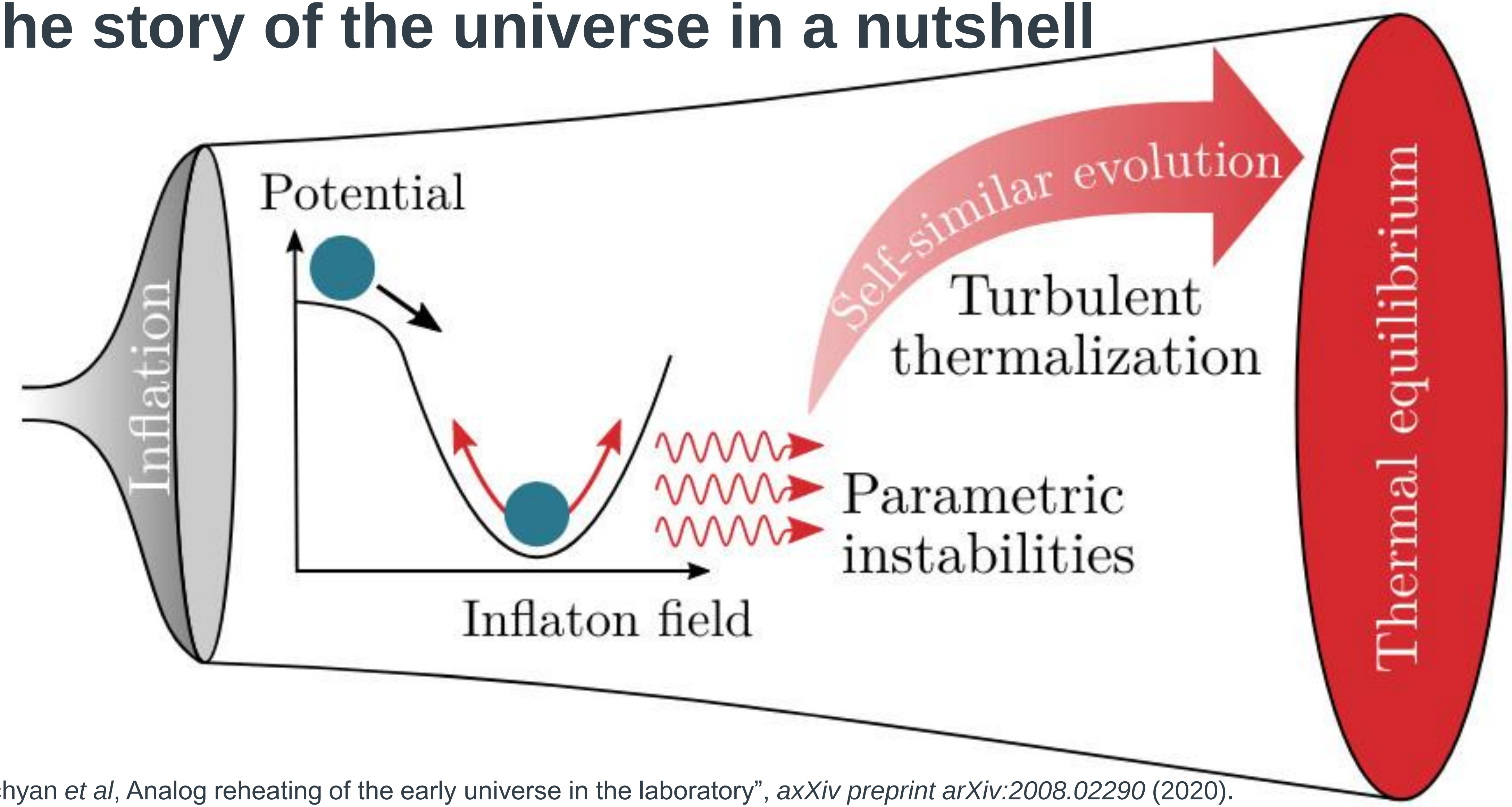
Creation and non-separability of phonon pairs in a modulated BEC

Victor Gondret

Quantum gases group meeting of July, 05th of 2021

The story of the universe in a nutshell



Chatrchyan *et al*, Analog reheating of the early universe in the laboratory", *axXiv preprint arXiv:2008.02290* (2020).
García–Bellido, J. (1999). The origin of matter and structure in the universe. *Philosophical Transactions of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences* 357.1763 (1999): 3237-3257.

The story of the universe in a nutshell

Inflation

The density of the universe drops down

The inflaton starts to oscillate

Preheating

Decay of the inflaton field into pairs of entangled particles

Particles interact

Reheating

Decoherence, redistribution, thermalization

The story of the universe in a nutshell

13 × 10⁹B.C.

