Onset of hydrodynamical behaviour in heavy ion collisions

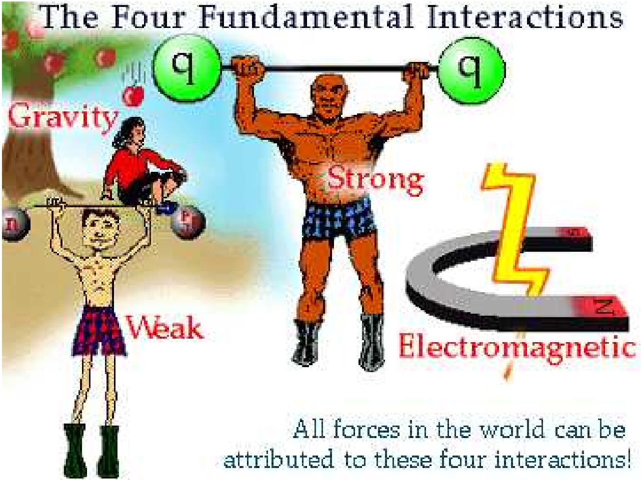
Frankfurt, 24th October 2013

Thomas EPELBAUM

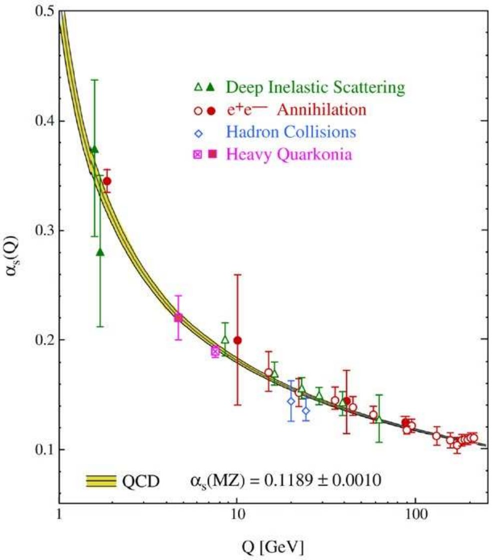
IPhT

OUTLINE

THE STRONG FORCE: A PECULIAR ONE

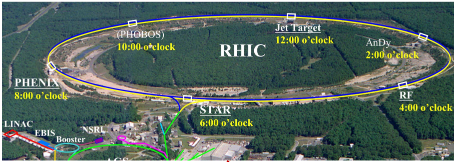


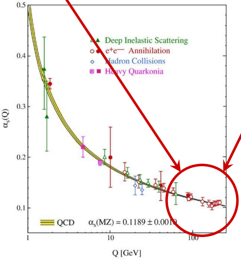
THE STRONG FORCE: A PECULIAR ONE

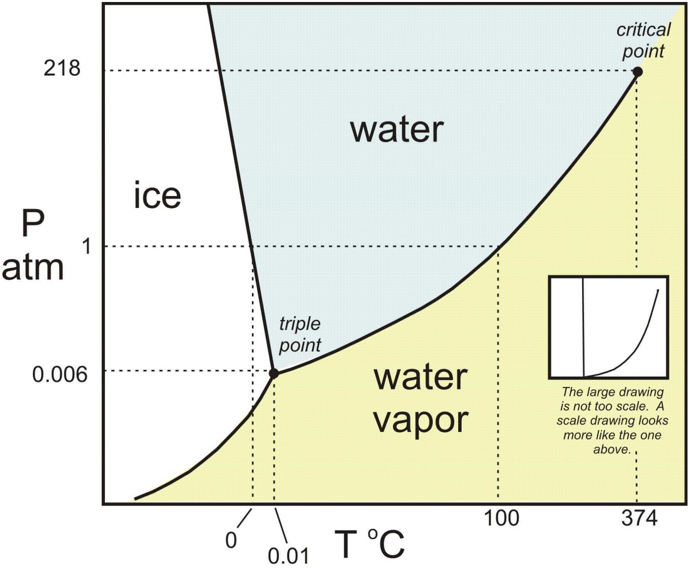


LHC **RHIC**

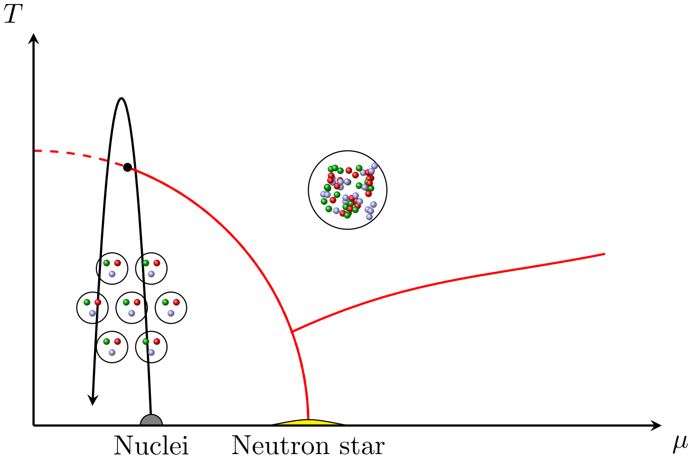

ALICE
CMS
ATLAS
LHCb

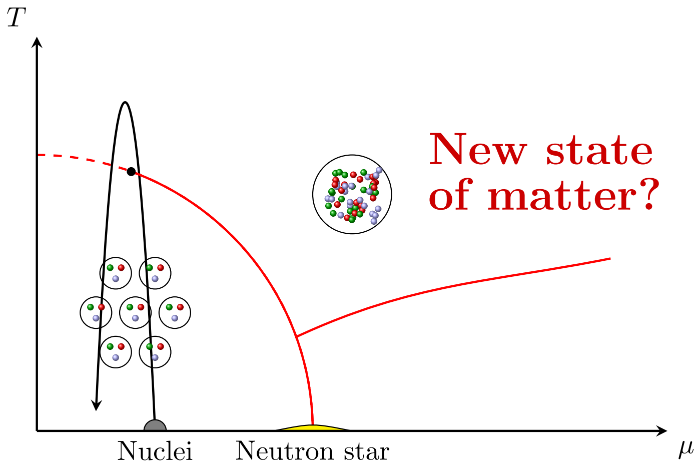

(PHOBOS)
10:00 o'clock
Jet Target
12:00 o'clock
RHIC
AnDy
2:00 o'clock
PHENIX
8:00 o'clock
RF
4:00 o'clock
STAR
6:00 o'clock
LINAC
EBIS
Booster
NSRL


Deep Inelastic Scattering
e+e— Annihilation
Hadron Collisions
Heavy Quarkonia
QCD
$\alpha_s(MZ) = 0.1189 \pm 0.0010$
$\alpha_s(Q)$
Q [GeV]
0.5
0.4
0.3
0.2
0.1
1
10
100



critical point
218
water
ice
P atm
1
triple point
0.006
water vapor
The large drawing is not too scale. A scale drawing looks more like the one above.
0
0.01
T °C
100
374

