Thermalization in classical statistical field theory

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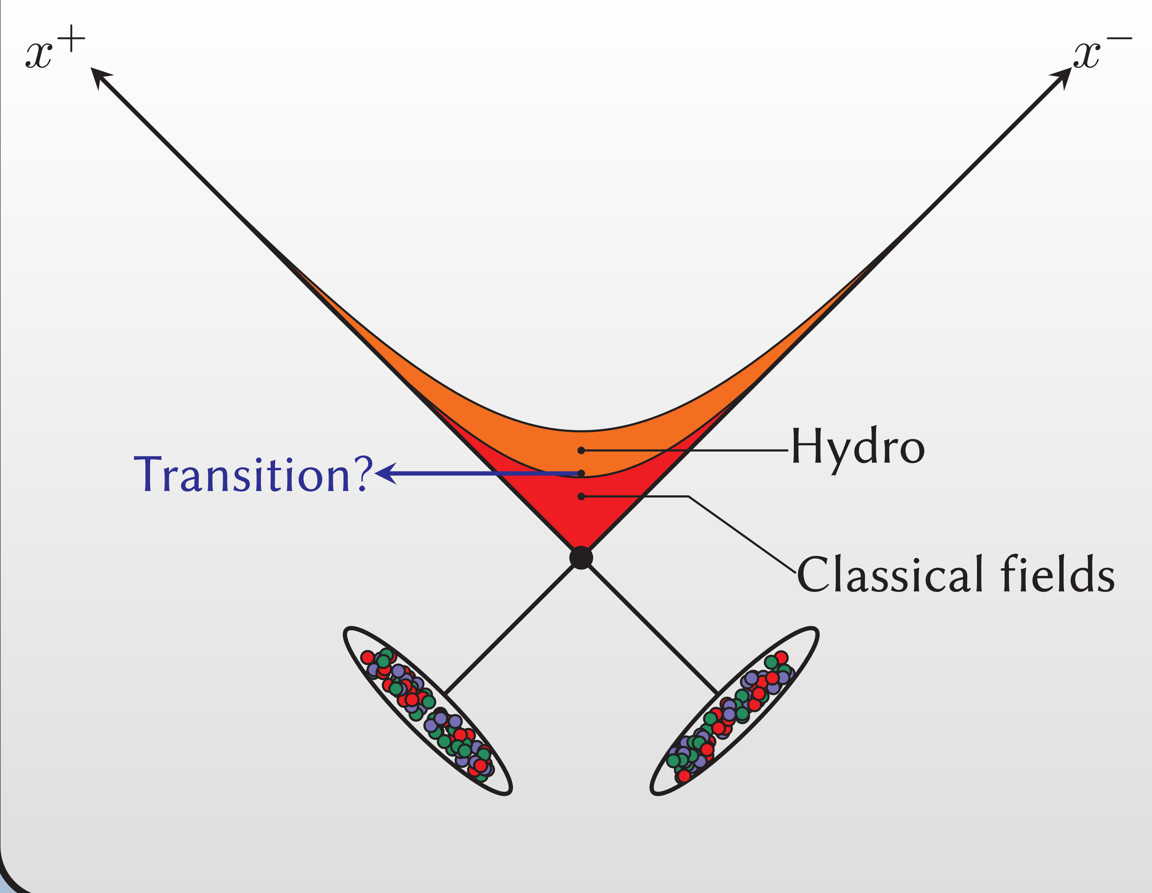
1. Heavy Ion Collision

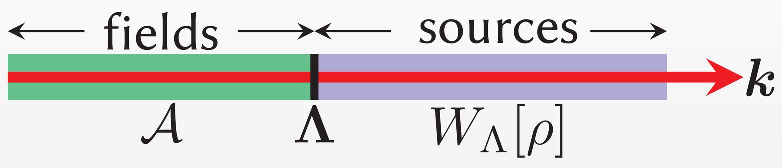


2. Goals

- Calculate $T^{\mu\nu}$, in order to provide initial conditions for hydrodynamics. This implies calculating the energy density ϵ and the pressures P_L, P_T
- Determine if the system reaches thermal equilibrium: equation of state (EOS)? Bose-Einstein (BE) distribution fonction?