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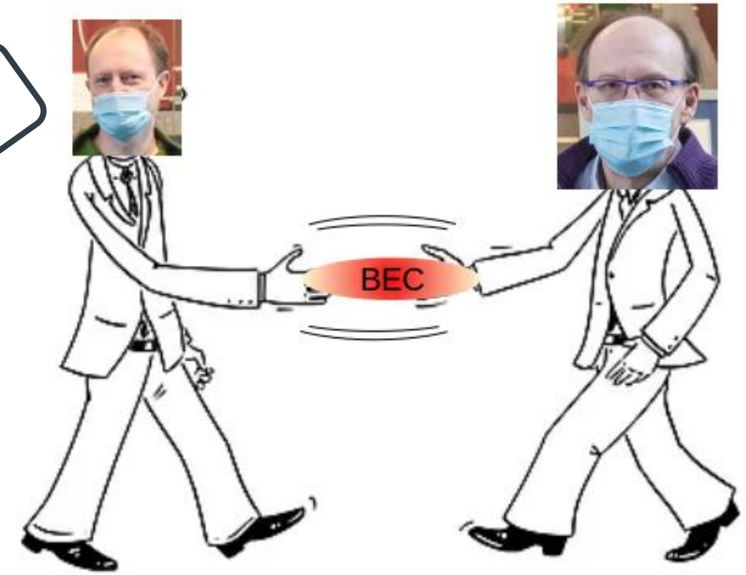
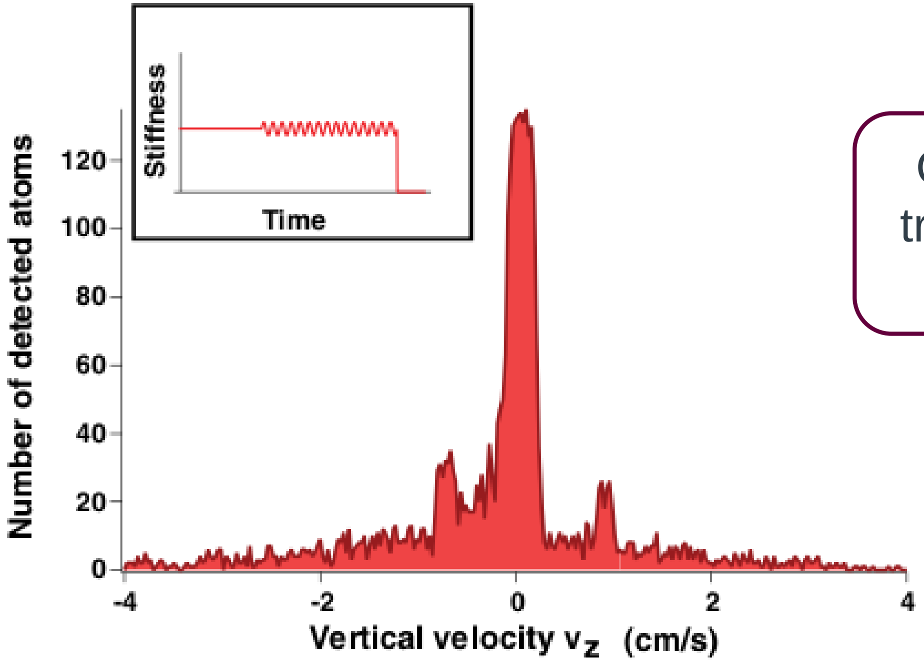
2012 A.D.





Jaskula,*et al*. (2012). PRL *109*(22), 220401.

Oscillations of the transverse trapping frequency $\omega_\perp$

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13×10^9 B.C.

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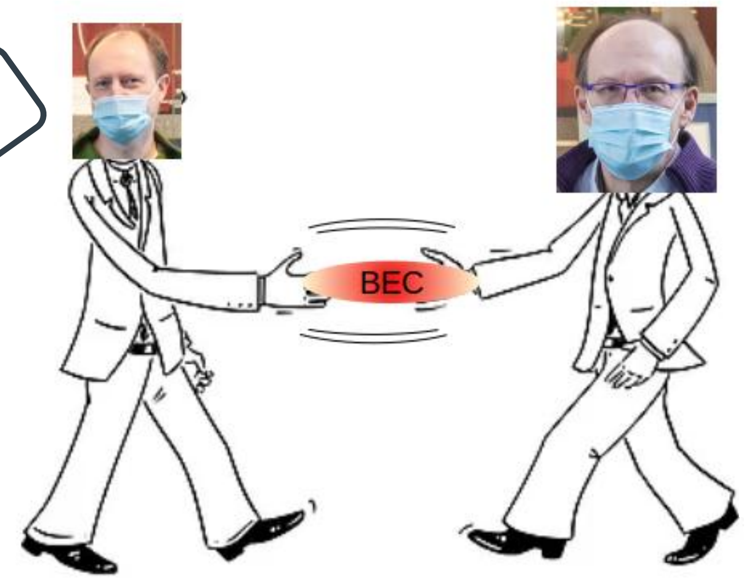
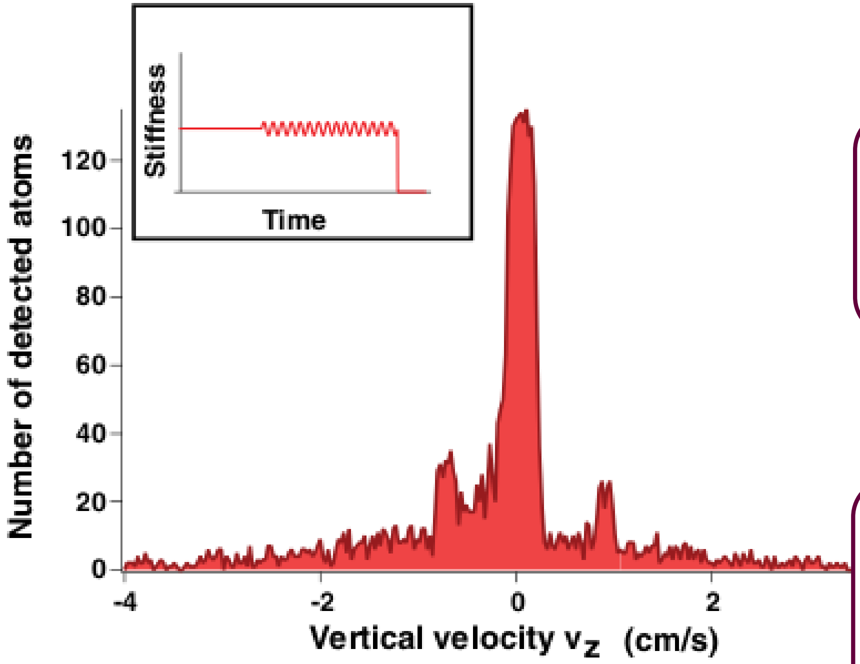