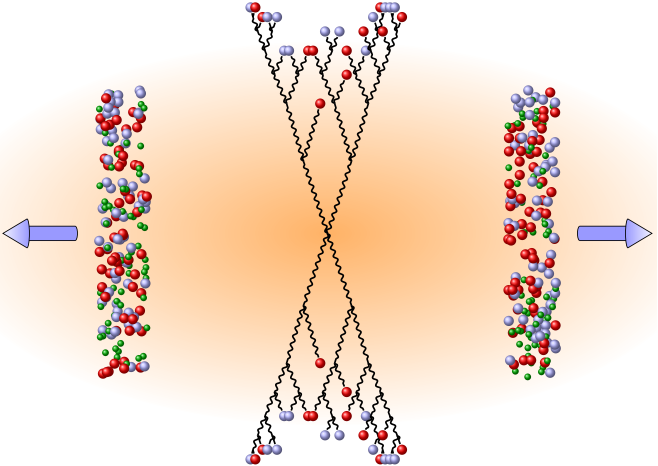

T
Nuclei
Neutron star
μ


T
New state
of matter?
Nuclei
Neutron star
μ

1 Motivation

A new state of matter: the Quark Gluon Plasma

A NEW STATE OF MATTER: THE QUARK GLUON PLASMA



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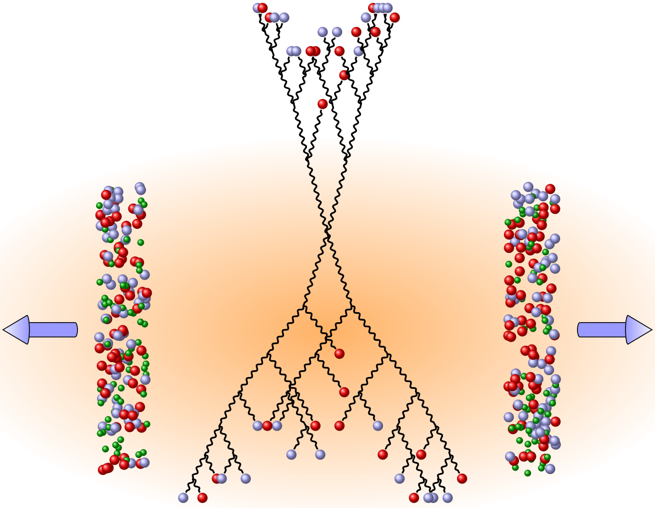

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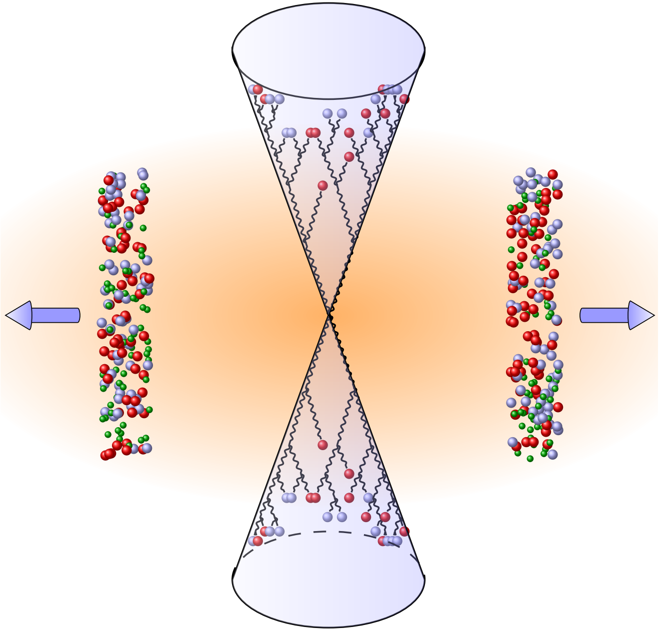

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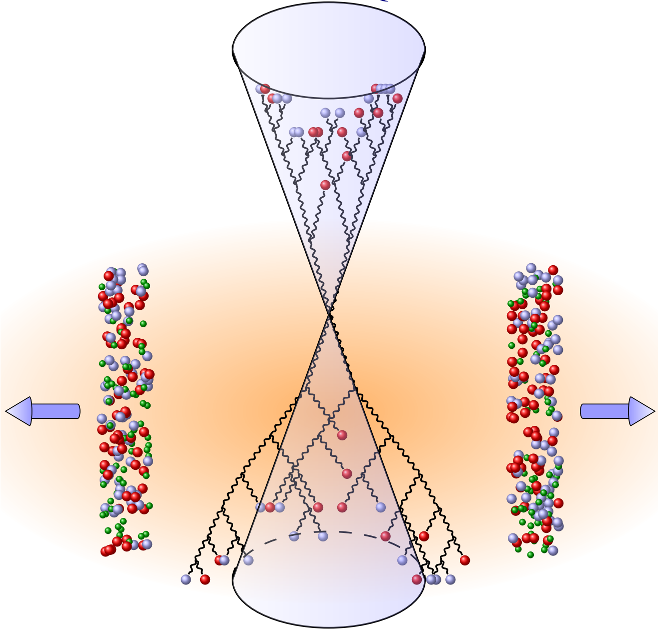

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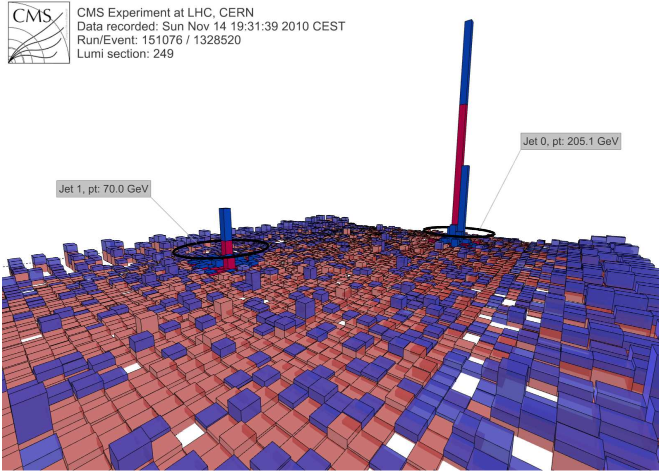
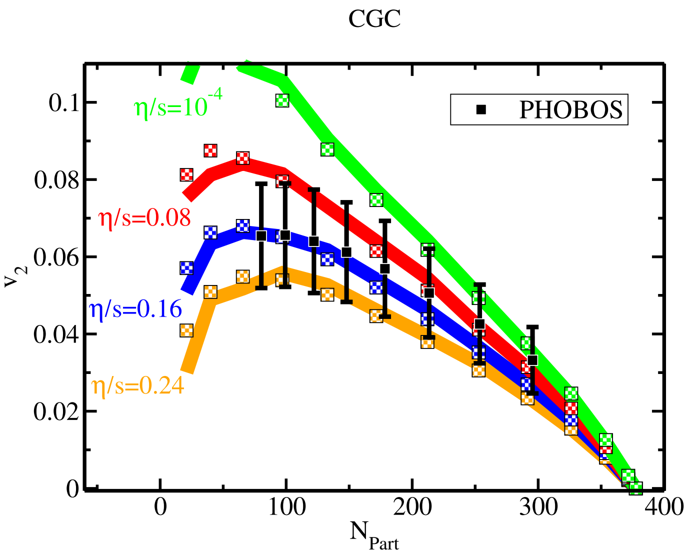


HOW TO DEAL WITH THE QGP?



[LUZUM, ROMATSCHKE (2008)]

Viscous Hydrodynamics

I) Macroscopic theory
II) Few parameters: P_L, P_T, ϵ, $\vec{u}$
III) Need input:

1) Equation of state $f(P_L, P_T) = \epsilon$
2) Small anisotropy
3) Initialization: $\epsilon(\tau_0)$, $P_L(\tau_0)$? ...
4) viscous coefficients: shear viscosity η,...
5) Short isotropization time

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None of this is easy to get from QCD

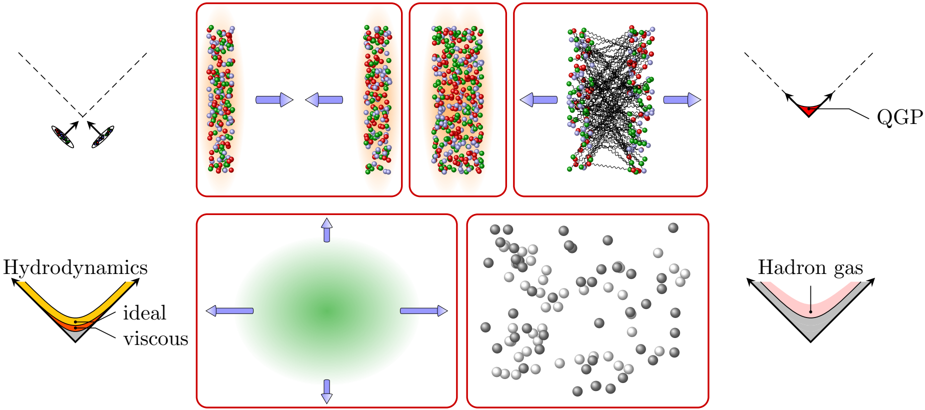
1 MOTIVATION

Probing the strong force

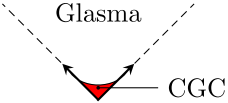
The Quark Gluon Plasma

How to deal with a Heavy Ion Collision

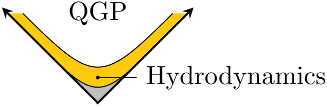
Heavy Ion Collisions: The general picture



Early transition: the problem



Isotropization?
Time scale?