3. Theoretical Framework

- Color Glass Condensate: a semi classical effective theory

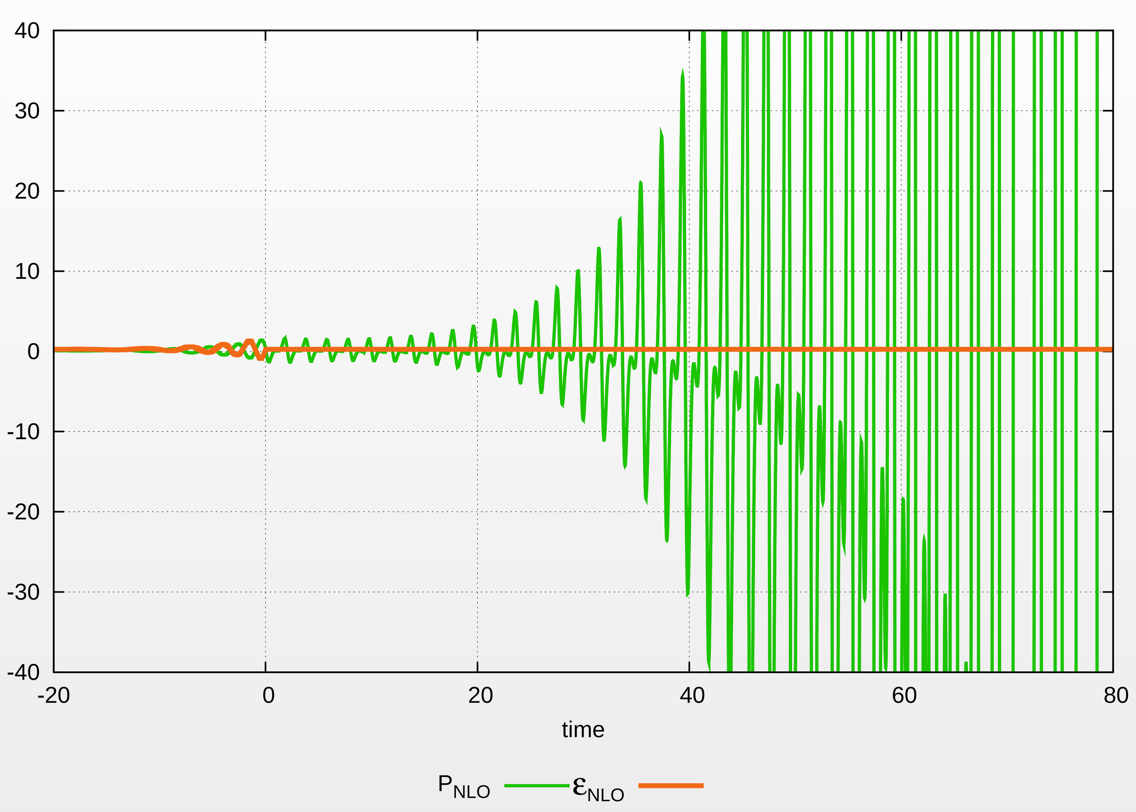


- Λ = Unphysical momentum cutoff between color fields and classical sources
- JIMWLK equation (renormalization group equation for the evolution with Λ)

$$\Lambda \frac{\partial}{\partial \Lambda} W_\Lambda[\rho] = -\mathcal{H} W_\Lambda[\rho]$$

4. Divergences & Resummation

- Secular divergences at NLO



- LO and NLO related by $T^{\mu\nu}_{\text{NLO}} = \widehat{O}\, T^{\mu\nu}_{\text{LO}}$
- Resummation:

$$T^{\mu\nu}_{\text{resum}} = e^{\widehat{O}}\, T^{\mu\nu}_{\text{LO}} = T^{\mu\nu}_{\text{LO}} + T^{\mu\nu}_{\text{NLO}} + \cdots$$

- Equivalent formulation

$$T^{\mu\nu}_{\text{resum}} = \int [Da] \underbrace{F[a]}_{\substack{\text{Gaussian}\\ \text{distribution}}} T^{\mu\nu}_{\text{LO}}[\varphi_0 + a]$$

5. Scalar fields in a fixed box

- Scalar field with a ϕ^4 potential

$$\mathcal{L}(\phi) = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \underbrace{\frac{g^2}{4!}\phi^4}_{V(\phi)}$$