Jaskula, *et al*. (2012). PRL *109*(22), 220401.

Oscillations of the transverse trapping frequency $\omega_\perp$

Creation of phonons pairs with opposite momenta

Simulating the early universe in the lab (no exaggeration of course…)

COSMOLOGY STANDARD MODEL

Oscillation of the inflaton → Creation of entangled particles pairs with opposite momenta → Interaction between particles, decoherence, thermalization

preheating *reheating*

LAB SIMULATION

Oscillation of the speed of sound in a BEC → Creation of entangled phonon pairs with opposite momenta → Interaction between phonons, decoherence, thermalization

Outline of the talk

Describing the background BEC

Phonon pair creation

Non separability of the phonon pair

Describing the background BEC

Decompose the field as

$$\hat{\Phi} = \Phi_0(1 + \hat{\phi})$$

where the mean field Φ_0 obeys the GP equation

$$i\partial_t \Phi_0 = -\frac{1}{2m}\nabla^2\Phi_0 + V\,\Phi_0 + g|\Phi_0|^2\Phi_0.$$

Assumptions :

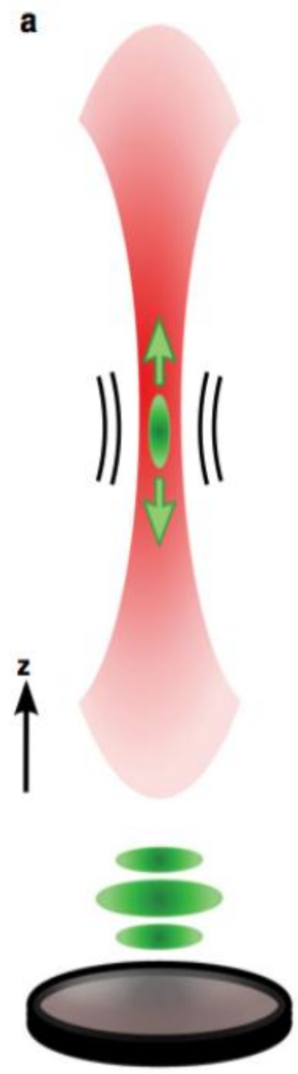
- $\omega_z = 0$ → condensate homogeneous in z of size L
- ansatz for the atom density

$$|\Phi_0(r,t)|^2 \sim \frac{n_1}{\pi\sigma^2}e^{-r^2/\sigma^2}$$

where n_1 is the constant linear density N/L.

Valid for $n_1 a_s \to 0$,with the scattering length $a_s = mg/4\pi$

Gerbier F., “Quasi-1D Bose-Einstein condensates in the dimensional crossover regime.” *EPL (Europhysics Letters) 66,6 (2004):771.*



$$V(r,z) = \frac{1}{2}m(\omega_\perp^2 r^2 + \omega_z^2 z^2)$$

with $\omega_\perp \gg \omega_z$

Describing the background BEC

Describe the condensed WF

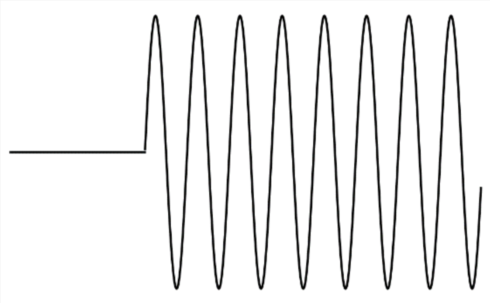
Plug the ansatz $|\Phi_0(r,t)|^2 \sim \frac{n_1}{\pi\sigma^2} e^{-r^2/\sigma^2}$ into the GP equation. When $\omega_\perp$ varies, σ does also as

$$m\ddot{\sigma} = -\partial_\sigma U(\sigma) \qquad \text{with} \quad U(\sigma) = \frac{m\omega_\perp^2(t)}{2}\sigma^2 + \frac{1+4n_1 a_s}{2m\sigma^2}$$

Shake it

$$\omega_\perp(t) = \begin{cases} \omega_{\perp,0} & \text{for } t < 0 \\ \omega_{\perp,0}(1 + A\sin\omega t) & \text{for } t > 0 \end{cases}$$

