Early Isotropization of the Quark Gluon Plasma

Santiago de Compostela, 29th October 2013

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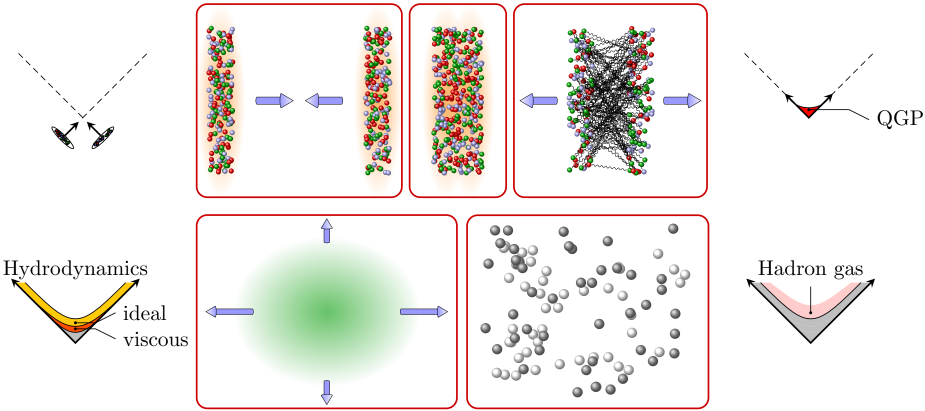
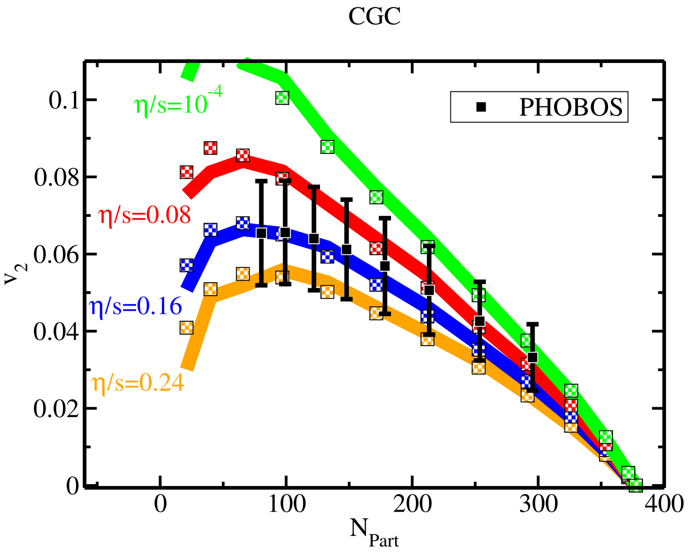
IPhT

Outline

Heavy Ion Collisions: The general picture





[LUZUM, ROMATSCHKE (2008)]

Viscous Hydrodynamics

I) Macroscopic theory
II) Few parameters: P_L, P_T, ϵ, $\vec{u}$
III) Need input:

1) Equation of state $f(P_L, P_T) = \epsilon$
2) Small anisotropy
3) Initialization: $\epsilon(\tau_0)$, $P_L(\tau_0)$? ...
4) viscous coefficients: shear viscosity η,...
5) Short isotropization time

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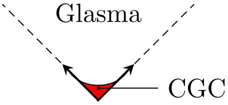
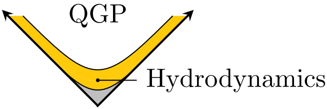
None of this is easy to get from QCD

Early transition: the problem



Isotropization?
Time scale?



Huge anisotropy (negative P_L)

Small anisotropy

Long time puzzle: Does (fast) isotropization occur?

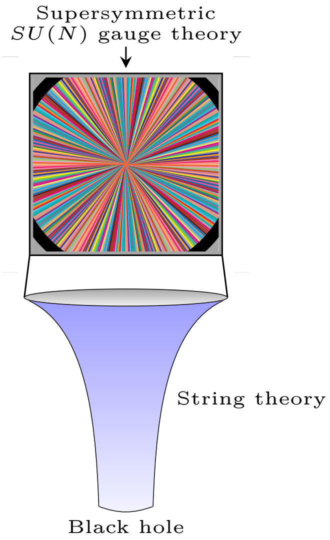
2 THEORETICAL FRAMEWORK

How to deal with a Heavy Ion Collision

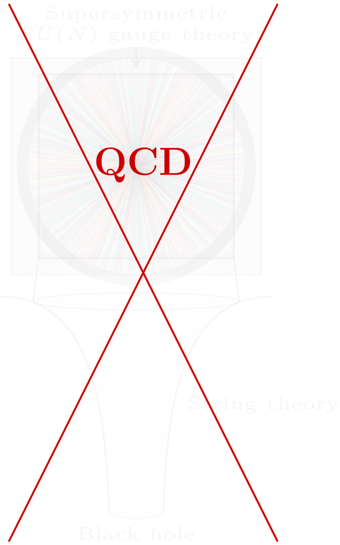
The Color Glass Condensate

The Classical statistical approximation

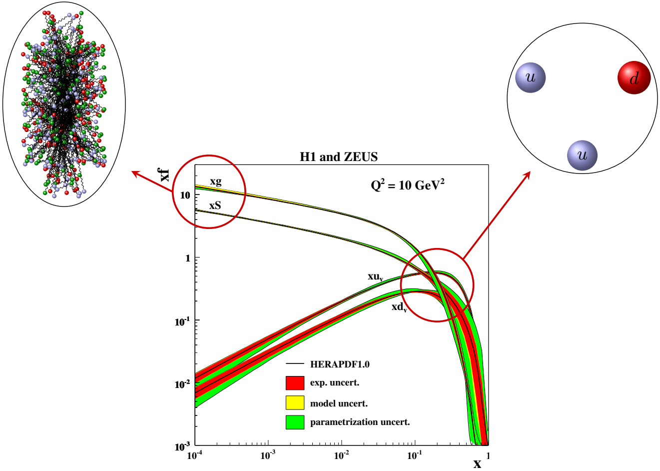
Strongly coupled method: AdS/QCD?



Weakly coupled method: QCD

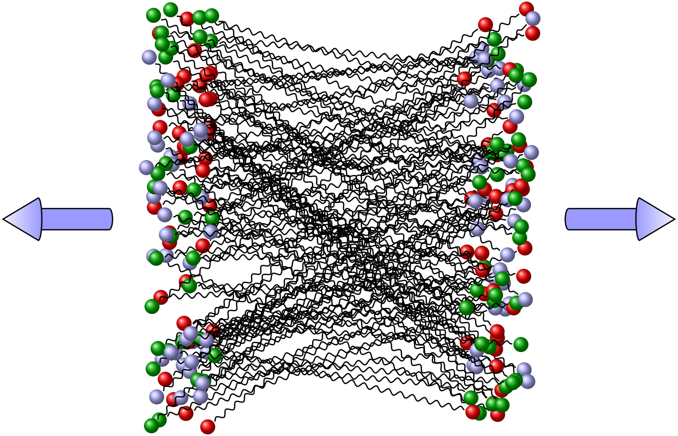


Weakly coupled QCD with only gluons

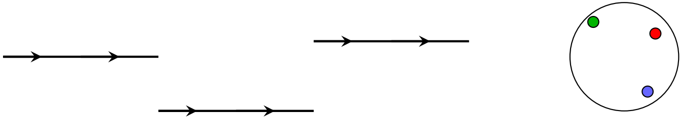


Weakly coupled method at dense regime:

$\alpha_s \ll 1$ **but** $f_{\text{gluon}} \sim \frac{1}{\alpha_s}$



Two additional features: saturation and time dilation



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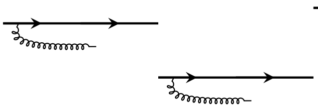

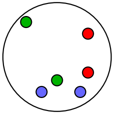

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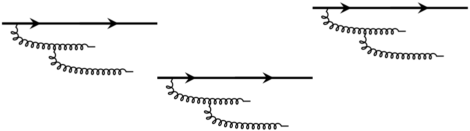