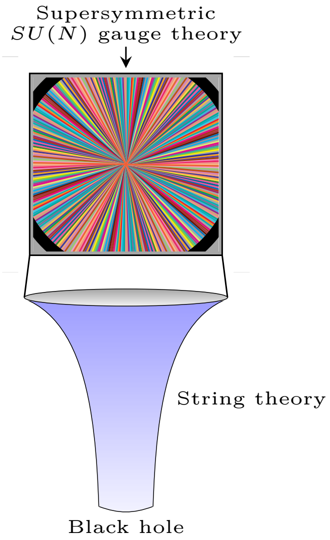


Huge anisotropy
(negative P_L)

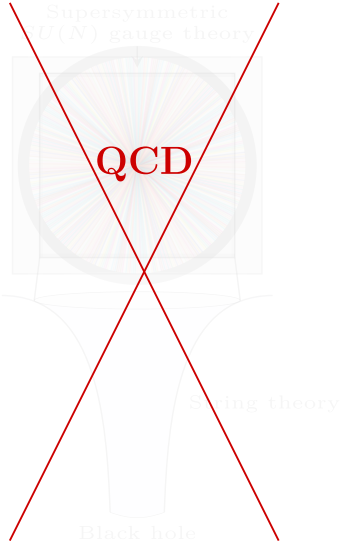
Small anisotropy

Long time puzzle: Does (fast) isotropization occur?

Strongly coupled method: AdS/QCD?

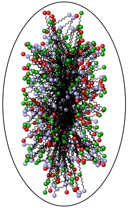


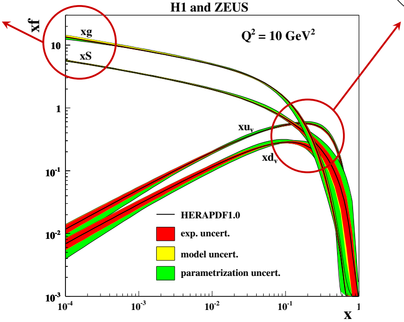
Weakly coupled method: QCD

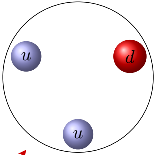


How to study the transition?

Weakly coupled QCD with only gluons

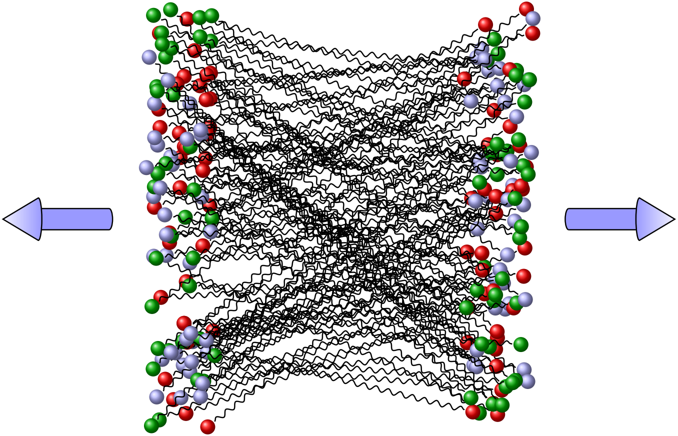




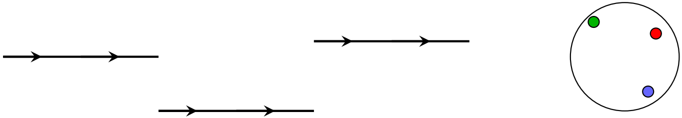


Weakly coupled method at dense regime:

$\alpha_s \ll 1$ **but** $f_{\mathrm{gluon}} \sim \frac{1}{\alpha_s}$



Two additional features: saturation and time dilation



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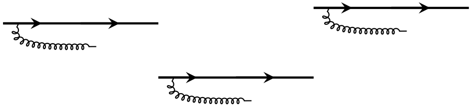

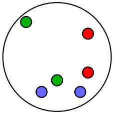

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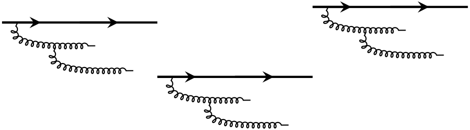

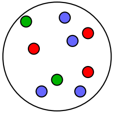

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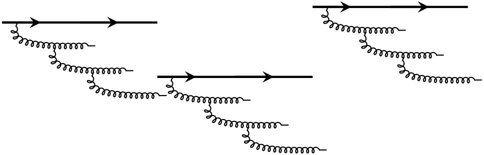

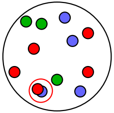

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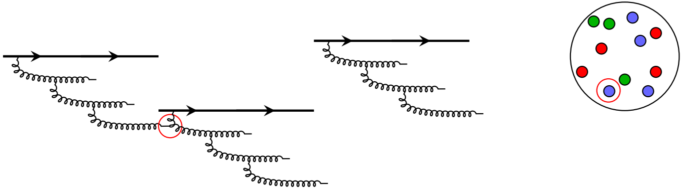

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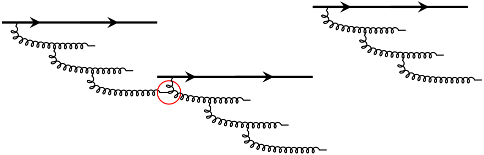
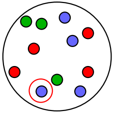
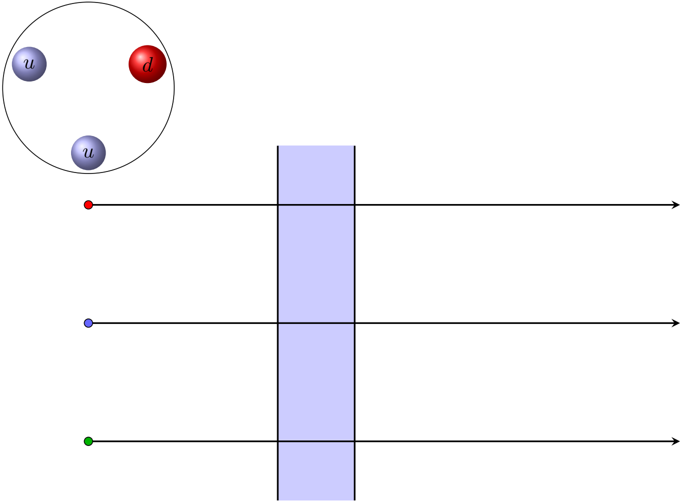




Gluon saturation when emission $=$ recombination
$\Rightarrow$ **Saturation scale** Q_s

Two additional features: saturation and time dilation



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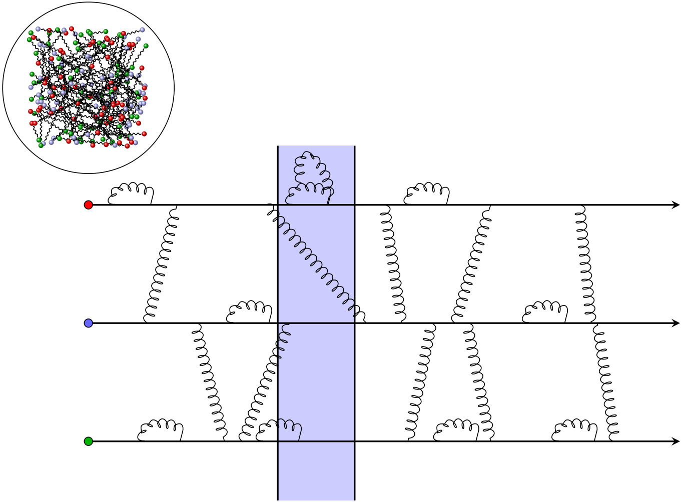