$\omega_\perp(t)$



Obtain $\sigma(t)$ and $\Phi_0(r,t)$

Describing the background BEC

Describe the condensed WF

Plug the ansatz $|\Phi_0(r,t)|^2 \sim \frac{n_1}{\pi\sigma^2} e^{-r^2/\sigma^2}$ into the GP equation. When $\omega_\perp$ varies, σ does also as

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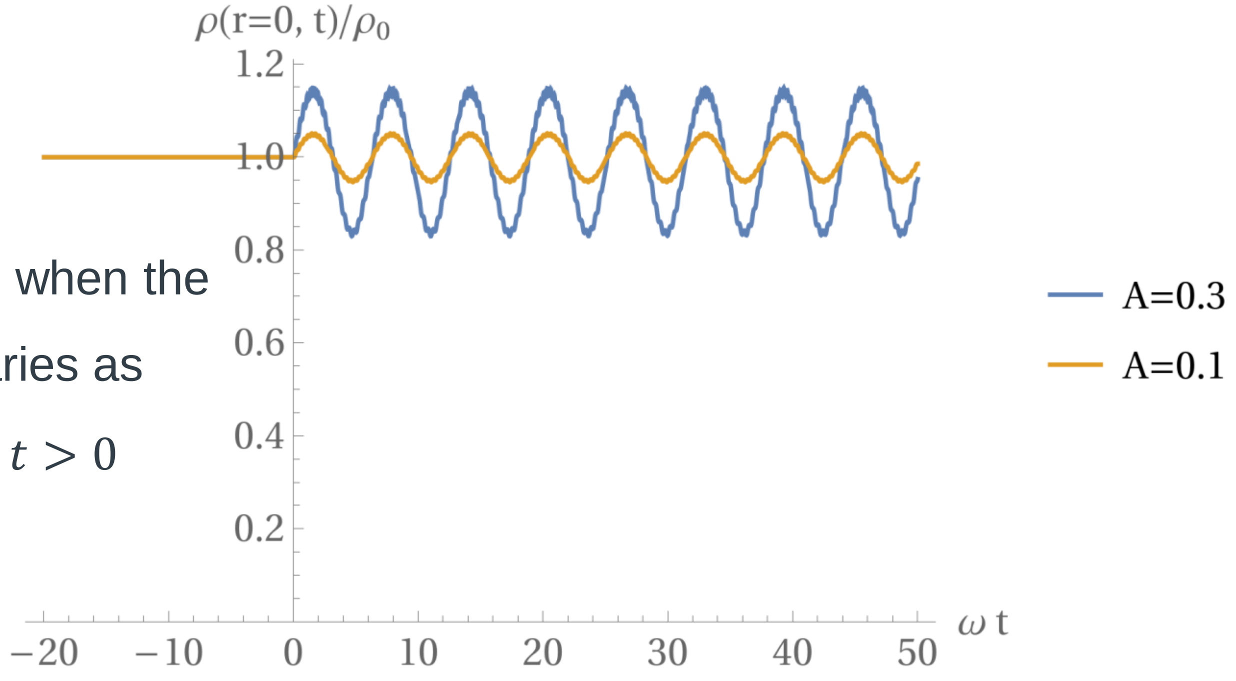
Density response of the condensate when the transverse trapping frequency varies as

$$\omega_\perp(t) = \omega_{\perp,0}(1 + A\sin\omega t) \quad \text{for} \;\; t > 0$$



Describing the background BEC

Describe the condensed WF

Plug the ansatz $|\Phi_0(r,t)|^2 \sim \frac{n_1}{\pi\sigma^2} e^{-r^2/\sigma^2}$ into the GP equation. When $\omega_\perp$ varies, σ does also as

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Obtain the BdG equation for longitudinal excitations

Come back to the total field $\hat{\Phi} = \Phi_0\left(1 + \hat{\phi}(z,t)\right)$

1. Plug $\hat{\Phi}$ into the GP equation and keep only 1st order terms in $\hat{\phi}(z,t)$
2. Integrate over r and obtain the Bogoliubov-de Gennes equation for $\hat{\phi}$:

$$i\partial_t\hat{\phi} = -\frac{1}{2m}\partial_{zz}\hat{\phi} + g_1 n_1(\hat{\phi} + \hat{\phi}^\dagger)$$

where $g_1(t) = g/2\pi\sigma^2(t)$

Robertson *et al*, Phys. Rev. D95, 065020 (2017)

Outline of the talk

Describing the background BEC

Phonon pair creation

Non separability of the phonon pair

Shake the system transversally at frequency ω

$\downarrow$

Perturbation field $\hat{\phi}(z,t)$ follows

$$i\partial_t\hat{\phi} = -\frac{1}{2m}\partial_{zz}\hat{\phi} + g_1 n_1(\hat{\phi} + \hat{\phi}^\dagger)$$

with modulated interaction strength g_1 at ω

Phonon pair creation

Start with the Bogoliubov de Gennes equation,

$$i\partial_t\hat{\phi} = -\frac{1}{2m}\partial_{zz}\hat{\phi} + g_1(t)n_1(\hat{\phi} + \hat{\phi}^\dagger)$$

Fourier transform with $\hat{\phi} = \sum_k \hat{\phi}_k\, e^{ikz}$ with $k \in 2\pi\mathbb{Z}/L$.