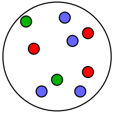

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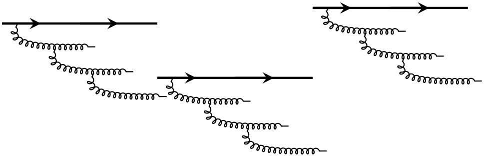

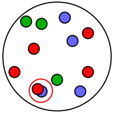

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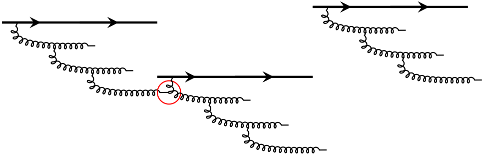

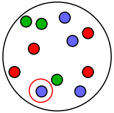

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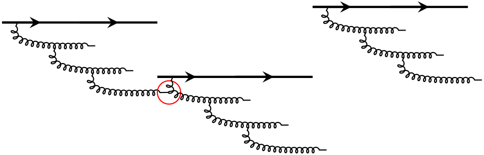
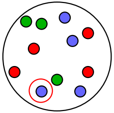
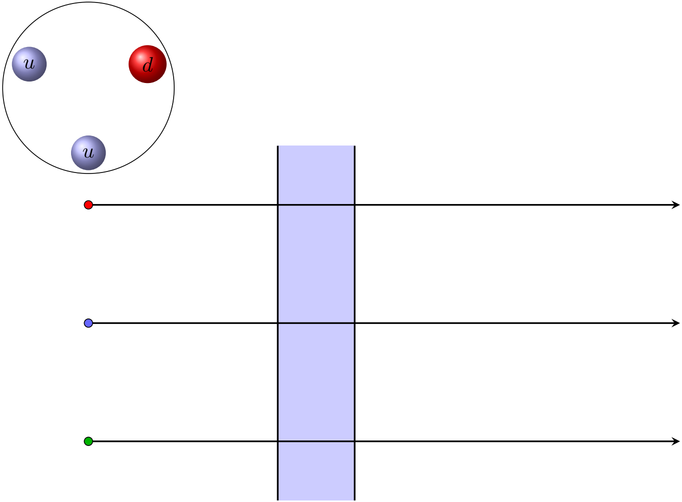




Gluon saturation when emission = recombination
$\Rightarrow$ Saturation scale Q_s

Two additional features: saturation and time dilation



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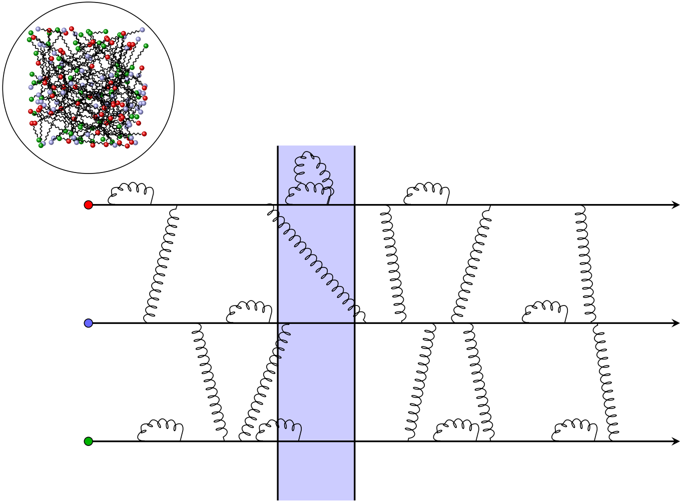

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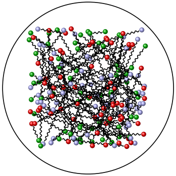

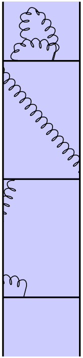

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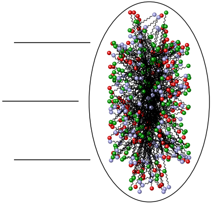

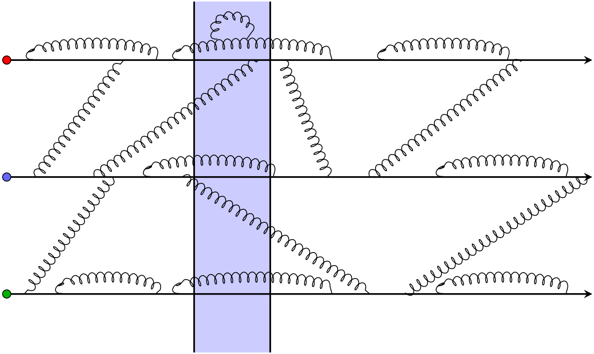

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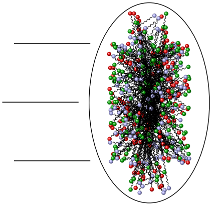
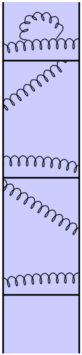




2 Theoretical framework

How to deal with a Heavy Ion Collision

The Color Glass Condensate

The Classical statistical approximation

The Color Glass Condensate [McLerran, Venugopalan (1993)]

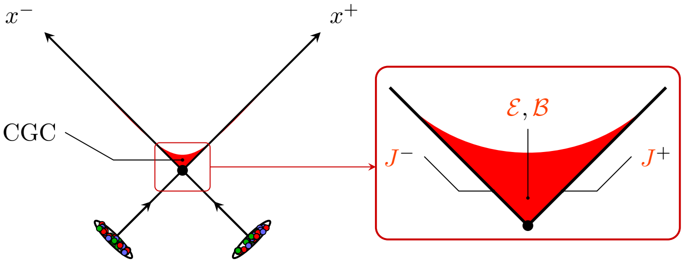
The main assumptions

- Fast gluons are "frozen" by time dilation.
- Described as static color sources J located on the light cone axis
- Small $x \to$ Gluon saturation $\to J \sim Q_s^3\, \alpha_s^{-1/2}$.
- Slow gluons are the standard gauge field $\mathcal{A}^\mu \sim Q_s\, \alpha_s^{-1/2}$.
- System boost-invariant $\to \mathcal{A}^\mu$ rapidity independant.

Langrangian of theory reads

$$\mathcal{L} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + J_\mu \mathcal{A}^\mu$$

Theoretical framework (Weakly coupled but strongly interacting)



LO: $\epsilon = \frac{1}{2} \underbrace{\left(\vec{\mathcal{E}}^2 + \vec{\mathcal{B}}^2\right)}_{\substack{\textbf{Classical} \\ \text{color fields}}}$ $\quad \mathcal{D}_\mu \mathcal{F}^{\mu\nu} = \underbrace{J^\nu}_{\substack{\text{Color sources} \\ \text{on the light cone}}}$

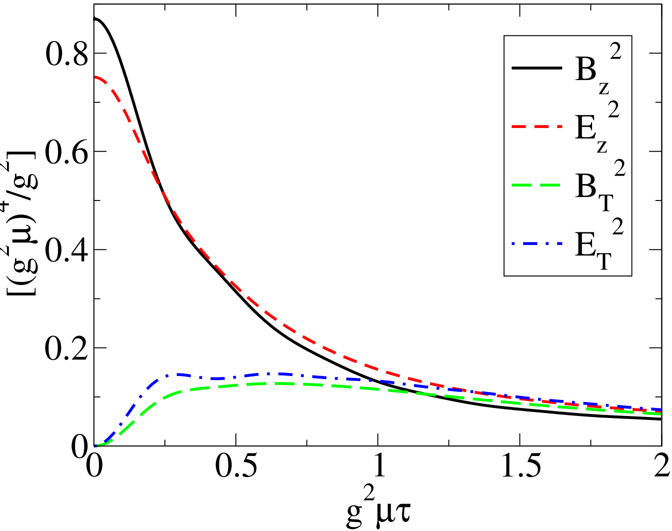
[KRASNITZ, VENUGOPALAN (1998)]

$$\epsilon = \mathcal{E}_\perp^2 + \mathcal{B}_\perp^2 + \mathcal{E}_L^2 + \mathcal{B}_L^2$$

$$P_T = \mathcal{E}_L^2 + \mathcal{B}_L^2$$

$$P_L = \mathcal{E}_\perp^2 + \mathcal{B}_\perp^2 - \mathcal{E}_L^2 - \mathcal{B}_L^2$$