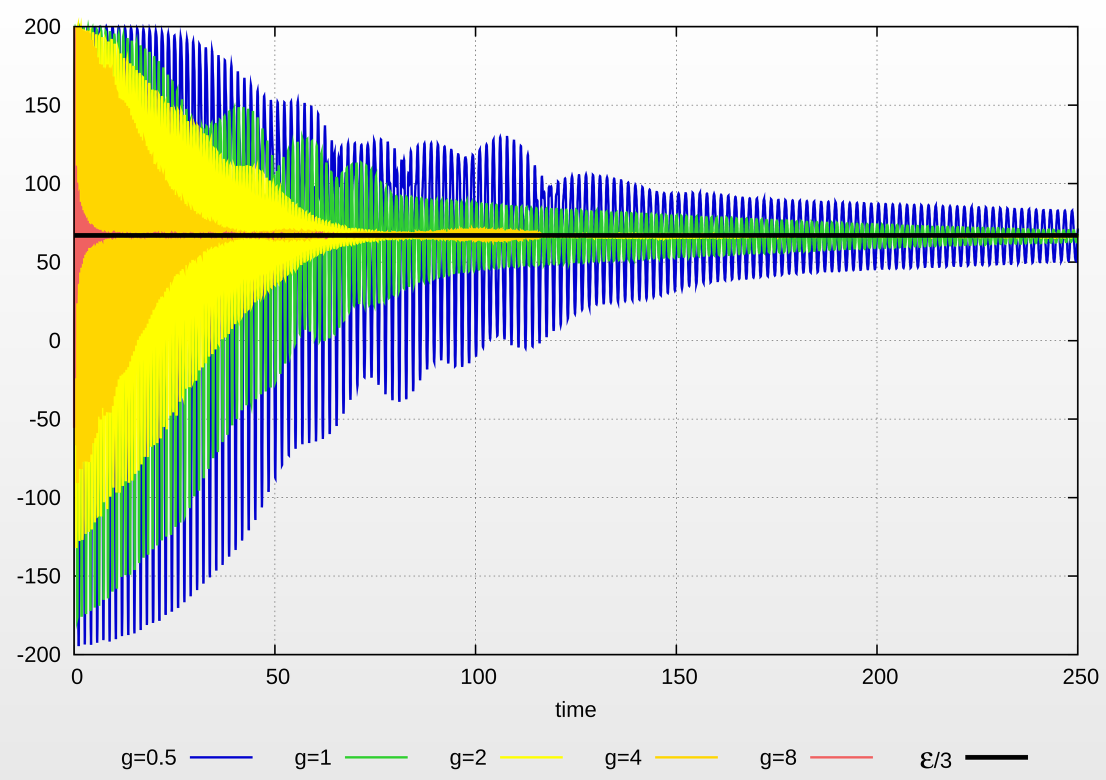
- The model shares some important features with QCD: **scale invariance** and the presence of **instabilities**
- Setup of the numerical computation

$$\phi_i(t_0, \boldsymbol{x}) = \varphi_0(\boldsymbol{x}) + \int_{\boldsymbol{k}} \text{Re}\left[c_{\boldsymbol{k}}\, e^{i\omega_{\boldsymbol{k}} t_0}\, v_{\boldsymbol{k}}(\boldsymbol{x})\right]$$

with

$$[-\Delta + V''(\varphi_0)]\, v_{\boldsymbol{k}}(\boldsymbol{x}) = \omega^2_{\boldsymbol{k}} v_{\boldsymbol{k}}(\boldsymbol{x}),$$
$$\langle c_{\boldsymbol{k}}\, c^*_{\boldsymbol{l}} \rangle = \delta(\boldsymbol{k} - \boldsymbol{l})$$