$$\mathrm{i}\partial_t\begin{pmatrix}\hat{\phi}_k \\ \hat{\phi}^\dagger_{-k}\end{pmatrix} = \begin{pmatrix}\frac{k^2}{2m} + g_1n_1 & g_1n_1 \\ -g_1n_1 & -\frac{k^2}{2m} - g_1n_1\end{pmatrix}\begin{pmatrix}\hat{\phi}_k \\ \hat{\phi}^\dagger_{-k}\end{pmatrix}$$

$\hat{\phi}_k$ annihilation operator for atoms

and perform Bogoliubov transformation to deal with phonons

$$\begin{pmatrix}\hat{\phi}_k \\ \hat{\phi}^\dagger_{-k}\end{pmatrix} = \begin{pmatrix}u_k & v_k \\ v_k & u_k\end{pmatrix}\begin{pmatrix}\hat{b}_k \\ \hat{b}^\dagger_{-k}\end{pmatrix}$$

$\hat{b}_k$ annihilation operator for phonons

Phonon pair creation

$$i\partial_t \begin{pmatrix} \hat{b}_k \\ \hat{b}^\dagger_{-k} \end{pmatrix} = \begin{pmatrix} \Omega_k & -i\partial_t\Omega_k/2\Omega_k \\ -i\partial_t\Omega_k/2\Omega_k & -\Omega_k \end{pmatrix} \begin{pmatrix} \hat{b}_k \\ \hat{b}^\dagger_{-k} \end{pmatrix} \qquad \text{with} \quad \Omega_k^2 = \frac{g_1 n_1 k^2}{m} + \left(\frac{k^2}{2m}\right)^2$$

- $\hat{b}_k$ is the annihilation operator for collective excitations (phonons)
- When $\partial_t\Omega_k = 0$: k and $-k$ modes evolve independently from each other
- When $\partial_t\Omega_k \neq 0$: mixing between k and $-k$ modes

Phonon pair creation

$$i\partial_t \begin{pmatrix} \hat{b}_k \\ \hat{b}^\dagger_{-k} \end{pmatrix} = \begin{pmatrix} \Omega_k & -i\partial_t\Omega_k/2\Omega_k \\ -i\partial_t\Omega_k/2\Omega_k & -\Omega_k \end{pmatrix} \begin{pmatrix} \hat{b}_k \\ \hat{b}^\dagger_{-k} \end{pmatrix} \qquad \text{with} \quad \Omega_k^2 = \frac{g_1 n_1 k^2}{m} + \left(\frac{k^2}{2m}\right)^2$$

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Why do we **create phonons**?

Phonon pair creation

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Why do we **create phonons**?

Define the number of phonons with momentum k : $n_k \equiv \langle \hat{b}_k^\dagger \hat{b}_k \rangle$

$$\hat{b}_k(t) = \alpha(t) \times \hat{b}_k(0) + \underbrace{\beta(t)}_{\neq 0 \text{ if } \partial_t\Omega_k \neq 0} \times \hat{b}^\dagger_{-k}(0)$$

Phonon pair creation

$$i\partial_t \begin{pmatrix} \hat{b}_k \\ \hat{b}^\dagger_{-k} \end{pmatrix} = \begin{pmatrix} \Omega_k & -i\partial_t\Omega_k/2\Omega_k \\ -i\partial_t\Omega_k/2\Omega_k & -\Omega_k \end{pmatrix} \begin{pmatrix} \hat{b}_k \\ \hat{b}^\dagger_{-k} \end{pmatrix} \qquad \text{with} \quad \Omega_k^2 = \frac{g_1 n_1 k^2}{m} + \left(\frac{k^2}{2m}\right)^2$$

- $\hat{b}_k$ is the annihilation operator for collective excitations (phonons)
- When $\partial_t\Omega_k = 0$: k and $-k$ modes evolve independently from each other
- When $\partial_t\Omega_k \neq 0$: mixing between k and $-k$ modes