The Color Glass Condensate [McLerran, Venugopalan (1993)]



[Lappi, McLerran (2006)]

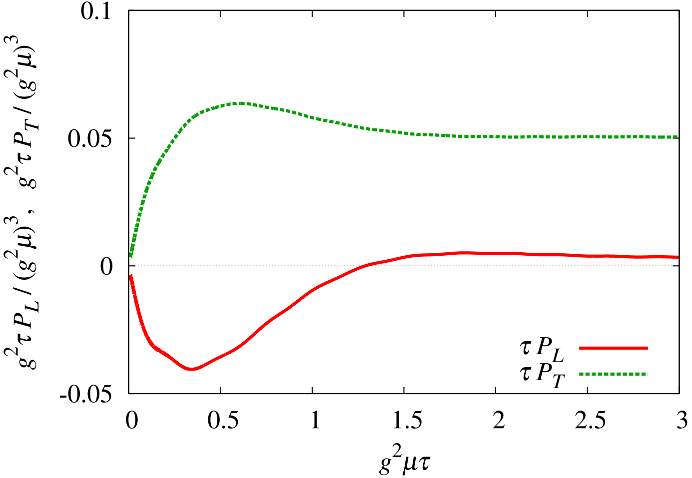
$$\epsilon = \underbrace{\mathcal{E}_\perp^2}_{0} + \underbrace{\mathcal{B}_\perp^2}_{0} + \mathcal{E}_L^2 + \mathcal{B}_L^2$$

$$P_T = \mathcal{E}_L^2 + \mathcal{B}_L^2$$

$$P_L = \underbrace{\mathcal{E}_\perp^2}_{0} + \underbrace{\mathcal{B}_\perp^2}_{0} - \mathcal{E}_L^2 - \mathcal{B}_L^2$$

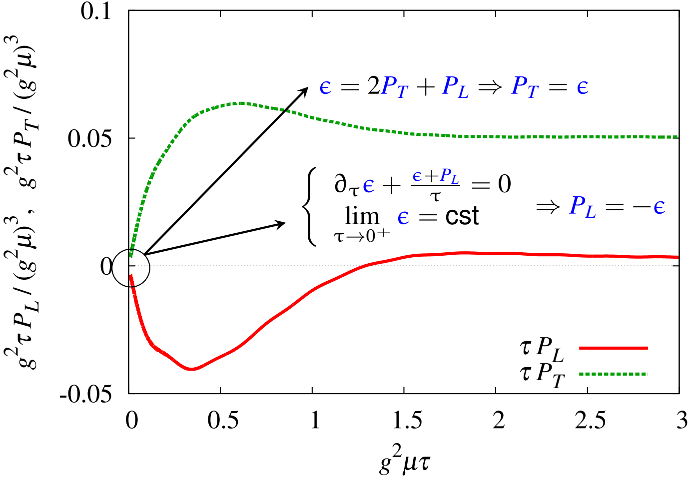
Initial $T^{\mu\nu}$ is $(\epsilon, \epsilon, \epsilon, -\epsilon)$!

Strong anisotropy at early time



[GELIS, FUKUSHIMA (2012)]

Strong anisotropy at early time

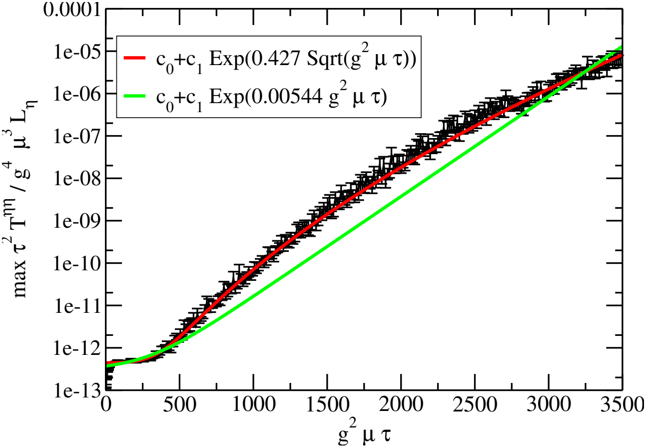


[GELIS, FUKUSHIMA (2012)]

$$E^2(x) = \underbrace{\mathcal{E}^2(x)}_{\text{LO}} + \underbrace{\frac{1}{2}\int_{\vec{k}} \left|e_{\vec{k}}(x)\right|^2}_{\text{NLO}} + \cdots$$

$e_{\vec{k}}(x)$ perturbation to $\mathcal{E}(x)$ created by a plane wave of momentum $\vec{k}$ in the remote past.

THE COLOR GLASS CONDENSATE AT NLO



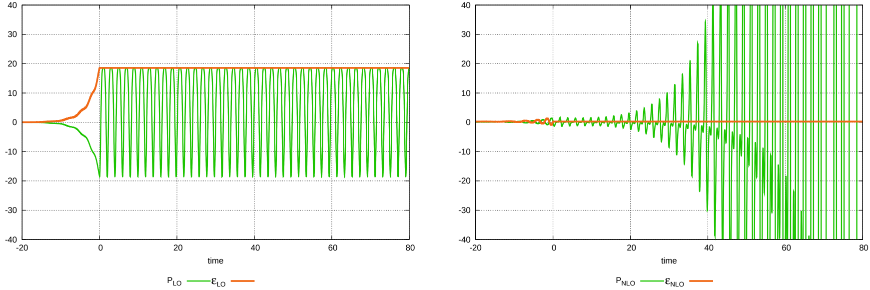
[ROMATSCHKE, VENUGOPALAN (2006)]

Small Fluctuations grow exponentially (Weibel instability)

THE COLOR GLASS CONDENSATE AT NLO

- Because of instabilities, the NLO correction eventually becomes as large as the LO ⇒ Important effect, should be included

- NLO alone will grow forever ⇒ unphysical effect, should be taken care of



- Such growing contributions are present at all orders of the perturbative expansion

How to deal with them?

2 THEORETICAL FRAMEWORK

How to deal with a Heavy Ion Collision

The Color Glass Condensate

The Classical statistical approximation

- At the initial time $\tau = \tau_0$, take:

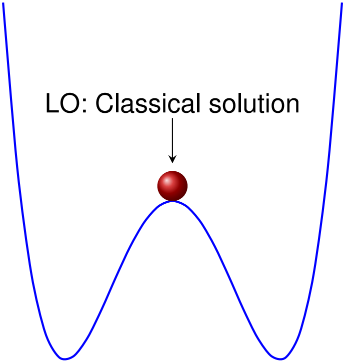
$$\vec{E}_0(\tau_0, \vec{x}) = \vec{\mathcal{E}}_0(\tau_0, \vec{x}) + \int_{\vec{k}} c_{\vec{k}} \vec{e}_{\vec{k}}(\tau_0, \vec{x})$$

  where $c_{\vec{k}}$ are random coefficients: $\left\langle c_{\vec{k}} c_{\vec{k}'} \right\rangle \sim \delta_{\vec{k}\vec{k}'}$

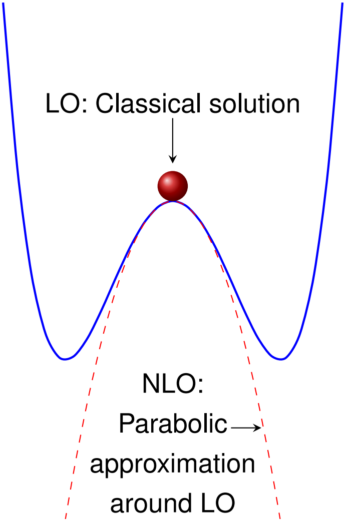
- Solve the **Classical** equation of motion $D_\mu F^{\mu\nu} = J^\nu$
- Compute $\left\langle \vec{E}^2(\tau, \vec{x}) \right\rangle$, where $\langle\rangle$ is the average on the $c_{\vec{k}}$ (Monte-Carlo)
- One can show that this resums all the fastest growing terms at each order, leading to a result that remain bounded when $\tau \to \infty$ [GELIS, LAPPI, VENUGOPALAN (2008)]

