My work is related to the high-energy regime of quantum-chromodynamics (QCD), and aims at describing from first principles the early stages of the heavy ion collisions that are currently performed at the relativistic heavy ion collider (RHIC) and the large hadron collider (LHC), where a new state of matter called the quark-gluon plasma (QGP) is formed. In this peculiar energy regime of QCD, one is allowed to use weak coupling techniques, since the strong coupling constant $\alpha_s \ll 1$. But because the two heavy nuclei are highly boosted, they emit many gluons until the regime of "gluon saturation" is reached. When this occurs, the gluon occupation number is of order α_s^{-1}, a property that completely changes the usual power counting of perturbative QCD: an infinite set of diagrams is now contributing at each order of the perturbative expansion. With my collaborators, we use an effective theory that can deal with this new situation in order to describe the formation of the QGP: the Color Glass Condensate (CGC). My goal is to understand the thermalization mechanisms that happen during the first instants of a heavy ion collision, by looking at the energy-momentum tensor of the system in order to see if it obeys an equation of state (EOS) and becomes isotropic, and by computing the occupation number to check if it follows the Bose-Einstein statistics. These properties are assumed to be true because of the success of viscous hydrodynamical simulations in describing the bulk properties of heavy ion collisions. However, proving these properties from first principles in a QCD framework has yet to be done. This is my main center of interest. During the course of my Ph.D., I have been involved in the following projects

1. In a first work [1, 2], we have studied a scalar field theory with a quartic coupling in a fixed volume, a situation where the pressure tensor is isotropic by construction. This theory, although much simpler than QCD, shares some important features with it: scale invariance at the classical level and the presence of instabilities, even if they are not of the same nature in the scalar theory (parametric resonance) and in QCD (Weibel instabilities, i.e. plasma instabilities). These instabilities cause secular divergences in the pressure of the system if one naively truncates the perturbative expansion at a finite loop order. In order to cure these divergences, we have used the classical statistical method, that resums an infinite set of higher order contributions. This amounts to letting the system evolve with the classical equations of motion, with fluctuating initial conditions whose variance is derived from a one-loop calculation. By doing so, we were able to study numerically the energy momentum tensor[1]. We observed a transient regime where the pressure oscillates, after which it converges towards an equilibrium value in agreement with the formation of an EOS. We physically understood this phenomenon as the decoherence of the initial coherent state, triggered by the instabilities. This idea is supported by additional studies that we have performed in [2], that showed a drastic increase of the statistical entropy, and an occupation number $f(k)$ that expands toward the higher momentum modes. At late times, it is well described by a classical equilibrium distribution. We also observed the presence of a chemical potential whose value is very close to the mass of the quasi-particles in the system, the latter being computed through the spectral function. An excess in the zero mode of $f(k)$, combined with this value of the chemical potential, led us to conclude that a Bose-Einstein condensate is formed in this system.

2. In a subsequent work [3], we extended our study of this scalar field theory to a longitudinally expanding geometry in order to mimic the QGP expansion. This amounts to using the proper time/rapidity coordinate system. We first computed analytically the one-loop spectrum in order to determine the initial fields in the classical statistical method. Like in the fixed volume case, we observed the rapid formation of an EOS. But at early times the pressure tensor remains anisotropic because the instabilities are not yet able to compete with the fast

expansion; the longitudinal pressure hence decreasing faster than the transverse one. After some time, the instabilities become strong enough to isotropize the system. We also compared the outcome of our simulations with a simple viscous hydrodynamical model, the two showing very similar behaviour when the pressure tensor is close to being isotropic. In addition, we computed an estimate of the shear viscosity over entropy ratio, obtaining a much smaller value than the leading order (LO) perturbative result.

3. We then proceeded to study the Yang-Mills case. We first had to compute the spectrum of initial fields in the CGC framework analytically [4]. This amounts to taking plane waves in the remote past, and propagating them on top of the classical fields generated by the two nuclei. With this spectrum of fluctuations, we then performed for the first time realistic numerical simulations of the very early stages of a heavy ion collision in the classical statistical framework [5]. This showed that for an already small value of α_s, the instabilities can compete with the expansion of the system and are able to almost isotropize the pressures on very short time scales, compatible with the ones used in hydrodynamical models. For $\alpha_s = 2.10^{-2}$, we observed a residual $30 - 40\%$ anisotropy at around 1 fm/c. We also crudely estimated the shear viscosity over entropy ratio, and obtained a value $\lesssim 1$, much smaller than the expected perturbative value. All these results support the use of viscous hydrodynamics.

In the near future, I plan to continue working on the thermalization problem in the CGC framework, as well as other possible approaches – hard thermal loop framework, kinetic theory, hydrodynamics – that can deal with the early life of the QGP. Specific questions I would like to explore at some point in the future:

1. The renormalization of the classical statistical simulations in scalar field theories. Firstly, the lattice regularization introduces a mass counterterm. Secondly, because we are interested in computing the energy density and the pressures, that are dimension four operators, we have to take into account the possibility of operator mixing with operators of lower dimensions. This study would be important in order to give more quantitative estimates of the isotropization and thermalization time scales at not so small couplings.

2. The renormalization of classical statistical simulations in QCD. In the continuum limit, a mass counterterm would be forbidden by gauge symmetry. However, on the lattice, differential operators become finite differences (i.e. non-local quantities) for which this constraint does not apply. In order to identify and substract these terms, one must perform a one-loop lattice perturbation theory calculation in the presence of a background field.

3. Evolution of the gluon distribution. This is a very natural extension of my work in order to study the thermalization of the system (or lack thereof), but is made difficult by the fact that there is no gauge invariant definition of the occupation number in non-abelian gauge theories.

4. Quantum corrections. For some scalar models, the 2PI formalism can be used to include quantum corrections coming from the time evolution of the system. These are not taken into account in the classical statistical method, since in this approach one makes the system evolve with the classical EOM. These quantum effects may speed up the thermalization process. Unfortunately, no gauge invariant 2PI formalism is available so far. In addition, the gluon occupation number being of order α_s^{-1}, one needs to compute an infinite number of diagrams at each order of the perturbative expansion. How to cope with this in the 2PI framework is unclear so far.

5. More accurate estimate of the isotropization time. This would require two natural extensions of our setup. The first one is simple yet costly numerically: it amounts to use the real QCD gauge group SU(3) instead of SU(2), that we have been using so far. The second one is more challenging both theoretically and numerically, and consists in including fermions in the description. At very early times, they should not play a big role because the fermion density is much lower than that of the gluons. But when the gluon field starts to decohere, the fermions could have a sizeable effect on the system.

6. Classical statistical methods as an input of viscous hydrodynamical simulations. Recent studies have combined LO CGC calculations with viscous hydrodynamics in order to fit the RHIC and LHC data. Since higher order corrections in the CGC framework play a central role in the isotropization of the system, it would be interesting to combine these improved calculations with hydrodynamics. This would be useful in assessing whether some significant flow is generated during the pre-hydro stage of the evolution.

7. Jet quenching. In principle, our framework is able to account for the energy loss of hard particles when they propagate in the glasma, which is the pre-equilibrium QGP phase. A way to properly describe a jet in classical statistical models is lacking right now, but may be of great interest for a better understanding of the jet asymmetries observed experimentally.

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