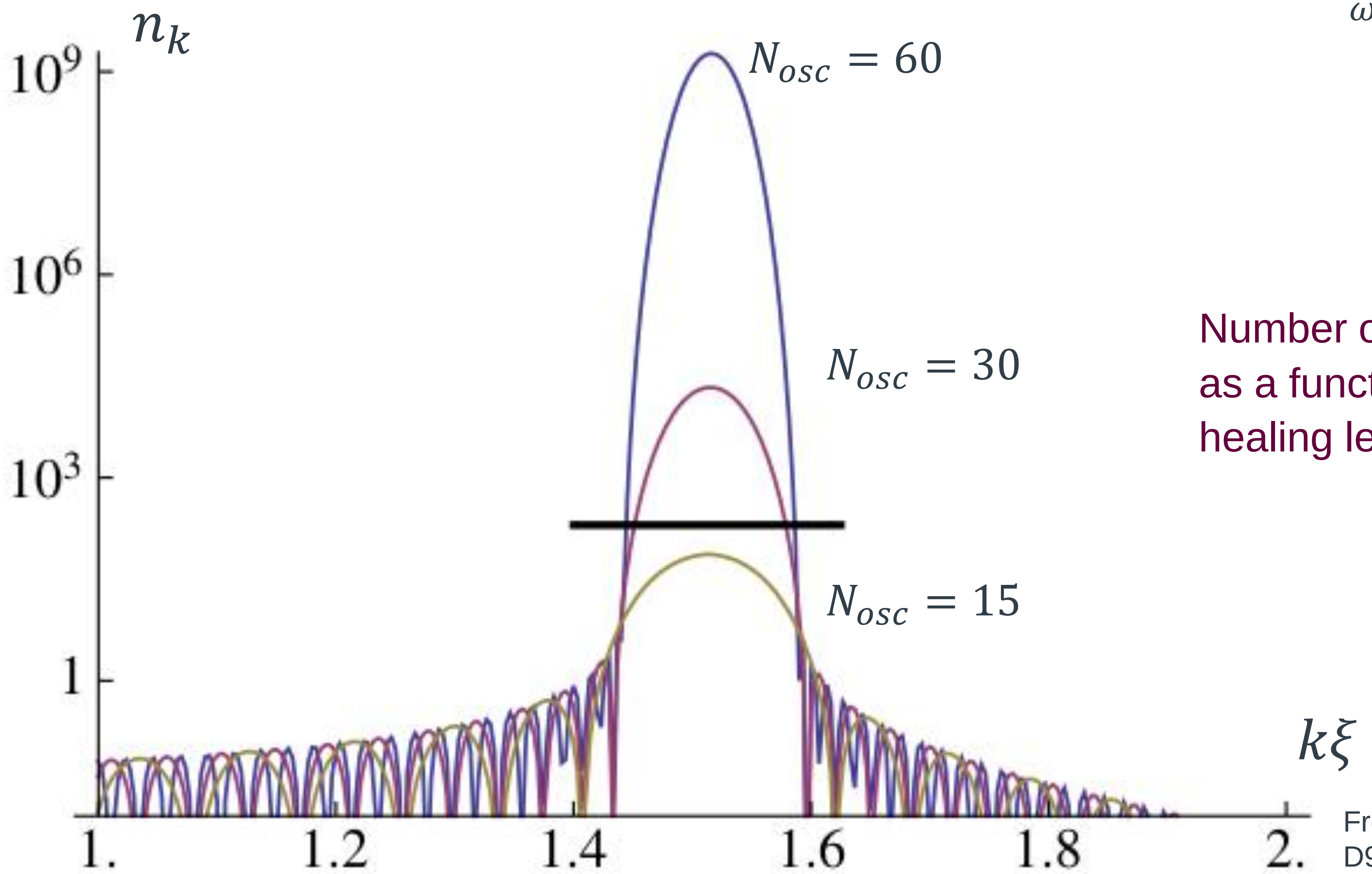
Why do we create **pairs** of phonons ?

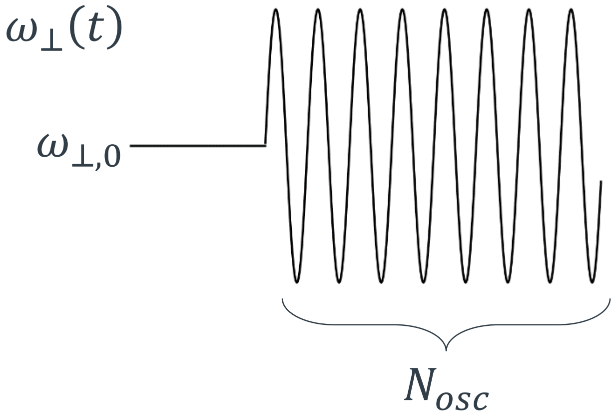
Define the number of phonons with momentum k : $n_k \equiv \langle \hat{b}_k^\dagger \hat{b}_k \rangle$

Show that

$$\partial_t(n_k - n_{-k}) = 0$$

Phonon pair creation





Number of phonons created n_k as a function of k in units of the healing length $\xi = 1/\sqrt{g_1 n_1 m}$.

From Robertson *et al*, Phys. Rev. D95, 065020 (2017)

Outline of the talk

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Phonon pair creation

Non separability of the phonon pair

Summary

Oscillations at ω of the transverse trapping frequency $\omega_\perp$

↓

Oscillations of the interaction strength g_1 of the 1D perturbative field $\hat{\phi}$

↓

Exponential creation of phonons pairs at energy $\omega/2$ and corresponding k.

Non separability of the phonons pair

A state of a 2 modes system is said separable if its density matrix can be written as

$$\hat{\rho}_{k,-k} = \sum_j P_j \; \hat{\rho}_k^j \otimes \; \hat{\rho}_{-k}^j$$

- $0 < P_j < 1$ and $\sum_j P_j = 1$
- $\hat{\rho}_k^j$ is the density matrix of a single-mode k subsystem

Separable state

$$\left|\langle \hat{b}_k \hat{b}_{-k} \rangle\right|^2 \le \langle \hat{b}_k^\dagger \hat{b}_k \rangle \langle \hat{b}_{-k}^\dagger \hat{b}_{-k} \rangle = n_k n_{-k}$$

In general

$$\left|\langle \hat{b}_k \hat{b}_{-k} \rangle\right|^2 \le n_k n_{-k} + \min\{n_k, n_{-k}\}$$

where $n_j \equiv \langle \hat{b}_j^\dagger \hat{b}_j \rangle$

Non separability of the phonon pair

In the experiment, we count the number of atoms arriving of the detector and compute

$$g^{(2)}(k,-k) = \langle \hat{n}_k \hat{n}_{-k} \rangle / \langle \hat{n}_k \rangle \langle \hat{n}_{-k} \rangle$$