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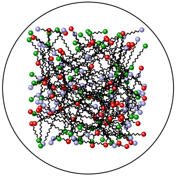

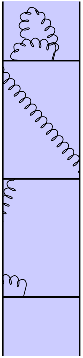

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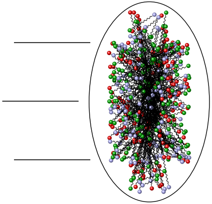

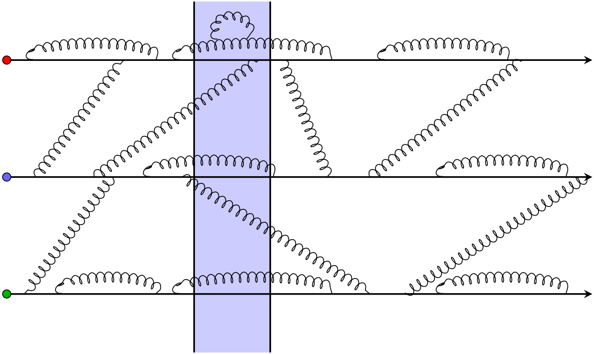

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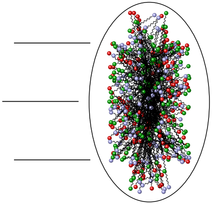
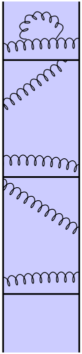




2. THEORETICAL FRAMEWORK

The Color Glass Condensate

The Classical statistical approximation

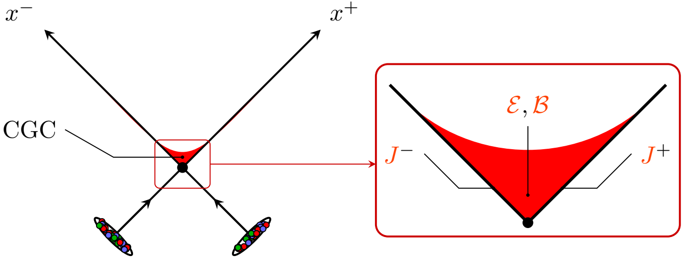
THE MAIN ASSUMPTIONS

- Fast gluons are "frozen" by time dilation.
- Described as static color sources J located on the light cone axis
- Small $x \rightarrow$ Gluon saturation $\rightarrow J \sim Q_s^3 \, \alpha_s^{-1/2}$.
- Slow gluons are the standard gauge field $\mathcal{A}^\mu \sim Q_s \, \alpha_s^{-1/2}$.
- System boost-invariant $\rightarrow \mathcal{A}^\mu$ rapidity independant.

Langrangian of theory reads

$$\mathcal{L} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + J_\mu \mathcal{A}^\mu$$

Theoretical framework (Weakly coupled but strongly interacting)



LO: $\epsilon = \frac{1}{2} \underbrace{\left(\vec{\mathcal{E}}^2 + \vec{\mathcal{B}}^2\right)}_{\substack{\textbf{Classical} \\ \text{color fields}}}$ $\quad \mathcal{D}_\mu \mathcal{F}^{\mu\nu} = \underbrace{J^\nu}_{\substack{\text{Color sources} \\ \text{on the light cone}}}$

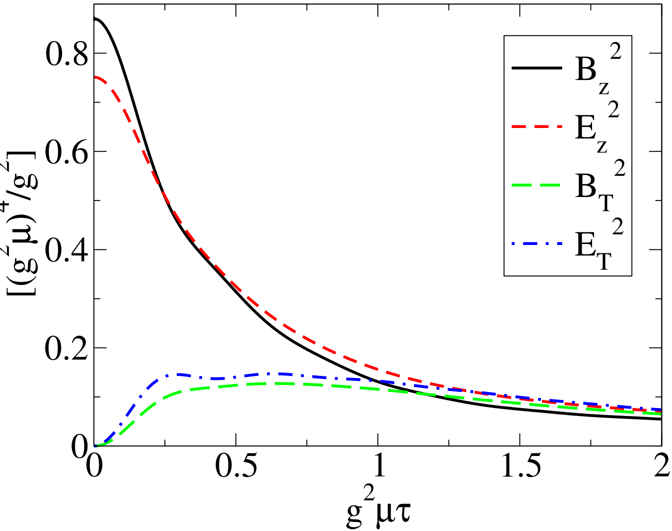
[KRASNITZ, VENUGOPALAN (1998)]

THE COLOR GLASS CONDENSATE [MCLERRAN, VENUGOPALAN (1993)]

$$\epsilon = \mathcal{E}_\perp^2 + \mathcal{B}_\perp^2 + \mathcal{E}_L^2 + \mathcal{B}_L^2$$

$$P_T = \mathcal{E}_L^2 + \mathcal{B}_L^2$$

$$P_L = \mathcal{E}_\perp^2 + \mathcal{B}_\perp^2 - \mathcal{E}_L^2 - \mathcal{B}_L^2$$



[LAPPI, MCLERRAN (2006)]

THE COLOR GLASS CONDENSATE [MCLERRAN, VENUGOPALAN (1993)]

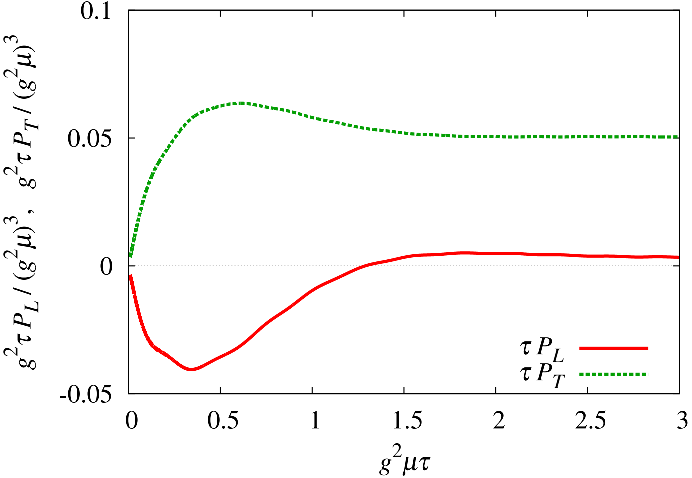
$$\epsilon = \underbrace{\mathcal{E}_\perp^2}_{0} + \underbrace{\mathcal{B}_\perp^2}_{0} + \mathcal{E}_L^2 + \mathcal{B}_L^2$$

$$P_T = \mathcal{E}_L^2 + \mathcal{B}_L^2$$

$$P_L = \underbrace{\mathcal{E}_\perp^2}_{0} + \underbrace{\mathcal{B}_\perp^2}_{0} - \mathcal{E}_L^2 - \mathcal{B}_L^2$$

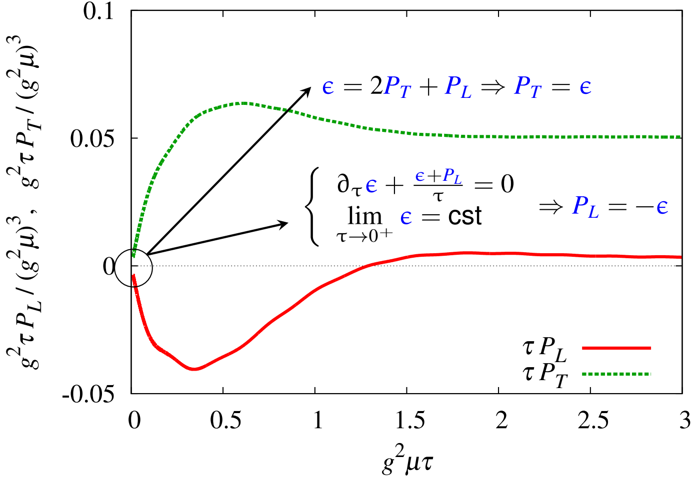
Initial $T^{\mu\nu}$ is $(\epsilon, \epsilon, \epsilon, -\epsilon)$!

