langle \hat{a}_k \hat{a}_{-k} \rangle|^2 + \cancel{|\langle \hat{a}_k^\dagger \hat{a}_{-k} \rangle|^2}$$

if the state is separable :  $\underbrace{\leq n_k n_{-k}}$

## Non separability criteria

$$g^{(2)}(k, -k) > 2$$

# First experimental result



2D color map of the normalized second order correlation function  $g^{(2)}(k_1, k_2)$

$$g^{(2)}(k_1, k_2) = \frac{\langle n_{k_1} n_{k_2} \rangle}{\langle n_{k_1} \rangle \langle n_{k_2} \rangle}$$

$\langle \dots \rangle$  = average over experimental realisations

PRELIMINARY

Correlations !

# Perspectives

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- Longer acquisition
- Try to modulate the effective mass of the atoms using a lattice
- Study the creation dynamics of the phonon pairs
- Check the non-separability criteria assumptions : Bragg diffraction + interferometer
- Study the time evolution of the phonon pair correlations / entanglement, the time evolution of the re-thermalization

# Thank you for your time !

## Some lecture

- Jaskula *et al.*, 2012. *Acoustic Analog to the Dynamical Casimir Effect in a Bose-Einstein Condensate*. Phys. Rev. Lett. 109, 220401.
- Robertson, Michel & Parentani, 2017. *Controlling and observing nonseparability of phonons created in time-dependent 1D atomic Bose condensates*. Phys. Rev. D 95, 065020
- Robertson, Michel & Parentani 2018. *Nonlinearities induced by parametric resonance in effectively 1D atomic Bose condensates*. Phys. Rev. D 98, 056003.
- Micheli & Robertson, 2022. *Phonon decay in 1D atomic Bose quasicondensates via Beliaev-Landau damping*. ArXiv.