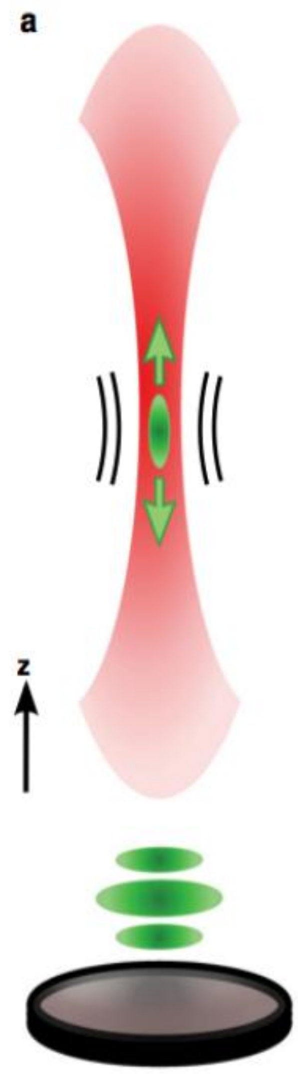
Noting that $\hat{n}_k = \hat{b}_k^\dagger \hat{b}_k$ and using Wick contraction

$$\langle \hat{n}_k \hat{n}_{-k} \rangle = \left\langle \hat{b}_k^\dagger \hat{b}_{-k}^\dagger \hat{b}_k \hat{b}_{-k} \right\rangle = n_k n_{-k} + \left| \left\langle \hat{b}_k \hat{b}_{-k} \right\rangle \right|^2 + \cancel{\left| \left\langle \hat{b}_k^\dagger \hat{b}_{-k} \right\rangle \right|^2}$$

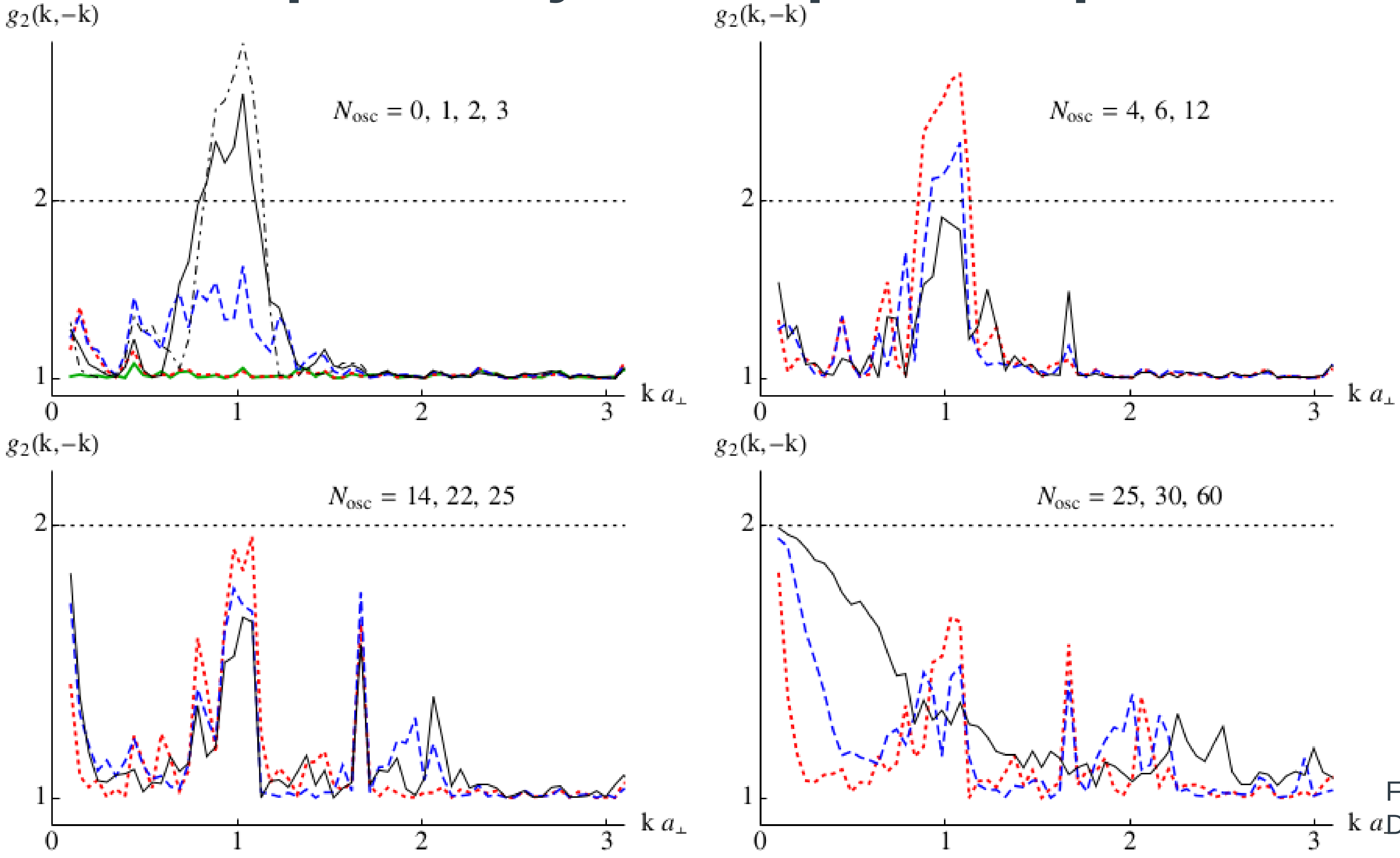
if the state is separable : $\overbrace{\leq n_k n_{-k}}$

Non separability criteria

$$g^{(2)}(k,-k) > 2$$



Non separability of the phonon pair



Here the nonlinearities are taken into account

Values of g^2 for different values of the number of oscillations.