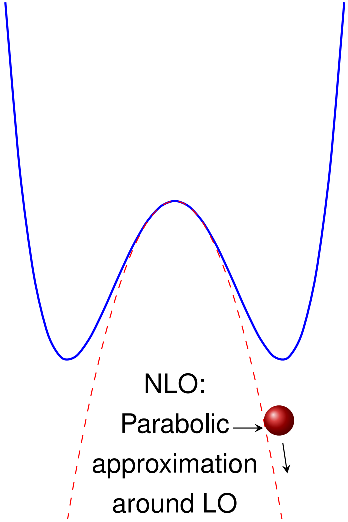
This gives: LO+NLO+Subset of higer orders

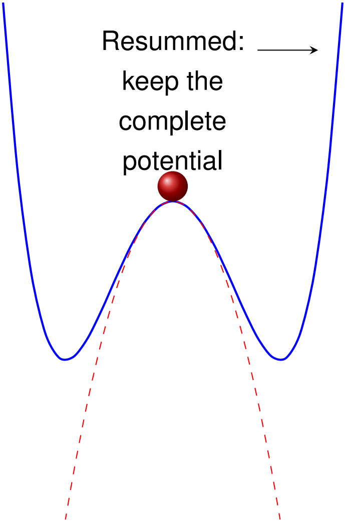
Implications of the classical-statistical method

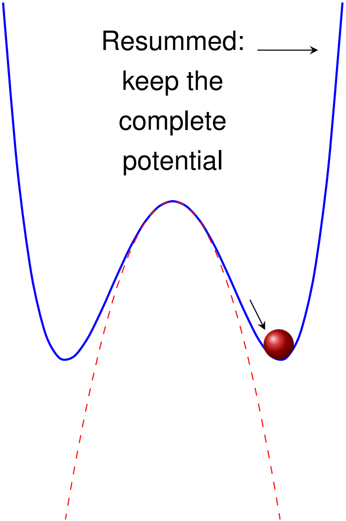


Implications of the classical-statistical method




NLO:
Parabolic
approximation
around LO


Resummed:
keep the
complete
potential


Resumed:
keep the
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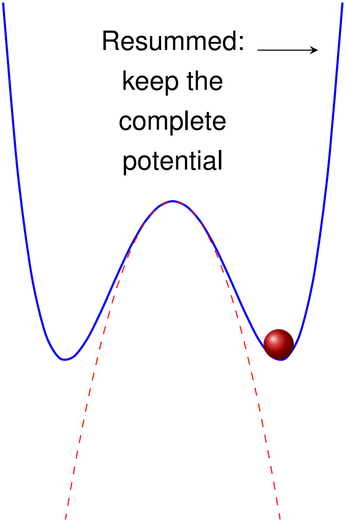
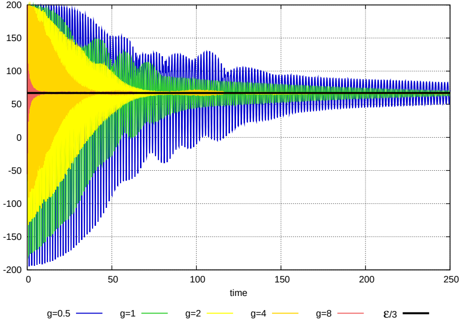
IMPLICATIONS OF THE CLASSICAL-STATISTICAL METHOD



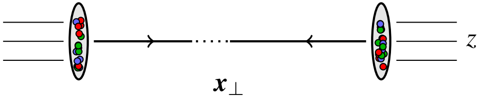


3 A PROOF OF CONCEPT: SCALAR FIELD THEORY

The Theory

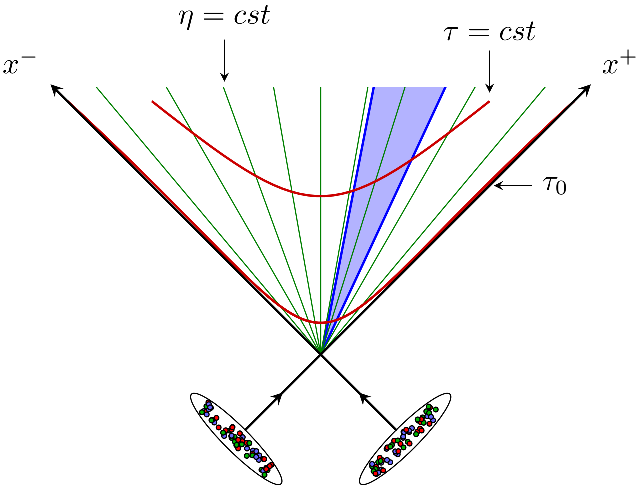
Numerical results

Adapted coordinate system to describe a Heavy Ion Collision?



System boost invariant in z direction

Proper time/rapidity coordinate system



The model

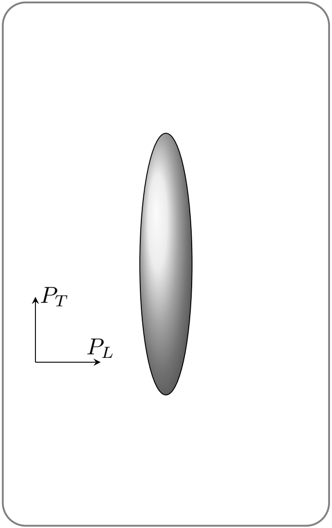
Initial conditions: classical statistical method

$$\phi(\tau_0, \boldsymbol{x}_\perp, \eta) = \varphi_0(\tau_0, \boldsymbol{x}_\perp) + \sum_{\boldsymbol{k}_\perp, \nu} c_{\nu \boldsymbol{k}_\perp} e^{i\nu\eta} a_{\nu, \boldsymbol{k}_\perp}(\tau_0, \boldsymbol{x}_\perp)$$

Time evolution: Klein Gordon equation

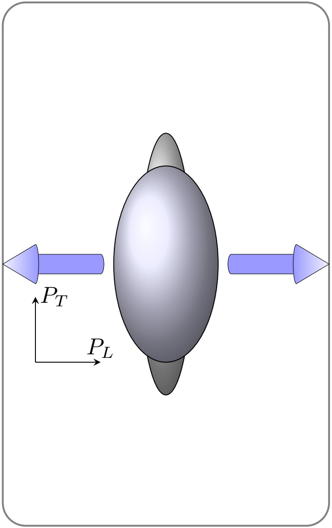
$$\underbrace{\left[\frac{\partial^2}{\partial\tau^2} + \frac{1}{\tau}\frac{\partial}{\partial\tau} - \boldsymbol{\nabla}_\perp^2 - \frac{1}{\tau^2}\frac{\partial^2}{\partial\eta^2}\right]}_{\square} \phi + \frac{g^2}{6}\phi^3 = 0$$

Initial anisotropy

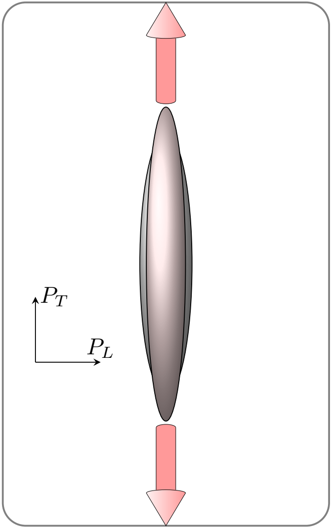


How interactions compete with expansion?

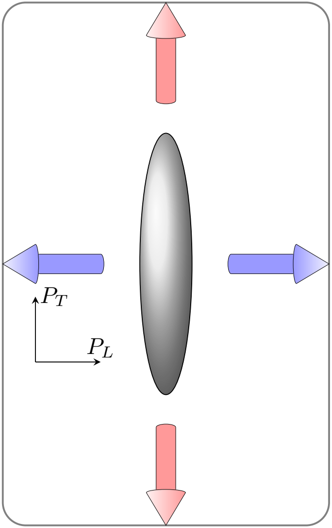
Interactions isotropize the system



Expansion dilutes the system



Expansion $\lesssim$ Interactions for realistic α_s?