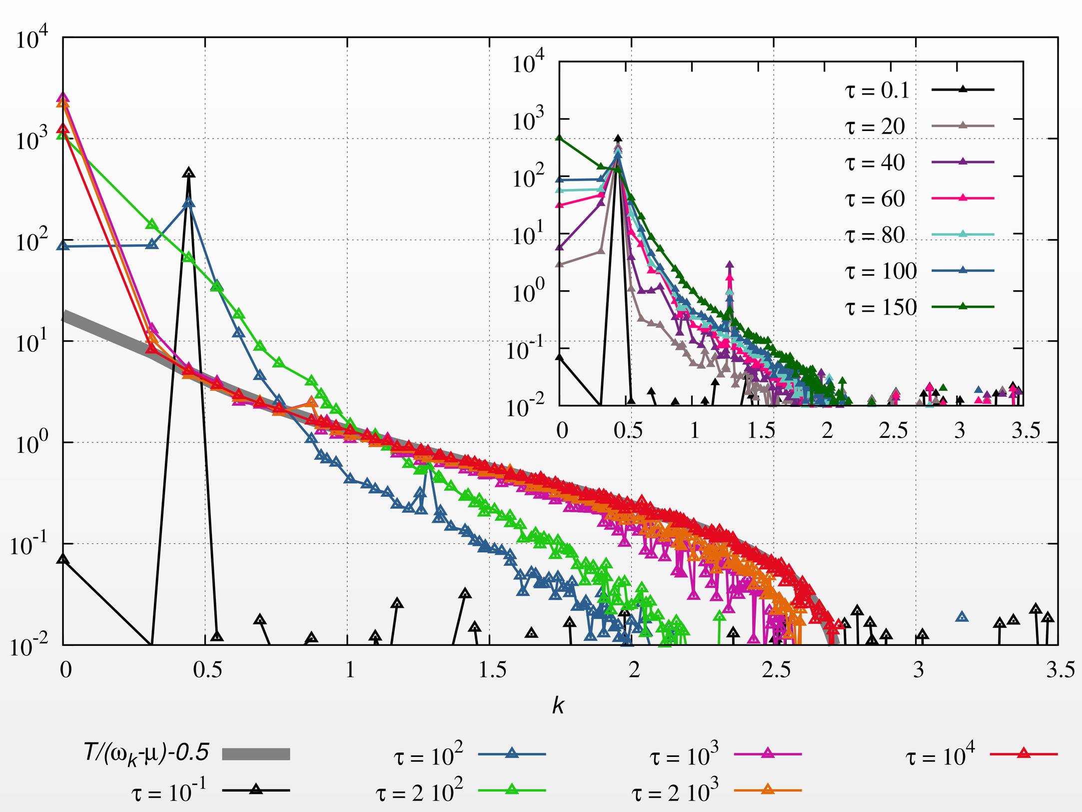
6. Resummed $T^{\mu\nu}$



The resummation cures the secular divergences, and one gets the expected **equation of state** in a 3+1D scale invariant theory

$$\boldsymbol{\epsilon = 3P}.$$

7. Distribution function



- At large times:

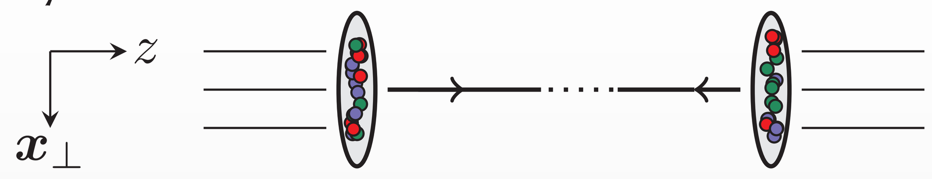
$$f_{\boldsymbol{k}} \approx \frac{T}{\omega_{\boldsymbol{k}} - \mu} - \frac{1}{2}.$$

Classical equilibrium distribution

- $\mu \neq 0$: slow chemical equilibration
- **Bose-Einstein condensation**: excess of particles at $\boldsymbol{k} = 0$ and $\mu \approx m$ (appears for initial conditions that have too many particles for a given energy)

8. Effects of longitudinal expansion

- System boost invariant in the z direction



- Manifest in terms of proper time/rapidity

$$\eta = \frac{1}{2}\ln\frac{t+z}{t-z}, \qquad \tau = \sqrt{t^2 - z^2}.$$

Boost invariance $\Leftrightarrow$ η independence

- EOM for a boost-invariant field $\varphi(\tau, \boldsymbol{x}_\perp)$

$$\left[\frac{\partial^2}{\partial \tau^2} + \frac{1}{\tau}\frac{\partial}{\partial \tau} - \boldsymbol{\nabla}^2_\perp\right]\varphi + V'(\varphi) = 0$$

- EOM for a small fluctuation $a(\tau, \boldsymbol{x}_\perp, \eta)$

$$\left[\frac{\partial^2}{\partial \tau^2} + \frac{1}{\tau}\frac{\partial}{\partial \tau} - \boldsymbol{\nabla}^2_\perp - \frac{1}{\tau^2}\frac{\partial^2}{\partial \eta^2} + V''(\varphi)\right] a = 0$$