## Fundings



On  
the  
experimental  
side



On  
the  
theory  
side

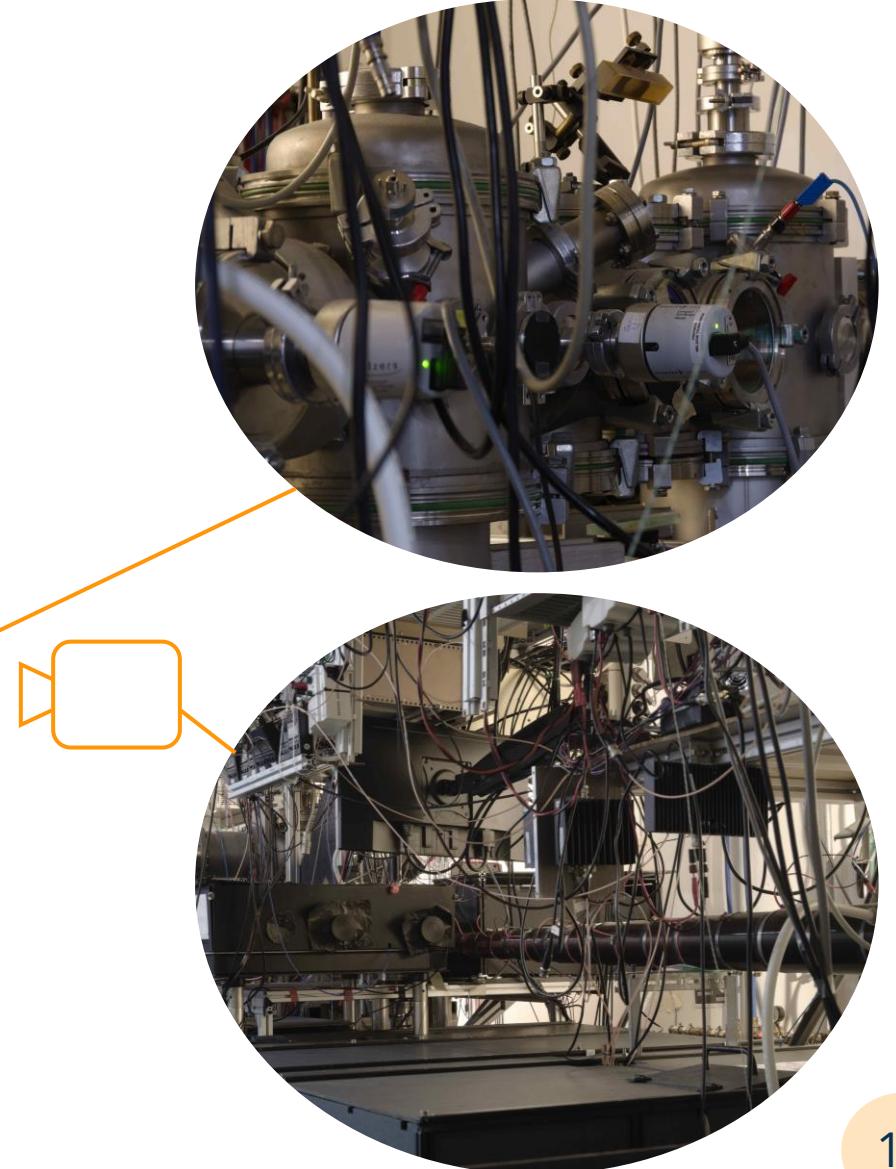
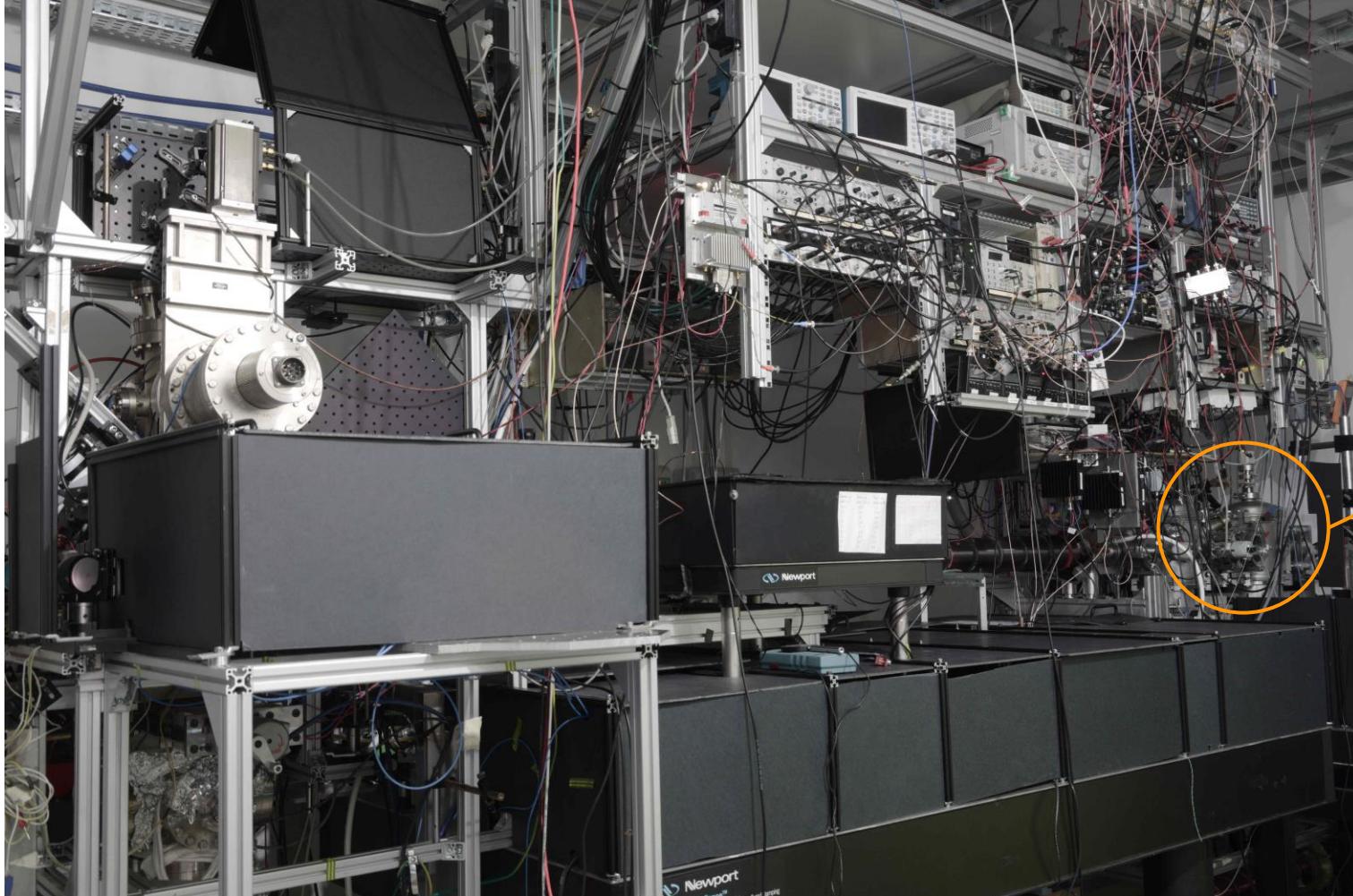