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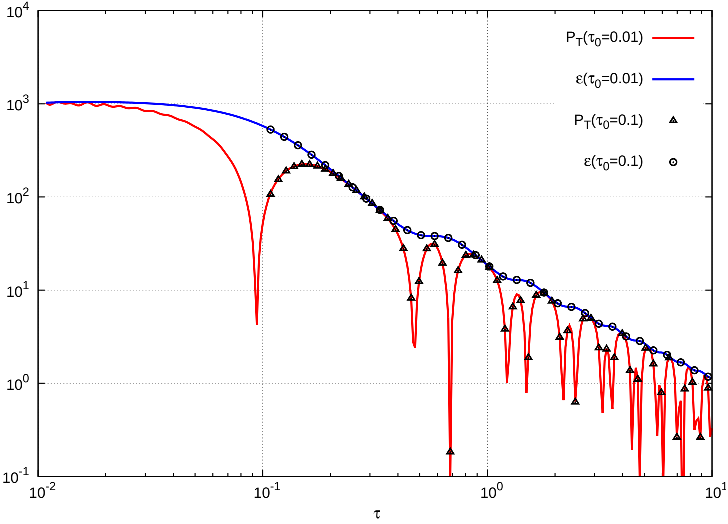
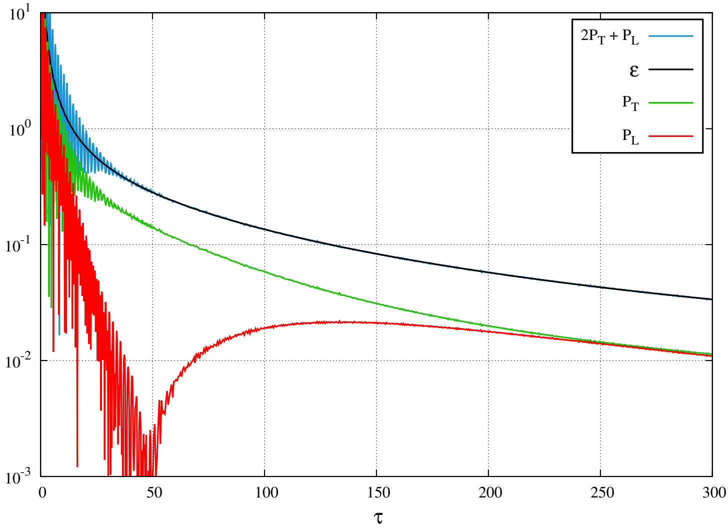
The Theory

Numerical results

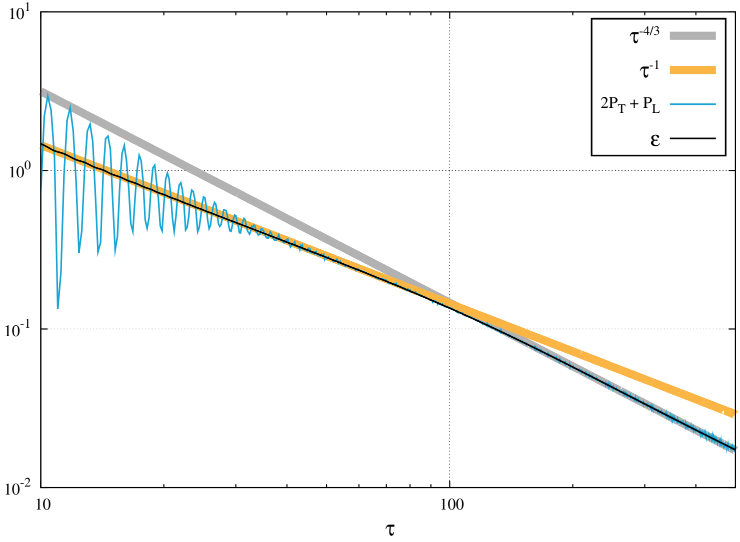
$T^{\mu\nu}_{\text{RESUM}}$ [Dusling, TE, Gelis, Venugopalan (2012]



$T^{\mu\nu}_{\text{RESUM}}$ [DUSLING, TE, GELIS, VENUGOPALAN (2012]



ϵ BEHAVIOUR



Bjorken Law: $\partial_\tau \epsilon + \frac{\epsilon + P_L}{\tau} = 0$

Equation of state: $\epsilon = 2P_L + P_T$

IDEAL HYDRO

Isotropic system

$$T^{\mu\nu}_{\text{ideal}} = \epsilon u^{\mu} u^{\nu} - P(g^{\mu\nu} - u^{\mu} u^{\nu})$$

Equation of state: $\epsilon = 2P_L + P_T$

IDEAL HYDRO

Isotropic system

$$T^{\mu\nu}_{\text{ideal}} = \epsilon u^\mu u^\nu - P(g^{\mu\nu} - u^\mu u^\nu)$$

VISCOUS HYDRO

Anisotropic system

$$T^{\mu\nu} = T^{\mu\nu}_{\text{ideal}} + \eta\, \pi^{\mu\nu}$$

In our case

$$P_T = \frac{\epsilon}{3} + \frac{2\eta}{3\tau} \qquad P_L = \frac{\epsilon}{3} - \frac{4\eta}{3\tau}$$

First order Viscous Hydrodynamics

BJORKEN's Law (coming from $\partial_\mu T^{\mu\nu} = 0$):

$$\partial_\tau \epsilon + \frac{\epsilon + P_L}{\tau} = 0 \;\rightarrow \partial_\tau \epsilon + \frac{4}{3}\frac{\epsilon}{\tau} = \frac{4}{3}\frac{\eta}{\tau^2}$$

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assuming $\eta = \frac{\eta_0}{\tau}$ and STEFAN-BOLTZMANN entropy $s \approx \epsilon^{\frac{3}{4}}$

$$\partial_\tau \epsilon + \frac{4}{3}\frac{\epsilon}{\tau} = \frac{4}{3} \underbrace{\frac{\eta}{s}}_{\text{cte}} \frac{\epsilon^{\frac{3}{4}}}{\tau^2}$$

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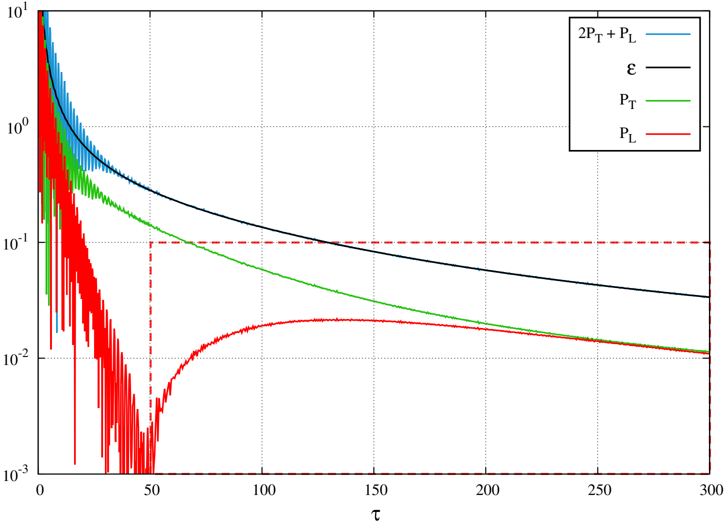
$$\partial_\tau \epsilon + \frac{4}{3}\frac{\epsilon}{\tau} = \frac{4}{3}\underbrace{\frac{\eta}{s}}_{\text{cte}}\frac{\epsilon^{\frac{3}{4}}}{\tau^2}$$

At a given time, knowing ϵ, P_T, P_L and assuming

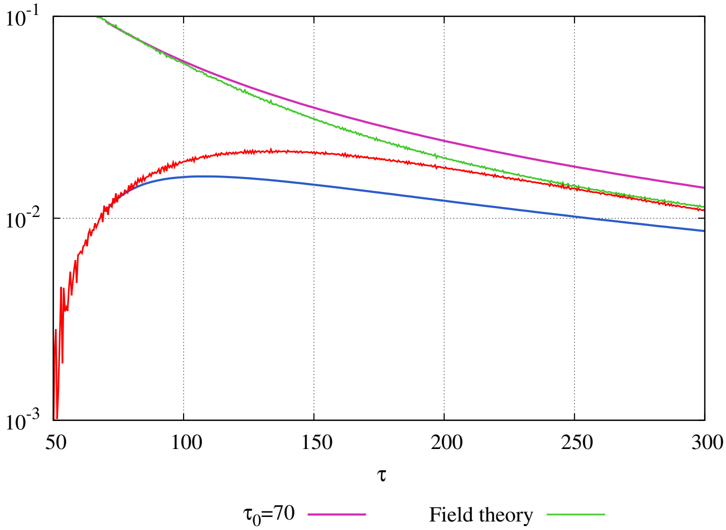
- an EOS
- STEFAN-BOLTZMANN entropy
- $\eta = \frac{\eta_0}{\tau}$
- $\frac{\eta}{s} =$ cte

gives a very simple hydro model.