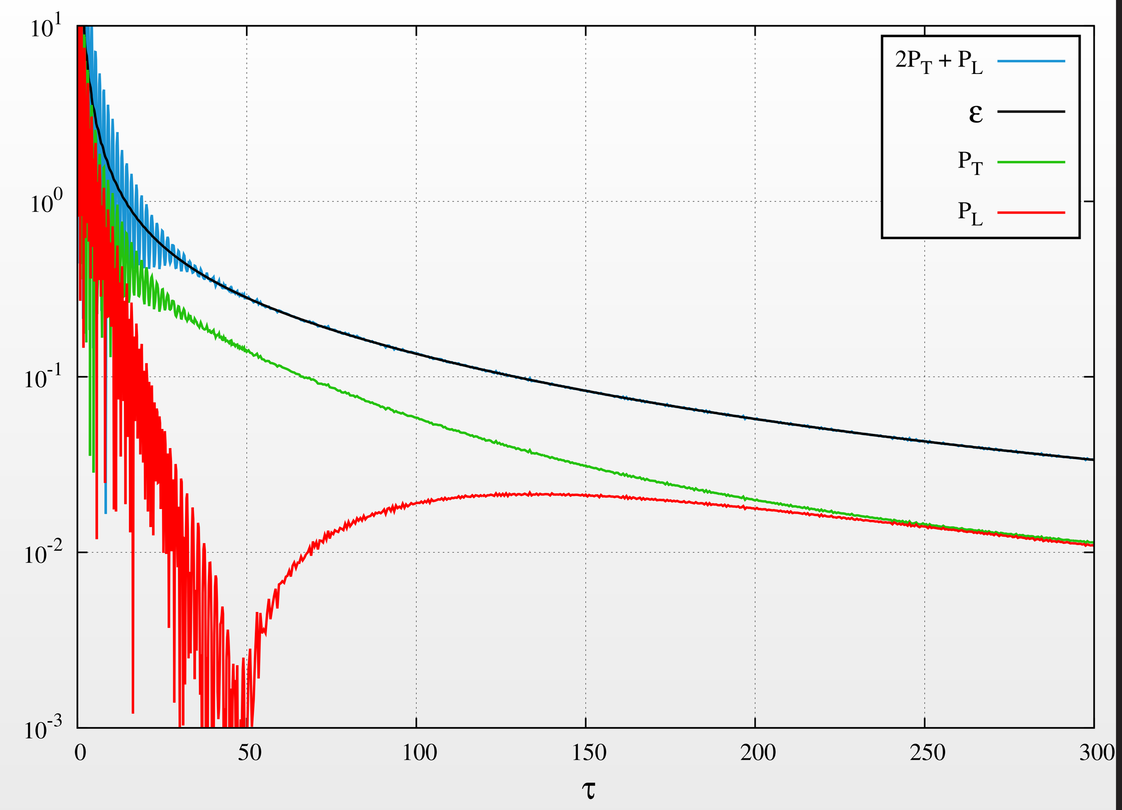
- Setup of the numerical calculation

$$\phi_i = \varphi_0 + \int_{\boldsymbol{k},\nu} \text{Re}\left[c_{\boldsymbol{k}\nu}\, H^{(2)}_{i\nu}(\omega_{\boldsymbol{k}}\tau_0)\, e^{i\nu\eta}\, v_{\boldsymbol{k}}(\boldsymbol{x}_\perp)\right]$$

with

$$[-\Delta_\perp + V''(\varphi_0)]\, v_{\boldsymbol{k}}(\boldsymbol{x}_\perp) = \omega^2_{\boldsymbol{k}} v_{\boldsymbol{k}}(\boldsymbol{x}_\perp),$$
$$\langle c_{\boldsymbol{k}\nu}\, c^*_{\boldsymbol{l}\mu} \rangle = \delta(\boldsymbol{k} - \boldsymbol{l})\delta(\nu - \mu)$$

9. Numerical results



- Equation of state: $\boldsymbol{\epsilon = 2P_T + P_L}$
- Later in the evolution $\boldsymbol{P_L \approx P_T}$
- $\epsilon \propto \tau^{-1}$ at early times and $\epsilon \propto \tau^{-\frac{4}{3}}$ later, in agreement with Bjorken law for respectively $P_L \approx 0$ and $P_L \approx \frac{\epsilon}{3}$.

10. Conclusion

- **Resummation→secular divergences cured**
- **Equation of state: $\boldsymbol{\epsilon = 2P_T + P_L}$**
- **Isotropization: $\boldsymbol{P_L \approx P_T}$**
- **Equilibration of the particle distribution**
- **Formation of a Bose-Einstein condensate**

References

[1] K. Dusling, T. Epelbaum, F. Gelis, R. Venugopalan. Nucl.Phys. A850 (2011) 69-109

[2] T. Epelbaum, F. Gelis. Nucl.Phys. A872 (2011) 210-244

[3] K. Dusling, T. Epelbaum, F. Gelis, R. Venugopalan. arXiv:1206.3336 [hep-ph]