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# Experimental setup



# From the Gross-Pitaevski to Bogoliubov-de Gennes

Laser Dipole Trap

$$\omega_{\perp} \propto \sqrt{I_{\text{LASER}}}$$

BEC

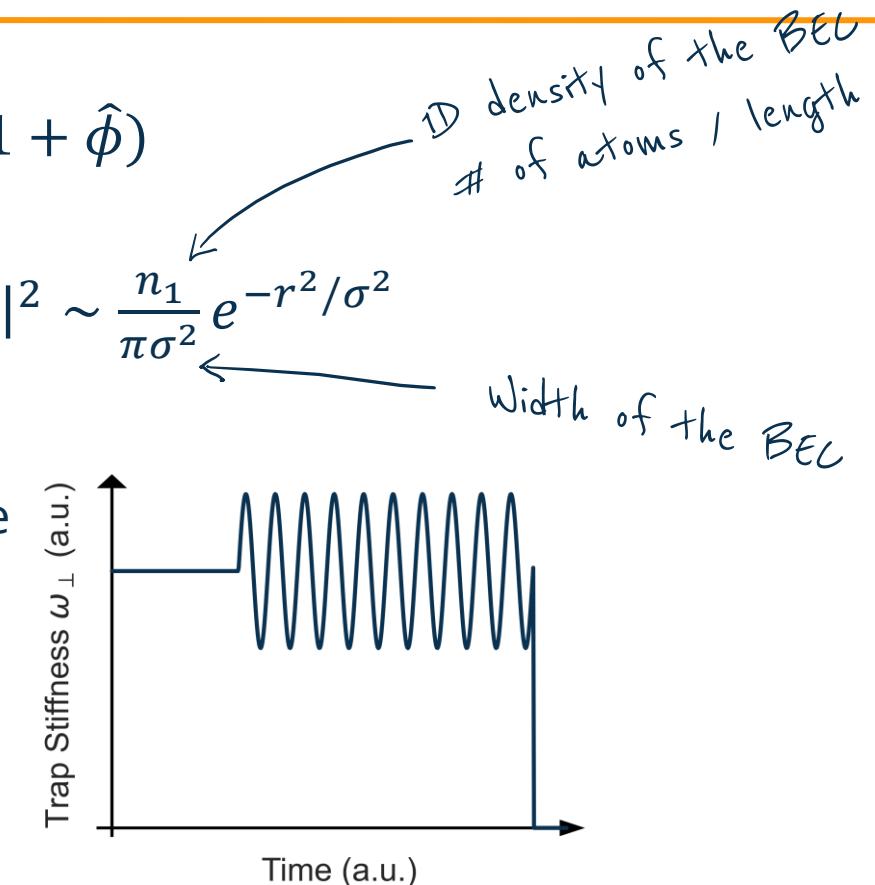
Describe the BEC as  $\hat{\Phi} = \Phi_0(1 + \hat{\phi})$

Ansatz for the density :  $|\Phi_0(r, t)|^2 \sim \frac{n_1}{\pi\sigma^2} e^{-r^2/\sigma^2}$

Oscillation of the transverse trap frequency

Oscillation of the BEC transverse width

The perturbative field follows the Bogoliubov – de Gennes equation



$$i\partial_t \hat{\phi} = -\frac{1}{2m} \partial_{zz} \hat{\phi} + g_1 n_1 (\hat{\phi} + \hat{\phi}^\dagger)$$

$$g_1(t) \sim 1/\sigma^2(t)$$

Robertson, S., Michel, F., Parentani, R., 2017. Controlling and observing nonseparability of phonons created in time-dependent 1D atomic Bose condensates. Phys. Rev. D 95, 065020.