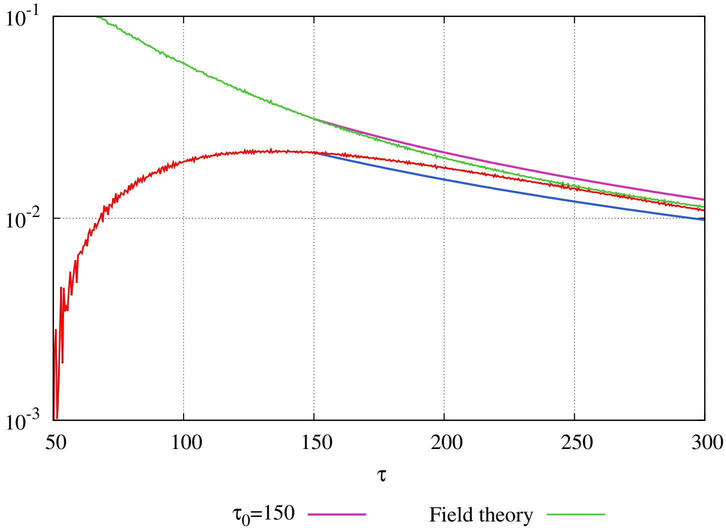
COMPARISON WITH HYDRO: ISOTROPIZATION



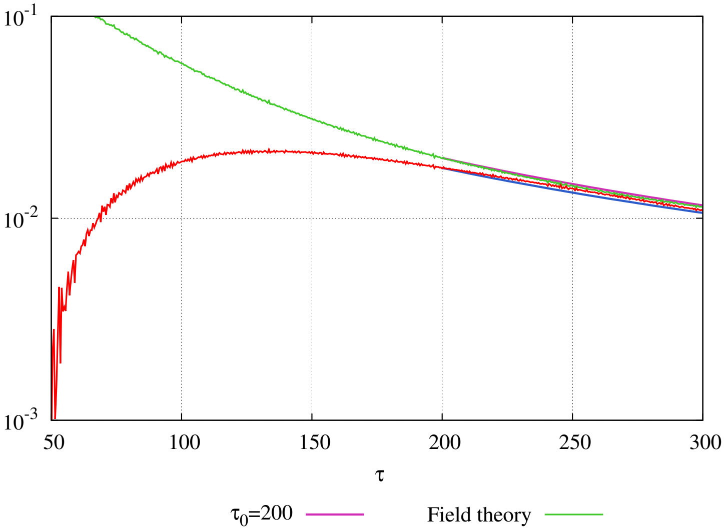
Comparison with Hydro: isotropization



Comparison with Hydro: isotropization

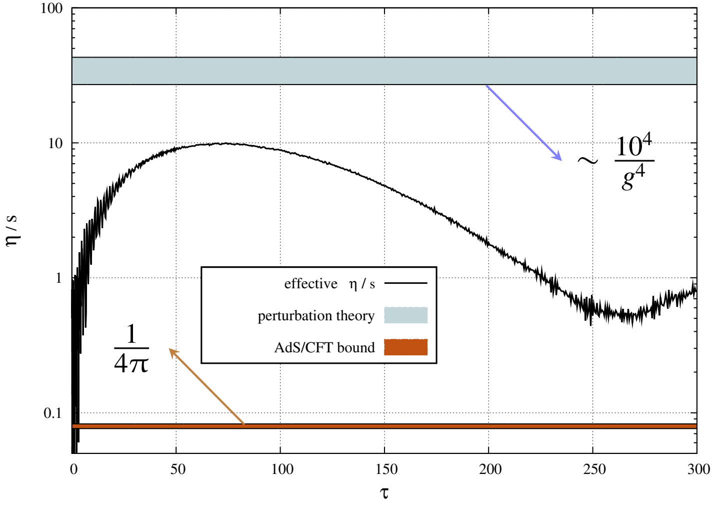


Comparison with Hydro: isotropization



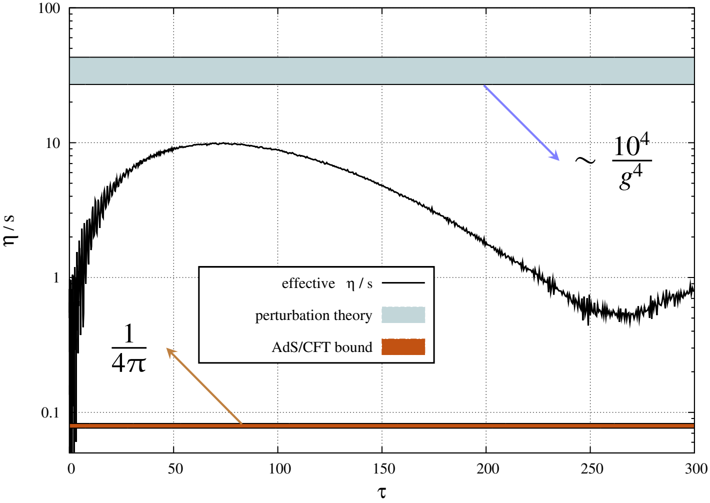
Comparison with Hydro: viscosity

$$P_T - P_L = \frac{2\eta}{\tau}$$



Comparison with Hydro: viscosity

see also [Asakawa, Bass, Muller (2006-07)]

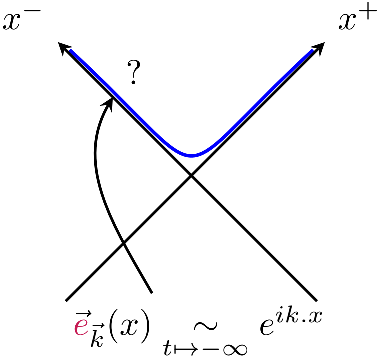


4 YANG-MILLS THEORY

The theory

Numerical results

- Need to know $\vec{e}_{\vec{k}}(\tau_0, \vec{x})$ at the time τ_0 we start the numerical simulation
- For practical reasons, we must start in the forward light cone $(\tau_0 > 0)$



This can be done analytically [TE,GELIS 1307:1765]

Result of [TE,GELIS 1307:1765]

$$e^{i}_{\nu\vec{k}_\perp} = i\nu\left(F^{i,-} - F^{i,+}\right) \qquad e^{\eta}_{\nu\vec{k}_\perp}(x) = \mathcal{D}^i\left(F^{i,-} - F^{i,+}\right)$$

with

$$F_k^{i,+}(x) \sim e^{i\nu\eta}\,\mathcal{U}_1^\dagger(\vec{x}_\perp)\int_{\vec{p}_\perp} e^{i\vec{p}_\perp\cdot\vec{x}_\perp}\,\widetilde{\mathcal{U}}_1(\vec{p}_\perp+\vec{k}_\perp)\left(\frac{p_\perp^2\tau}{2k_\perp}\right)^{i\nu}\left[\delta^{ij} - \frac{2p^i_\perp p^j_\perp}{p_\perp^2}\right]\epsilon^j_{k\lambda}\ .$$

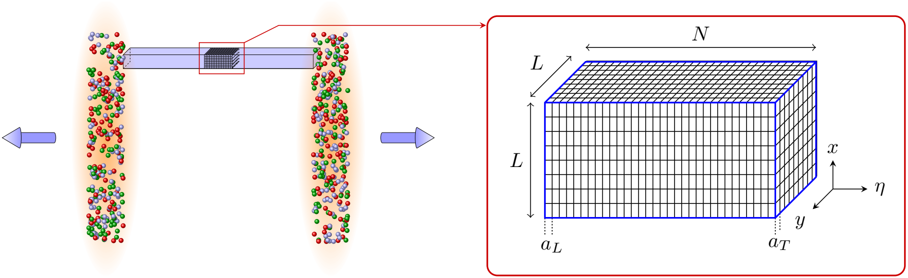
- $\mathcal{U}_1^\dagger$ depends on the color source J^+ of the first nucleus
- Analogous formula for $F^{i,-}$.