Strong anisotropy at early time



[GELIS, FUKUSHIMA (2012)]

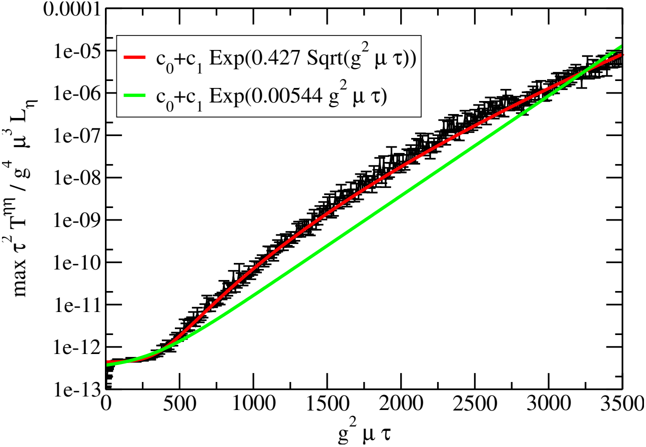
Strong anisotropy at early time



[GELIS, FUKUSHIMA (2012)]

$$E^2(x) = \underbrace{\mathcal{E}^2(x)}_{\text{LO}} + \underbrace{\frac{1}{2}\int_{\vec{k}} \left|e_{\vec{k}}(x)\right|^2}_{\text{NLO}} + \cdots$$

$e_{\vec{k}}(x)$ perturbation to $\mathcal{E}(x)$ created by a plane wave of momentum $\vec{k}$ in the remote past.

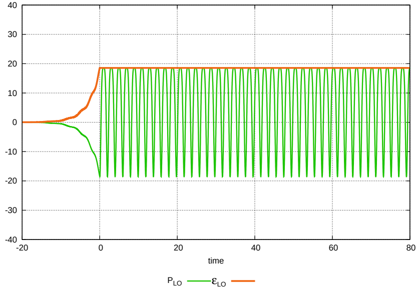


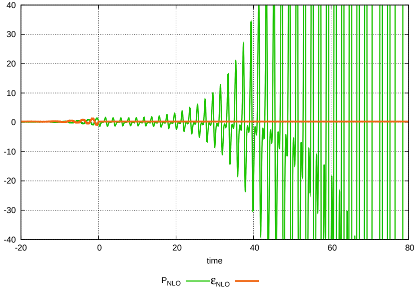
[ROMATSCHKE, VENUGOPALAN (2006)]

Small Fluctuations grow exponentially (Weibel instability)

- Because of instabilities, the NLO correction eventually becomes as large as the LO ⇒ Important effect, should be included

- NLO alone will grow forever ⇒ unphysical effect, should be taken care of





- Such growing contributions are present at all orders of the perturbative expansion

How to deal with them?

2 THEORETICAL FRAMEWORK

The Color Glass Condensate

The Classical statistical approximation

- At the initial time $\tau = \tau_0$, take:

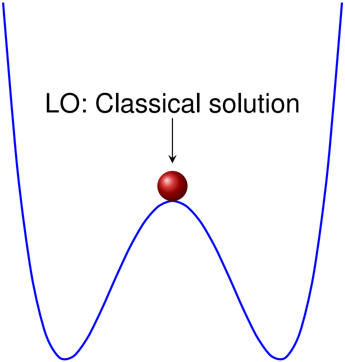
$$\vec{E}_0(\tau_0, \vec{x}) = \vec{\mathcal{E}}_0(\tau_0, \vec{x}) + \int_{\vec{k}} c_{\vec{k}} \vec{e}_{\vec{k}}(\tau_0, \vec{x})$$

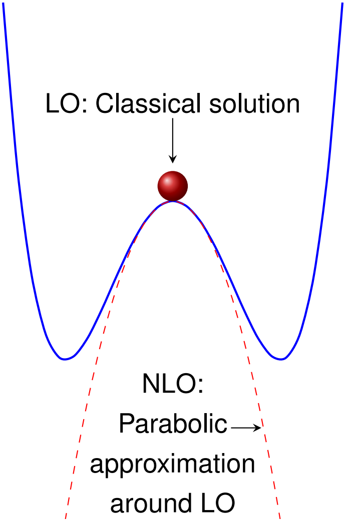
  where $c_{\vec{k}}$ are random coefficients: $\left\langle c_{\vec{k}} c_{\vec{k}'} \right\rangle \sim \delta_{\vec{k}\vec{k}'}$

- Solve the **Classical** equation of motion $D_\mu F^{\mu\nu} = J^\nu$
- Compute $\left\langle \vec{E}^2(\tau, \vec{x}) \right\rangle$, where $\langle\rangle$ is the average on the $c_{\vec{k}}$ (Monte-Carlo)
- One can show that this resums all the fastest growing terms at each order, leading to a result that remain bounded when $\tau \to \infty$ [GELIS, LAPPI, VENUGOPALAN (2008)]

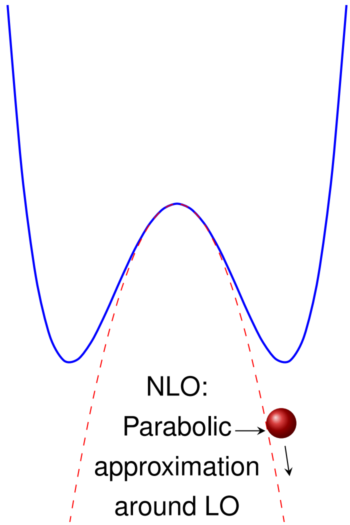
This gives: LO+NLO+Subset of higer orders

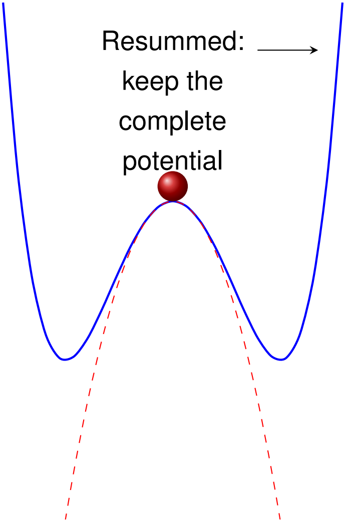
Implications of the classical-statistical method

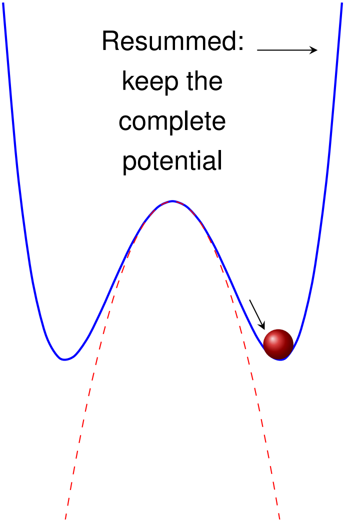



LO: Classical solution
NLO:
Parabolic
approximation
around LO

IMPLICATIONS OF THE CLASSICAL-STATISTICAL METHOD




Resummed:
keep the
complete
potential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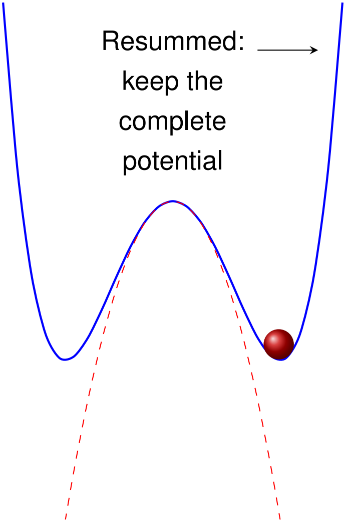
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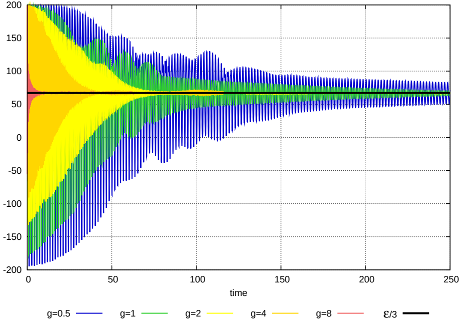
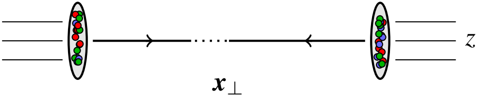

