

NOM :
Prénom :

Interrogation Écrite n°6 : corrigé

PTSI B Lycée Eiffel

25 mars 2014

1. $f(x) = \ln(1+x^2) = x^2 - \frac{x^4}{2} + o(x^5)$.
2. $g(x) = \operatorname{ch}(x) \operatorname{sh}(x) = \left(1 + \frac{x^2}{2} + \frac{x^4}{24} + o(x^5)\right) \left(x + \frac{x^3}{6} + \frac{x^5}{120} + o(x^5)\right) = x + \frac{x^3}{2} + \frac{x^5}{24} + \frac{x^3}{6} - \frac{x^5}{12} + \frac{x^5}{120} + o(x^5) = x + \frac{2x^3}{3} + \frac{2x^5}{15} + o(x^5)$.
3. $h(x) = \sqrt{1-x^2} = 1 - \frac{x^2}{2} - \frac{x^4}{8} + o(x^5)$.
4. $i(x) = (1+x)^x = e^{x \ln(1+x)} = e^{x(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + o(x^4))} = e^{x^2 - \frac{x^3}{2} + \frac{x^4}{3} - \frac{x^5}{4} + o(x^5)} = 1 + x^2 - \frac{x^3}{2} + \frac{x^4}{3} - \frac{x^5}{4} + \frac{1}{2} \left(x^2 - \frac{x^3}{2} + \frac{x^4}{3} - \frac{x^5}{4}\right)^2 + o(x^5) = 1 + x^2 - \frac{x^3}{2} + \frac{x^4}{3} - \frac{x^5}{4} + \frac{x^4}{2} - \frac{x^5}{2} + o(x^5) = 1 + x^2 - \frac{x^3}{2} + \frac{5x^4}{6} - \frac{3x^5}{4} + o(x^5)$.
5. $j(x) = \frac{x+1}{x^2+1} = (x+1)(1-x^2+x^4+o(x^5)) = 1+x-x^2-x^3+x^4+x^5+o(x^5)$.