4 YANG-MILLS THEORY

The theory

Numerical results

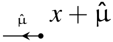
Gauge potential $A^\mu \to$ link variables (exact gauge invariance on the lattice)



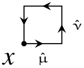
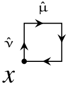
Numerical parameters

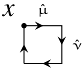
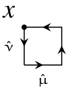
- Transverse lattice size $L = 64$, transverse lattice spacing $Q_s a_T = 1$
- Longitudinal lattice size $N = 128$, longitudinal lattice spacing $a_L = 0.016$
- Number of configurations for the Monte-Carlo $N_{\text{conf}} = 200$ to 2000
- Initial time $Q_s \tau_0 = 0.01$

Writing

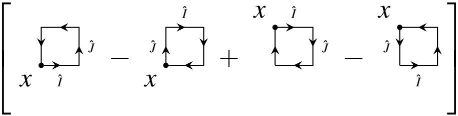
$E^{\mu}(x) =$  $\quad U_{\mu}(x) =$  $\quad U^{\dagger}_{\mu}(x) =$ 

and

$U_{\mu\nu}(x) =$  $\quad U^{\dagger}_{\mu\nu}(x) =$ 

$U_{\mu-\nu}(x) =$  $\quad U^{\dagger}_{\mu-\nu}(x) =$  .

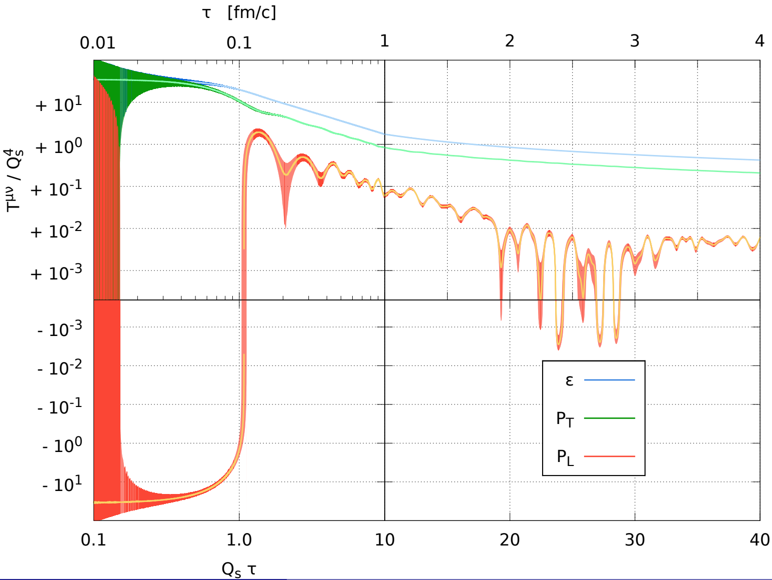
We can therefore rewrite the EOM as

∂_τ  $= \frac{-i}{2ga_I a_J^2} \sum_J$ 

∂_τ  $= -i\, g\, a_I$ 

NUMERICAL RESULTS [TE,GELIS 1307:2214]

$$\alpha_s = 8\,10^{-4}\ (g = 0.1)$$



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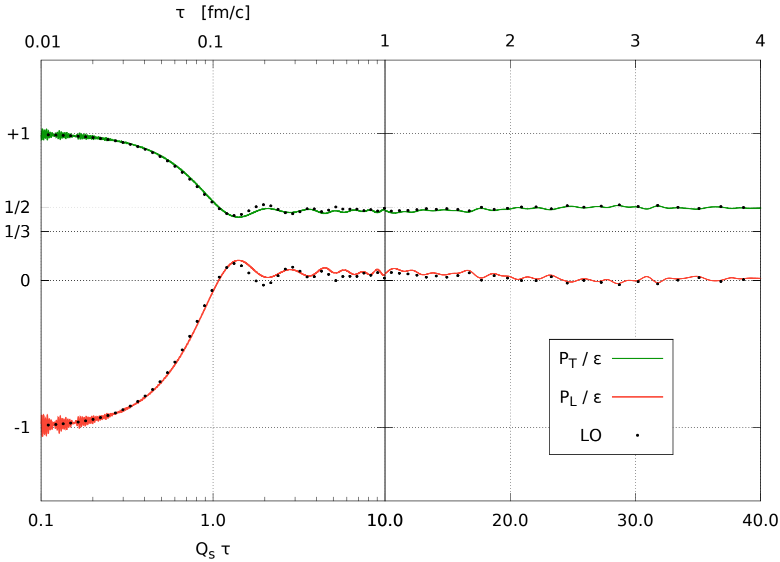
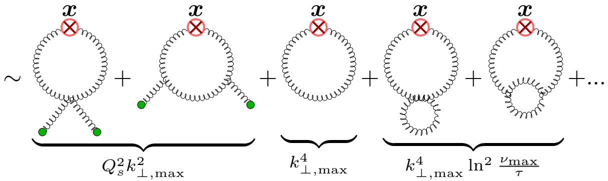


$$\left\langle E_{L,\mathrm{div}}^2 \right\rangle \sim Q_s^2 k_{\perp,\max}^2 + k_{\perp,\max}^4 + k_{\perp,\max}^4 \ln^2 \frac{\nu_{\max}}{\tau} + \ldots$$



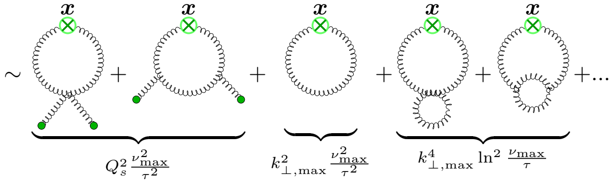
3 last diagrams can be substracted with a simulation where

$$A_\mu^a(x) = 0 + a_\mu^a(x)$$

E_L^2 "fine" (B_L^2 too)

RENORMALIZATION PROCEDURE

$$\left\langle E_{T,\mathrm{div}}^2 \right\rangle \sim Q_s^2 \frac{\nu_{\max}^2}{\tau^2} + k_{\perp,\max}^2 \frac{\nu_{\max}^2}{\tau^2} + k_{\perp,\max}^4 \ln^2 \frac{\nu_{\max}}{\tau} + \ldots$$



3 last diagrams can be substracted with a simulation where

$$A_\mu^a(x) = 0 + a_\mu^a(x)$$

How to deal with the first 2? $\rightarrow$ ad-hoc fit for the time being.

Otherwise E_L^2 and B_L^2 behaves as τ^{-2} at early time.

Renormalization procedure

$$\epsilon = E_T^2 + B_T^2 + \underbrace{E_L^2}_{\text{fine}} + \underbrace{B_L^2}_{\text{fine}}$$

$$P_T = \underbrace{E_L^2}_{\text{fine}} + \underbrace{B_L^2}_{\text{fine}}$$

$$P_L = E_T^2 + B_T^2 - \underbrace{E_L^2}_{\text{fine}} - \underbrace{B_L^2}_{\text{fine}}$$

Renormalization procedure

$$\langle P_T \rangle_{\text{phys.}} = \langle P_T \rangle_{\substack{\text{backgd.}\\ +\ \text{fluct.}}} - \langle P_T \rangle_{\substack{\text{fluct.}\\ \text{only}}}$$

$$\langle \epsilon, P_L \rangle_{\text{phys.}} = \underbrace{\langle \epsilon, P_L \rangle_{\substack{\text{backgd.}\\ +\ \text{fluct.}}}}_{\text{computed}} - \underbrace{\langle \epsilon, P_L \rangle_{\substack{\text{fluct.}\\ \text{only}}}}_{\text{computed}} + \underbrace{A\,\tau^{-2}}_{\text{fitted}} \ .$$

Ad-hoc term only one to satisfy Bjorken law and EOS:

$$\partial_\tau \tau^{-\alpha} + 2\tau^{-\alpha-1} = 0$$

How come that problematic divergent diagrams behaves as mass terms?