200
150
100
50
0
-50
-100
-150
-200
0
50
100
150
200
250
time
g=0.5
g=1
g=2
g=4
g=8
ε/3

3 A PROOF OF CONCEPT: SCALAR FIELD THEORY

The Theory

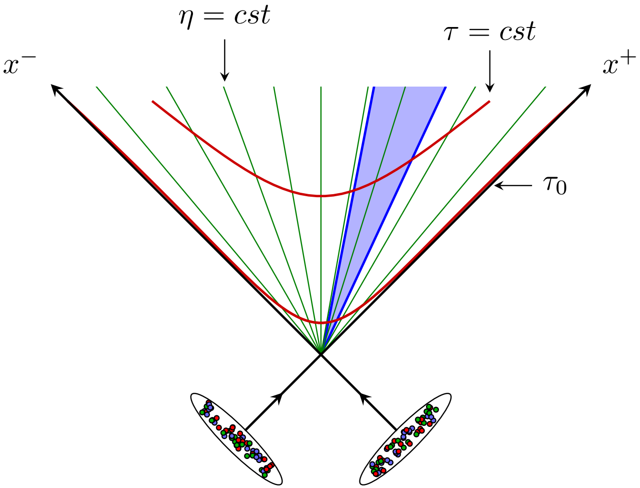
Numerical results

Adapted coordinate system to describe a Heavy Ion Collision?



System boost invariant in z direction

Proper time/rapidity coordinate system



The model

Initial conditions: classical statistical method

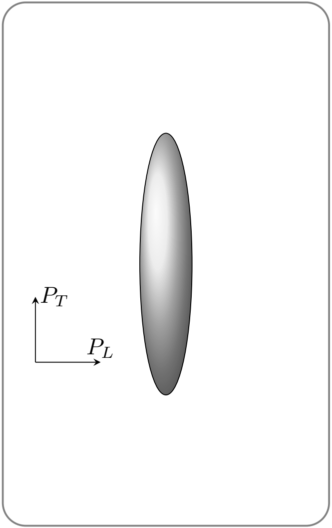
$$\phi(\tau_0, \boldsymbol{x}_\perp, \eta) = \varphi_0(\tau_0, \boldsymbol{x}_\perp) + \sum_{\boldsymbol{k}_\perp, \nu} c_{\nu \boldsymbol{k}_\perp} e^{i\nu\eta} a_{\nu, \boldsymbol{k}_\perp}(\tau_0, \boldsymbol{x}_\perp)$$

Time evolution: Klein Gordon equation

$$\underbrace{\left[\frac{\partial^2}{\partial\tau^2} + \frac{1}{\tau}\frac{\partial}{\partial\tau} - \boldsymbol{\nabla}_\perp^2 - \frac{1}{\tau^2}\frac{\partial^2}{\partial\eta^2}\right]}_{\square} \phi + \frac{g^2}{6}\phi^3 = 0$$

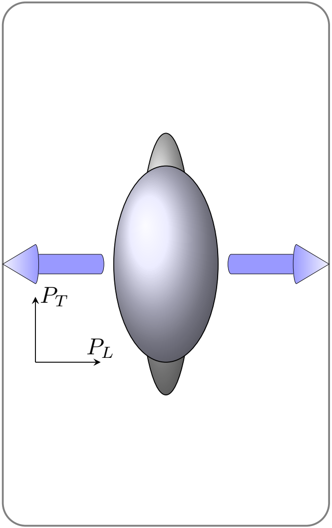
HOW INTERACTIONS COMPETE WITH EXPANSION?

Initial anisotropy

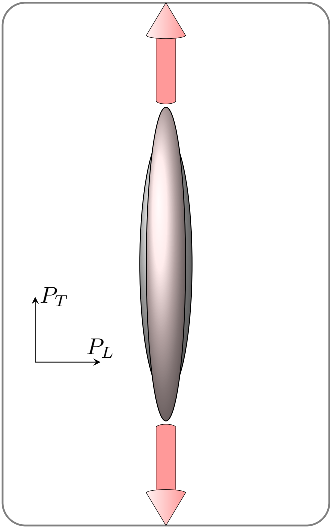


How interactions compete with expansion?

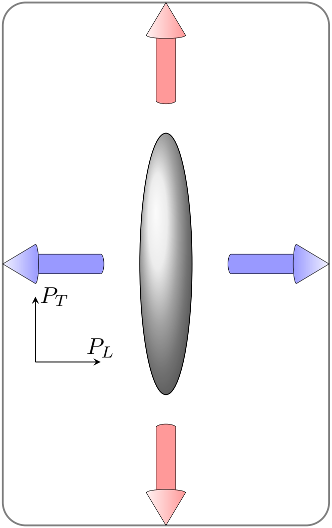
Interactions isotropize the system



Expansion dilutes the system

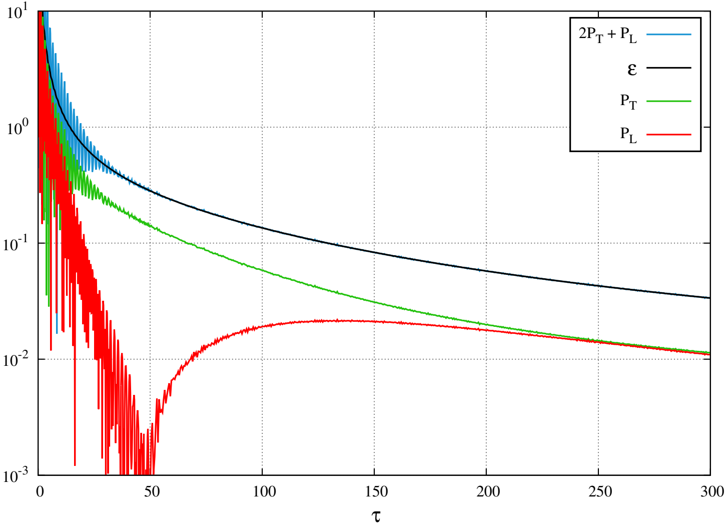


Expansion $\lessgtr$ Interactions for realistic α_s?

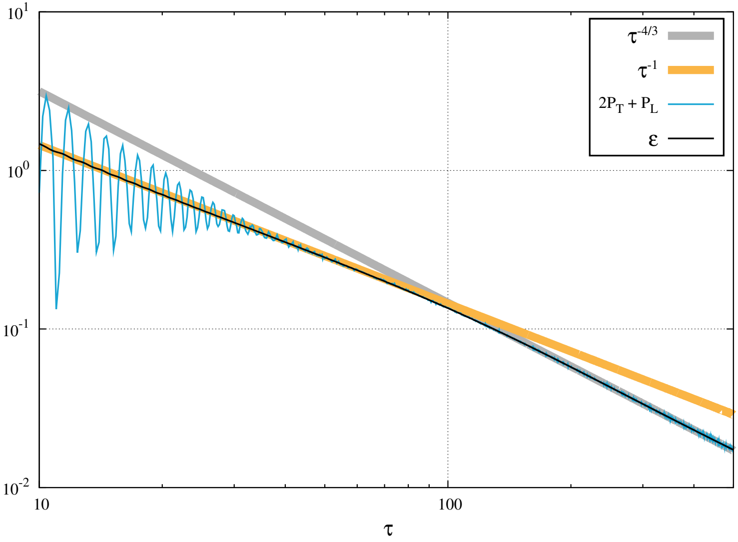


3. A PROOF OF CONCEPT: SCALAR FIELD THEORY
   - The Theory
   - Numerical results

$T^{\mu\nu}_{\rm RESUM}$ [Dusling, TE, Gelis, Venugopalan (2012)]



