The onset of hydrodynamical flow in high energy heavy ion collisions

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OUTLINE

Introduction

Theoretical tools

Numerical results

Conclusion

Viscous Hydrodynamics

I) Macroscopic theory
II) Few parameters: P_L, P_T, ϵ, $\vec{u}$
III) Need input:

1) Equation of state $f(P_L, P_T) = \epsilon$
2) Small anisotropy
3) Initialization: $\epsilon(\tau_0)$, $P_L(\tau_0)$? ...
4) viscous coefficients: shear viscosity η,...
5) Short isotropization time

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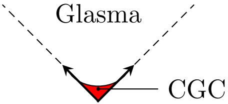
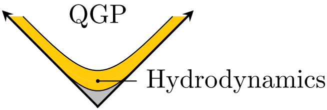
None of this is easy to get from QCD

Early transition: the problem



Isotropization?
Time scale?



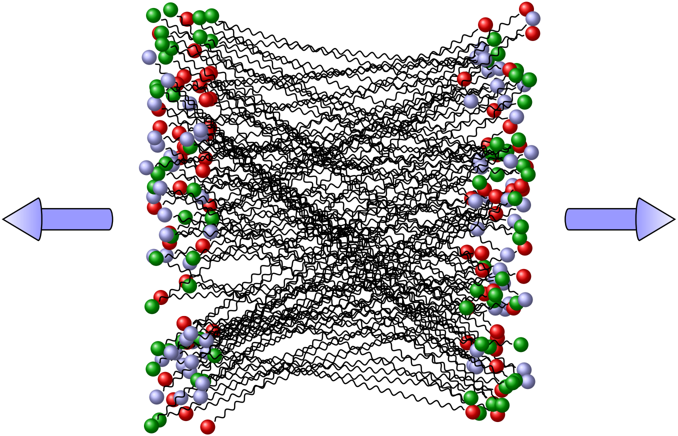
Huge anisotropy (negative P_L)

Small anisotropy

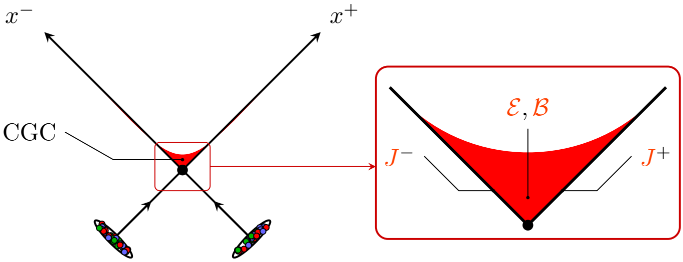
Long time puzzle: Does (fast) isotropization occur?

Weakly coupled method at dense regime:

$\alpha_s \ll 1$ **but** $f_{\text{gluon}} \sim \frac{1}{\alpha_s}$



Theoretical framework (Weakly coupled but strongly interacting)

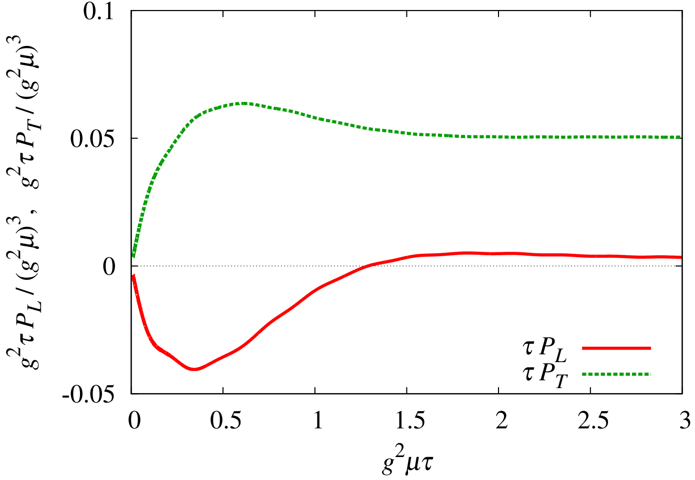


LO:

$$\epsilon = \frac{1}{2}\underbrace{\left(\vec{\mathcal{E}}^2 + \vec{\mathcal{B}}^2\right)}_{\text{Classical color fields}} \qquad \mathcal{D}_\mu \mathcal{F}^{\mu\nu} = \underbrace{J^\nu}_{\text{Color sources on the light cone}}$$