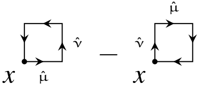
In the continuum limit, they don't exist local gauge invariant operators of dimension two.

On the lattice though, they coud be terms like

$$g^2 \frac{\nu_{\max}^2}{k_{\perp,\max}^2 \tau^2} \mathrm{Tr} F^2,$$

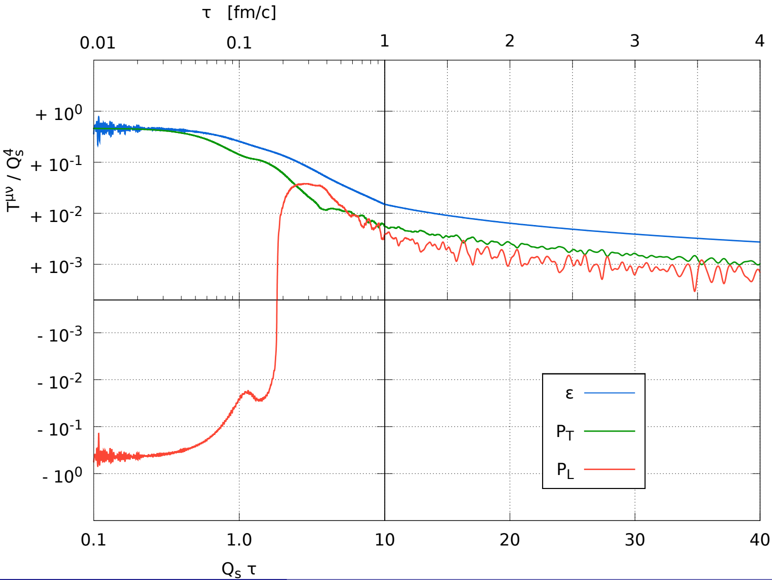
where

$$F_{\mu\nu}(x) \sim$$



NUMERICAL RESULTS [TE,GELIS 1307:2214]

$\alpha_s = 2\,10^{-2}$ $(g = 0.5)$



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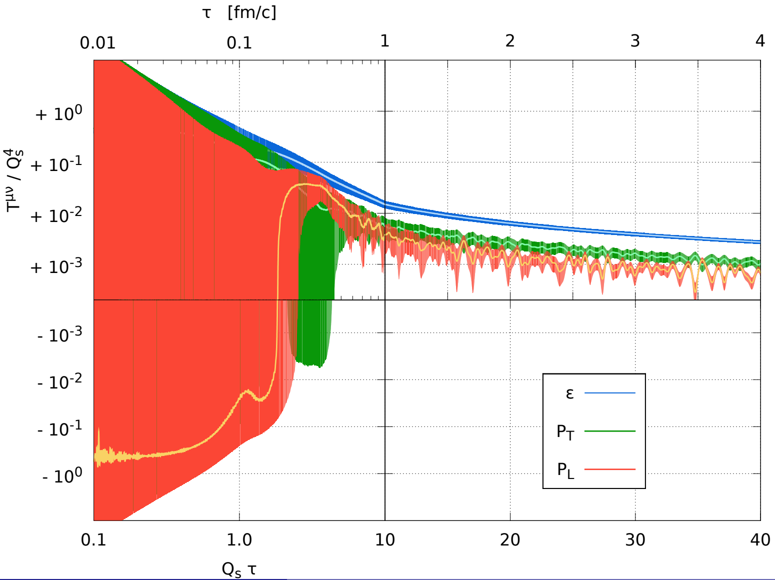

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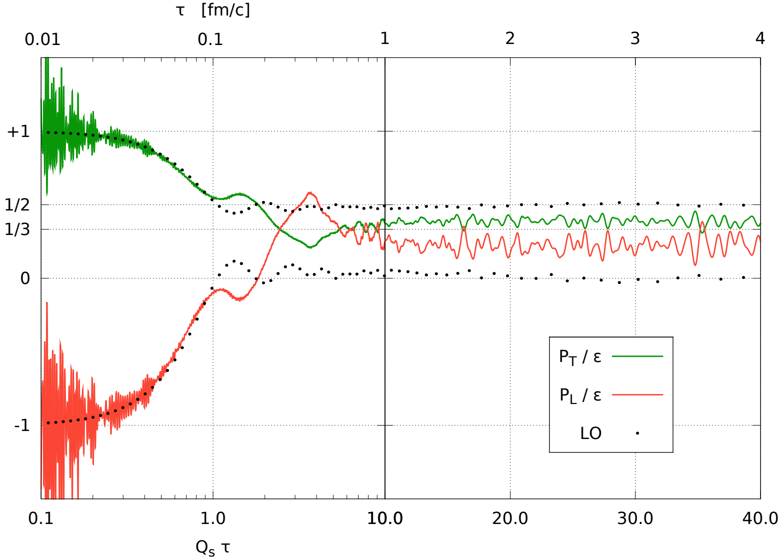

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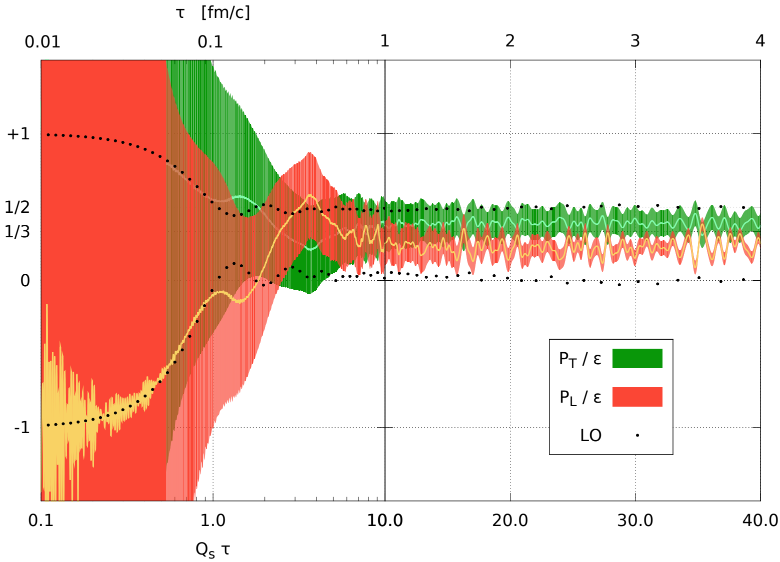

Assuming simple first order viscous hydrodynamics

$$\epsilon \approx \underbrace{\epsilon_0 \tau^{-\frac{4}{3}}}_{\text{Ideal hydro}} - \underbrace{2\eta_0 \tau^{-2}}_{\text{first order correction}}$$

we can compute the dimensionless ratio ($\eta = \eta_0 \tau^{-1}$)

$$\eta \epsilon^{-\frac{3}{4}} \lesssim 1$$

In contrast, perturbation theory at LO gives $\eta \epsilon^{-\frac{3}{4}} \sim 300$.

If the system is closed from being thermal

$$\epsilon^{-\frac{3}{4}} \sim s \Longrightarrow \frac{\eta}{s} \text{ Not far from } \frac{1}{4\pi}$$

- Correct NLO spectrum from first principles
- Fixed anisotropy for $g = 0.5$ at $\tau \sim 1fm/c$
- No need for strong coupling to get isotropization
- Compatible with viscous hydrodynamical expansion
- Assuming simple first order viscous hydrodynamics

$$\eta\epsilon^{-\frac{3}{4}} \lesssim 1$$

Viscous Hydrodynamics

I) Macroscopic theory
II) Few parameters: P_L, P_T, ϵ, $\vec{u}$
III) Need input:

1) Equation of state $f(P_L, P_T) = \epsilon$
2) Small anisotropy
3) Initialization: $\epsilon(\tau_0)$, $P_L(\tau_0)$? ...
4) viscous coefficients: shear viscosity η,...
5) Short isotropization time