# Phonon pair creation

$$i\partial_t \hat{\phi} = -\frac{1}{2m} \partial_{zz} \hat{\phi} + g_1(t) n_1 (\hat{\phi} + \hat{\phi}^\dagger)$$



1. Fourier Transform
2. Bogoliubov transformation

$$i\partial_t \begin{pmatrix} \hat{a}_k \\ \hat{a}_{-k}^\dagger \end{pmatrix} = \begin{pmatrix} \Omega_k & -i\partial_t \Omega_k / 2\Omega_k \\ -i\partial_t \Omega_k / 2\Omega_k & -\Omega_k \end{pmatrix} \begin{pmatrix} \hat{a}_k \\ \hat{a}_{-k}^\dagger \end{pmatrix} \quad \text{with } \Omega_k^2 = \frac{g_1 n_1 k^2}{m} + \left(\frac{k^2}{2m}\right)^2$$

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- When  $\partial_t \Omega_k = 0$ :  $k$  and  $-k$  modes evolve independently from each other
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# Phonon pair creation

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- $\hat{a}_k$  is the annihilation operator for collective excitations (phonons)
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Why do we **create phonons**?

Define the number of phonons with momentum  $k$ :  $n_k \equiv \langle \hat{a}_k^\dagger \hat{a}_k \rangle$

$$\hat{a}_k(t) = \alpha(t) \times \hat{a}_k(0) + \beta(t) \times \hat{a}_{-k}^\dagger(0)$$

$\overbrace{\neq 0}$  if  $\partial_t\Omega_k \neq 0$

# Phonon pair creation

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$$i\partial_t \begin{pmatrix} \hat{a}_k \\ \hat{a}_{-k}^\dagger \end{pmatrix} = \begin{pmatrix} \Omega_k & -i\partial_t\Omega_k/2\Omega_k \\ -i\partial_t\Omega_k/2\Omega_k & -\Omega_k \end{pmatrix} \begin{pmatrix} \hat{a}_k \\ \hat{a}_{-k}^\dagger \end{pmatrix} \quad \text{with } \Omega_k^2 = \frac{g_1 n_1 k^2}{m} + \left(\frac{k^2}{2m}\right)^2$$

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Why do we create **pairs** of phonons ?

Define the number of phonons with momentum  $k$  :  $n_k \equiv \langle \hat{a}_k^\dagger \hat{a}_k \rangle$

Show that

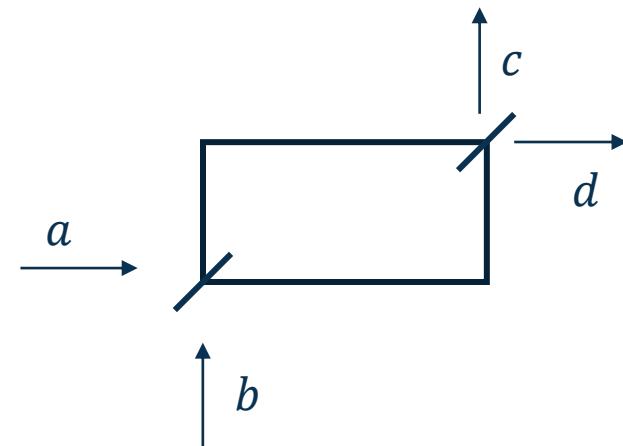
$$\partial_t(n_k - n_{-k}) = 0$$

## Supplement : checking the non-separability criteria

$$\langle \hat{n}_k \hat{n}_{-k} \rangle = \langle \hat{b}_k^\dagger \hat{b}_{-k}^\dagger \hat{b}_k \hat{b}_{-k} \rangle = n_k n_{-k} + |\langle \hat{b}_k \hat{b}_{-k} \rangle|^2 + |\langle \hat{b}_k^\dagger \hat{b}_{-k} \rangle|^2$$

if the state is separable :  $\leq \overbrace{n_k n_{-k}}^{0 \text{ ? ? ? ?}}$

$$c = \cos(\phi) a + \sin(\phi) b$$



$$n_c = \langle c^\dagger c \rangle = \cos(\phi)^2 a^\dagger a + \sin(\phi)^2 b^\dagger b + \cos \phi \sin \phi (a^\dagger b + b^\dagger a)$$

→ Check that this term is zero.