From Robertson *et al*, Phys. Rev. D98(5) 056003 (2018)

Conclusion

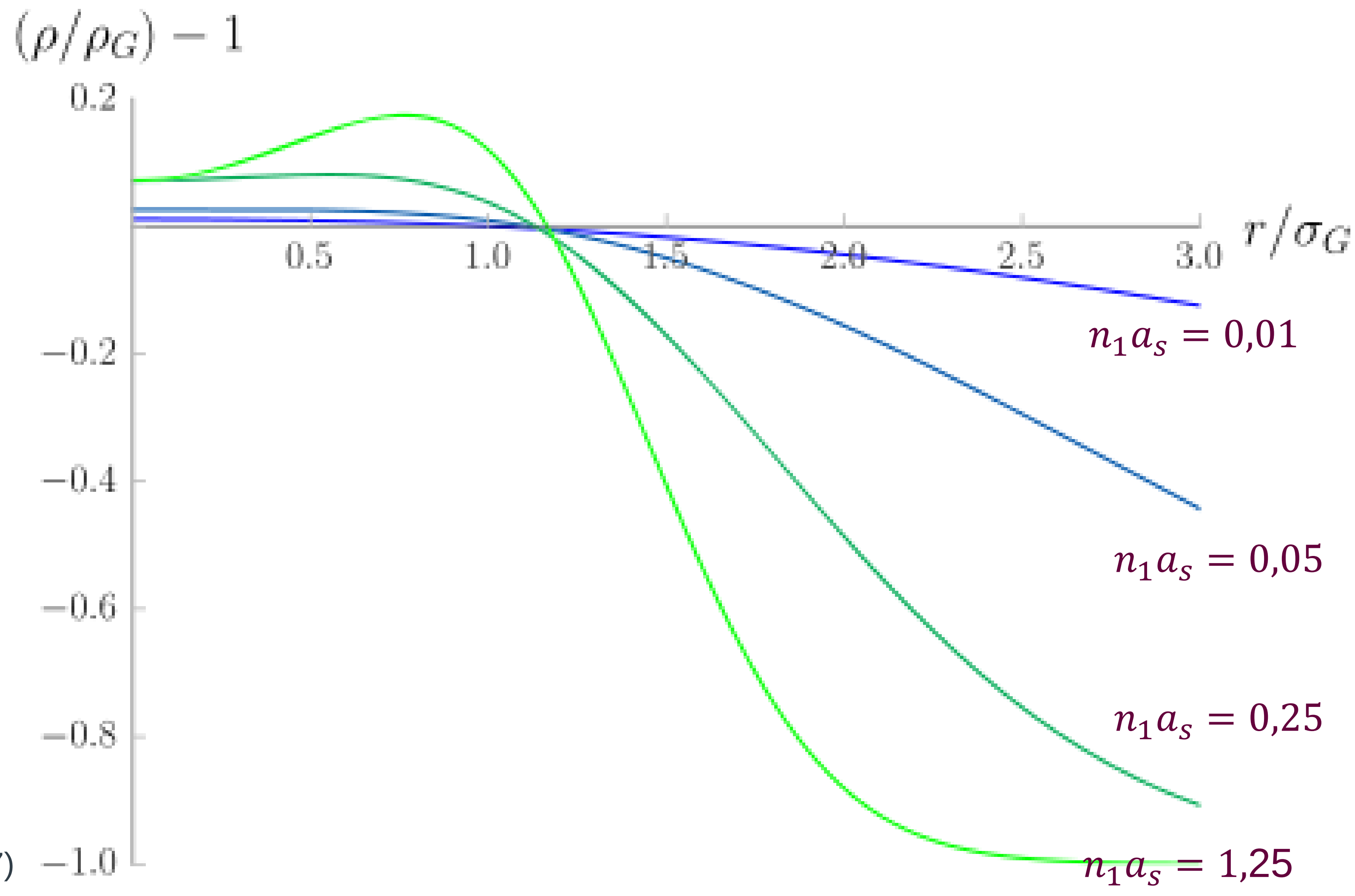
- **Oscillations of the transverse frequency of a BEC induces oscillations of the BEC speed of sound**

- **Modulation of *c* creates pair of entangled phonons**

- **Nonseparability of the phonon pair can be witnessed by the value of $\boldsymbol{g^{(2)}(k,-k)}$**


Helium$^{\times}$ ONE
Thank you for your time

Validity of the Gaussian ansatz



Relative difference between the numerically computed density profile and the Gaussian approximation.

Robertson *et al*, Phys. Rev. D95, 065020 (2017)