The onset of hydrodynamical flow in high energy heavy ion collisions

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Thomas EPELBAUM
IPhT
OUTLINE

Introduction

Theoretical tools

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Conclusion
Viscous Hydrodynamics

I) Macroscopic theory
II) Few parameters: $P_L, P_T, \epsilon, \vec{u}$
III) Need input:

1) Equation of state $f(P_L, P_T) = \epsilon$
2) Small anisotropy
3) Initialization: $\epsilon(\tau_0), P_L(\tau_0)$? ...
4) Viscous coefficients: shear viscosity $\eta$, ...
5) Short isotropization time
Viscous Hydrodynamics

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None of this is easy to get from QCD.
**Early transition: the problem**

- **Isotropization?**
- **Time scale?**

**Huge anisotropy**
(negative $P_L$)

**Small anisotropy**

**Long time puzzle: Does (fast) isotropization occur?**
How to study the transition?

Weakly coupled method at dense regime:
\[ \alpha_s \ll 1 \text{ but } f_{\text{gluon}} \sim \frac{1}{\alpha_s} \]
The Color Glass Condensate [McLerran, Venugopalan (1993)]

Theoretical framework (Weakly coupled but strongly interacting)

\[ x^- \quad \text{CGC} \quad x^+ \]

**LO:**
\[ \epsilon = \frac{1}{2} \left( \varepsilon^2 + B^2 \right) \]
\[ \mathcal{D}_\mu \mathcal{F}^{\mu\nu} = J^\nu \]

Classical color fields

Color sources on the light cone

[Krasnitz, Venugopalan (1998)]
Strong anisotropy at early time
Strong anisotropy at early time

\[ \epsilon = 2P_T + P_L \Rightarrow P_T = \epsilon \]

\[ \lim_{\tau \to 0^+} \epsilon = \text{cst} \Rightarrow P_L = -\epsilon \]
\[ E^2(x) = \mathcal{E}^2(x_\perp) + \frac{1}{2} \int \frac{d^2k}{(2\pi)^2} |e_{\vec{k}}(x)|^2 + \cdots \]

\( e_{\vec{k}}(x) \) perturbation to \( \mathcal{E}(x) \) created by a plane wave of momentum \( \vec{k} \) in the remote past.

Obtained by solving the linearized equation of motions.
Small Fluctuations grow exponentially (Weibel instability)
Because of instabilities, the NLO correction eventually becomes as large as the LO $\Rightarrow$ Important effect, should be included

NLO alone will grow forever $\Rightarrow$ unphysical effect, should be taken care of

Such growing contributions are present at all orders of the perturbative expansion

How to deal with them?
The classical-statistical method

- At the initial time $\tau = \tau_0$, take:

$$\vec{E}_0(\tau_0, \vec{x}) = \vec{E}_0(\tau_0, \vec{x}) + \int k \, c_k \vec{e}_k(\tau_0, \vec{x})$$

where $c_k$ are random coefficients: $\langle c_k c_{k'} \rangle \sim \delta_{kk'}$

- Solve the Classical equation of motion $D_\mu F^{\mu \nu} = J^\nu$

- Compute $\langle \vec{E}^2(\tau, \vec{x}) \rangle$, where $\langle \rangle$ is the average on the $c_k$ (Monte-Carlo)

- One can show that this resums all the fastest growing terms at each order, leading to a result that remain bounded when $\tau \to \infty$

[ELIS, LAPP, VENUGOPALAN (2008)]

This gives: LO+NLO+Subset of higher orders
The classical-statistical method

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The NLO spectrum

- Need to know $\vec{e}_k(\tau_0, \vec{x})$ at the time $\tau_0$ we start the numerical simulation
- For practical reasons, we must start in the forward light cone ($\tau_0 > 0$)

This can be done analytically [TE, Gélis 1307:1765]
THE NLO SPECTRUM

Result

\[ e^{i \nu \vec{k}_\perp} = i \nu (F_i^-, - F_i^+) \]
\[ e^{\eta \vec{k}_\perp} (x) = \mathcal{D}^i (F_i^-, - F_i^+) \]

with

\[ F_{k}^{i, +} (x) \sim e^{i \nu \eta} \mathcal{U}^\dagger_1 (\vec{x}_\perp) \int_{\vec{p}_\perp} e^{i \vec{p}_\perp \cdot \vec{x}_\perp} \tilde{\mathcal{U}}_1 (\vec{p}_\perp + \vec{k}_\perp) \left( \frac{p^2 \tau}{2 k_\perp} \right)^i \nu \left[ \delta^{ij} - \frac{2 p_i^j p_\perp^i}{p^2_\perp} \right] e^{j}_k \lambda . \]

• \( \mathcal{U}^\dagger_1 \) depends on the color source \( J^+ \) of the first nucleus

• Analogous formula for \( F_i^-, - \).
YM on a lattice

Gauge potential $A^\mu \rightarrow$ link variables (exact gauge invariance on the lattice)

Numerical parameters

- Transverse lattice size $L = 64$, transverse lattice spacing $Q_s a_T = 1$
- Longitudinal lattice size $N = 128$, longitudinal lattice spacing $a_L = 0.016$
- Number of configurations for the Monte-Carlo $N_{\text{conf}} = 200$ to 2000
- Initial time $Q_s \tau_0 = 0.01$
Numerical results [TE, Gelis 1307:2214]

\[ \alpha_s = 8 \times 10^{-4} \ (g = 0.1) \]
**Numerical Results [TE, Gelis 1307:2214]**

\[ \alpha_s = 2 \times 10^{-2} \ (g = 0.5) \]
Numerical results [TE,Gelis 1307:2214]

\[ \alpha_s = 2 \times 10^{-2} \quad (g = 0.5) \]
ANOMALOUSLY SMALL VISCOSITY

Assuming simple first order viscous hydrodynamics

\[ \epsilon \approx \epsilon_0 \tau^{-\frac{4}{3}} - 2\eta_0 \tau^{-2} \]

Ideal hydro first order correction

we can compute the dimensionless ratio \((\eta = \eta_0 \tau^{-1})\)

\[ \eta \epsilon^{-\frac{3}{4}} \lesssim 1 \]

In contrast, perturbation theory at LO gives \(\eta \epsilon^{-\frac{3}{4}} \sim 300\).

If the system is closed from being thermal

\[ \epsilon^{-\frac{3}{4}} \sim s \implies \frac{\eta}{s} \text{ Not far from } \frac{1}{4\pi} \]
Conclusion

- Correct NLO spectrum from first principles
- Fixed anisotropy for $g = 0.5$ at $\tau \sim 1 \text{fm}/c$
- No need for strong coupling to get isotropization
- Compatible with viscous hydrodynamical expansion
- Assuming simple first order viscous hydrodynamics

$$\eta \epsilon^{-\frac{3}{4}} \lesssim 1$$
\[ \langle A \rangle \sim 0, \quad \langle E \rangle \sim 0 \]
\[ \langle A^2 \rangle - \langle A \rangle^2 \sim \frac{Q_s^2}{g^2} \]
\[ \langle E^2 \rangle - \langle E \rangle^2 \sim \frac{Q_s^4}{g^2} \]

May give correct answer at LO
Not correct at NLO

\[ \langle A \rangle \sim \frac{Q_s}{g}, \quad \langle E \rangle \sim \frac{Q_s^2}{g} \]
\[ \langle A^2 \rangle - \langle A \rangle^2 \sim Q_s^2 \]
\[ \langle E^2 \rangle - \langle E \rangle^2 \sim Q_s^4 \]
give correct answer at LO
give correct answer at NLO