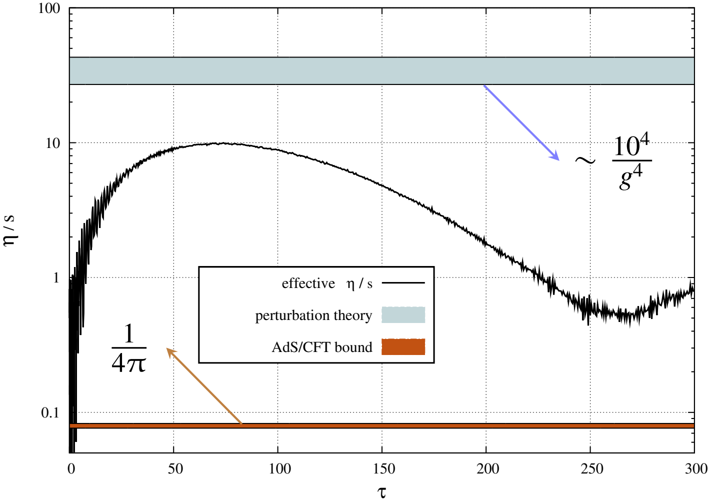
ϵ BEHAVIOUR



Bjorken Law: $\partial_\tau \epsilon + \frac{\epsilon + P_L}{\tau} = 0$

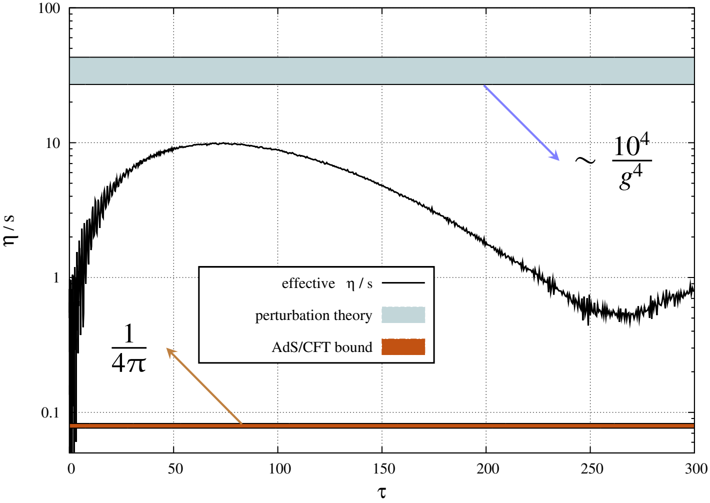
Comparison with Hydro: viscosity

$$P_T - P_L = \frac{2\eta}{\tau}$$



Comparison with Hydro: viscosity

see also [Asakawa, Bass, Muller (2006-07)]

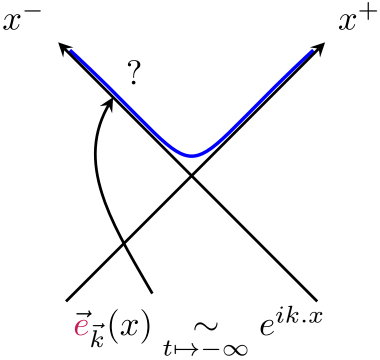


4 YANG-MILLS THEORY

The theory

Numerical results

- Need to know $\vec{e}_{\vec{k}}(\tau_0, \vec{x})$ at the time τ_0 we start the numerical simulation
- For practical reasons, we must start in the forward light cone $(\tau_0 > 0)$



This can be done analytically [TE,GELIS 1307:1765]

Result of [TE,GELIS 1307:1765]

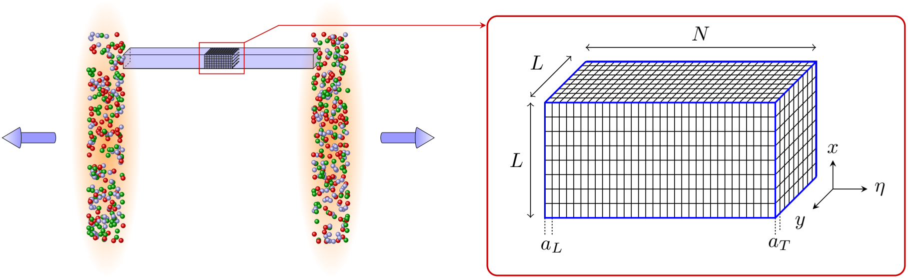
$$e^{i}_{\nu\vec{k}_\perp} = i\nu\left(F^{i,-} - F^{i,+}\right) \qquad\qquad e^{\eta}_{\nu\vec{k}_\perp}(x) = \mathcal{D}^{i}\left(F^{i,-} - F^{i,+}\right)$$

with

$$F^{i,+}_{k}(x) \sim e^{i\nu\eta}\,\mathcal{U}^{\dagger}_{1}(\vec{x}_\perp)\int_{\vec{p}_\perp} e^{i\vec{p}_\perp\cdot\vec{x}_\perp}\,\widetilde{\mathcal{U}}_{1}(\vec{p}_\perp+\vec{k}_\perp)\left(\frac{p_\perp^{2}\tau}{2k_\perp}\right)^{i\nu}\left[\delta^{ij} - \frac{2p^{i}_{\perp}p^{j}_{\perp}}{p_\perp^{2}}\right]\epsilon^{j}_{k\lambda}\,.$$

- $\mathcal{U}^{\dagger}_{1}$ depends on the color source J^{+} of the first nucleus
- Analogous formula for $F^{i,-}$.

Gauge potential $A^\mu \to$ link variables (exact gauge invariance on the lattice)

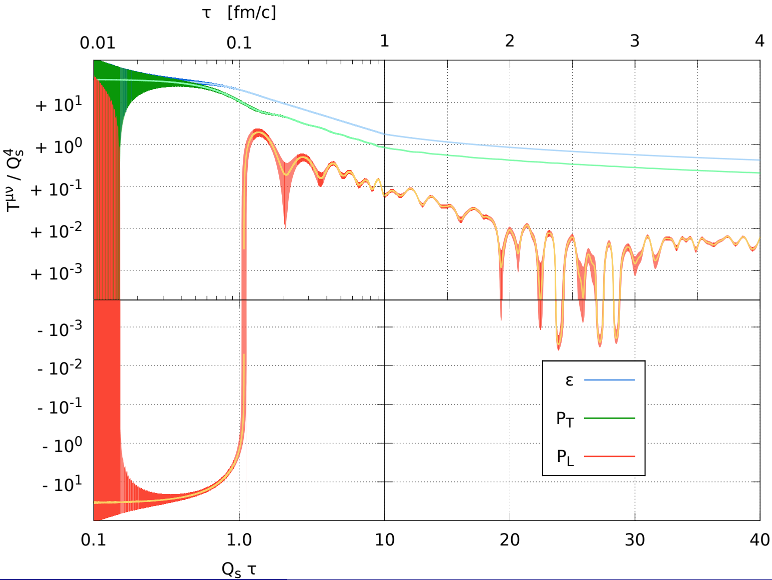


Numerical parameters

- Transverse lattice size $L = 64$, transverse lattice spacing $Q_s a_T = 1$
- Longitudinal lattice size $N = 128$, longitudinal lattice spacing $a_L = 0.016$
- Number of configurations for the Monte-Carlo $N_{\text{conf}} = 200$ to 2000
- Initial time $Q_s \tau_0 = 0.01$

NUMERICAL RESULTS [TE,GELIS 1307:2214]

$$\alpha_s = 8\,10^{-4}\ (g = 0.1)$$



NUMERICAL RESULTS [TE,GELIS 1307:2214]

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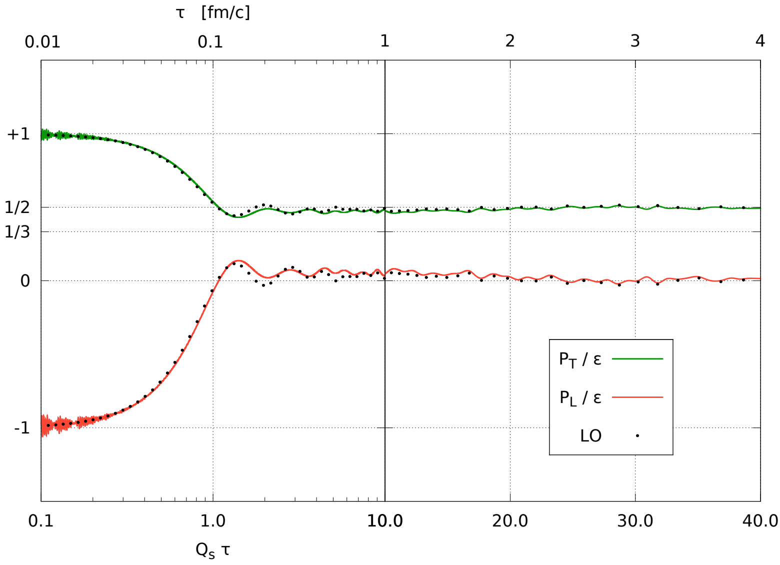
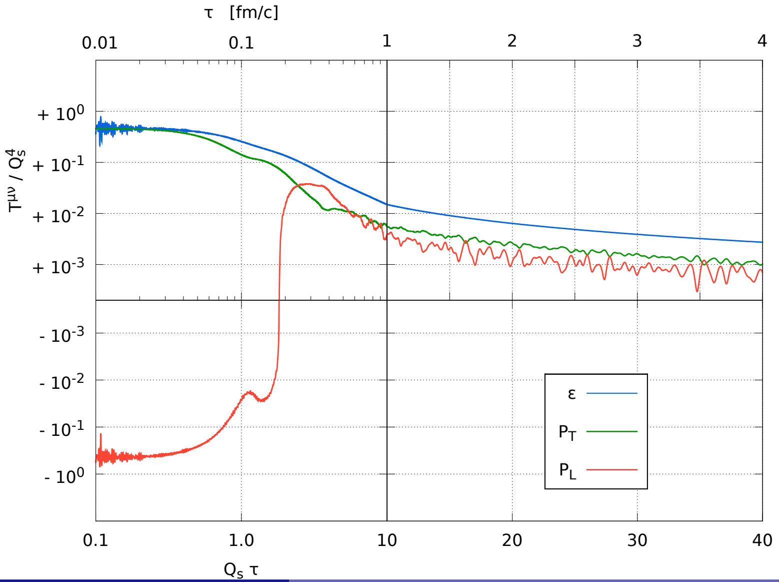


NUMERICAL RESULTS [TE,GELIS 1307:2214]

$\alpha_s = 2\,10^{-2}$ $(g = 0.5)$



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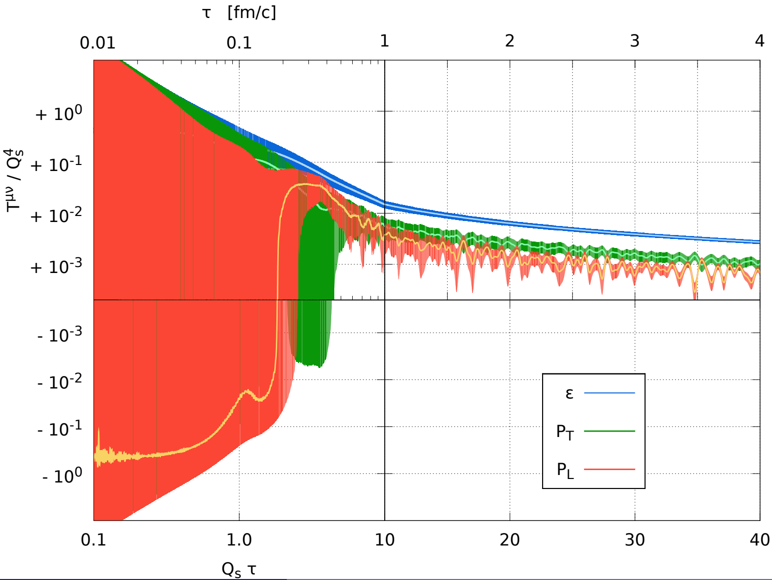

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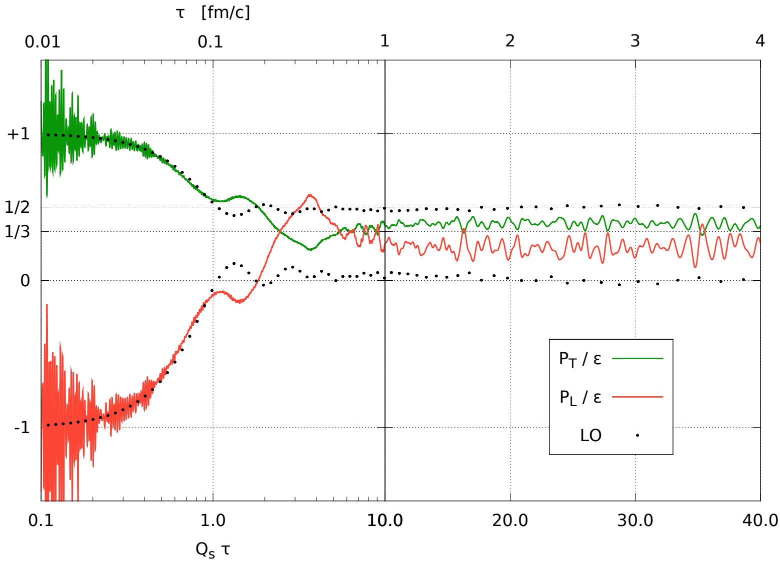

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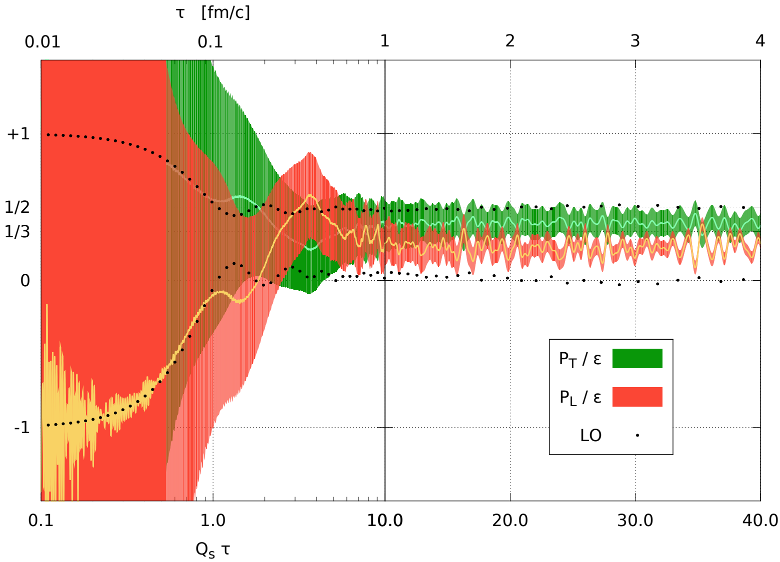

Assuming simple first order viscous hydrodynamics

$$\epsilon \approx \underbrace{\epsilon_0 \tau^{-\frac{4}{3}}}_{\text{Ideal hydro}} - \underbrace{2\eta_0 \tau^{-2}}_{\text{first order correction}}$$

we can compute the dimensionless ratio ($\eta = \eta_0 \tau^{-1}$)

$$\eta \epsilon^{-\frac{3}{4}} \lesssim 1$$

In contrast, perturbation theory at LO gives $\eta \epsilon^{-\frac{3}{4}} \sim 300$.

If the system is closed from being thermal

$$\epsilon^{-\frac{3}{4}} \sim s \Longrightarrow \frac{\eta}{s} \text{ Not far from } \frac{1}{4\pi}$$

Conclusion

- Correct NLO spectrum from first principles
- Fixed anisotropy for $g = 0.5$ at $\tau \sim 1 fm/c$
- No need for strong coupling to get isotropization
- Assuming simple first order viscous hydrodynamics

$$\eta\epsilon^{-\frac{3}{4}} \lesssim 1$$

- Compatible with viscous hydrodynamical expansion

Viscous Hydrodynamics

I) Macroscopic theory
II) Few parameters: P_L, P_T, ϵ, $\vec{u}$
III) Need input:

1) Equation of state $f(P_L, P_T) = \epsilon$
2) Small anisotropy
3) Initialization: $\epsilon(\tau_0)$, $P_L(\tau_0)$? ...
4) viscous coefficients: shear viscosity η,...
5) Short isotropization time