[KRASNITZ, VENUGOPALAN (1998)]

Strong anisotropy at early time



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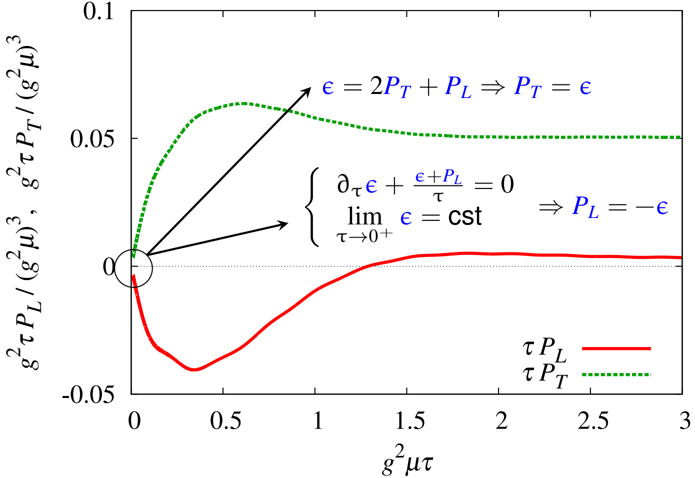
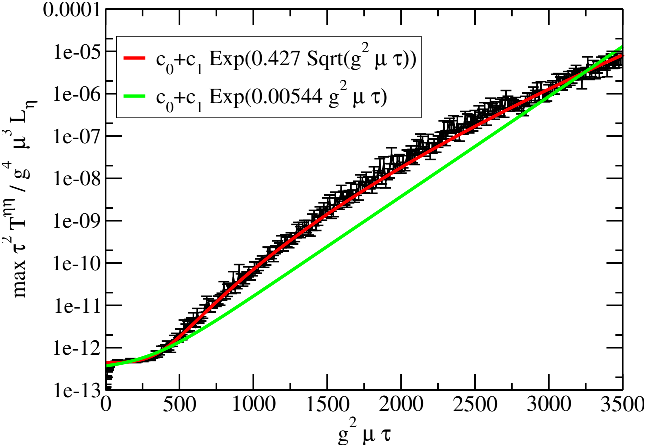


$$E^2(x) = \underbrace{\mathcal{E}^2(\boldsymbol{x}_\perp)}_{\text{LO}} + \underbrace{\frac{1}{2}\int_{\vec{k}} \left|e_{\vec{k}}(x)\right|^2}_{\text{NLO}} + \cdots$$

$e_{\vec{k}}(x)$ perturbation to $\mathcal{E}(x)$ created by a plane wave of momentum $\vec{k}$ in the remote past.

Obtained by solving the linearized equation of motions.

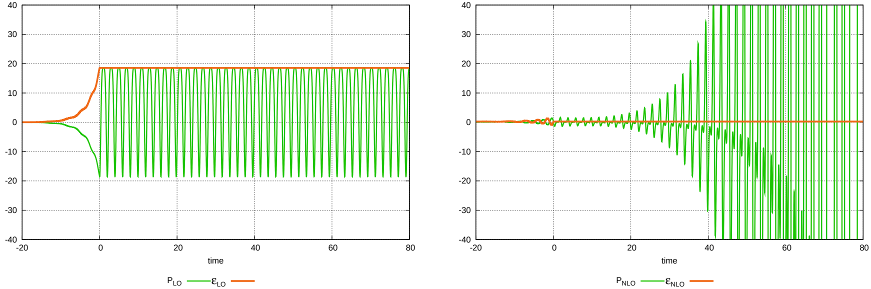


[ROMATSCHKE, VENUGOPALAN (2006)]

Small Fluctuations grow exponentially (Weibel instability)

- Because of instabilities, the NLO correction eventually becomes as large as the LO ⇒ Important effect, should be included

- NLO alone will grow forever ⇒ unphysical effect, should be taken care of



- Such growing contributions are present at all orders of the perturbative expansion

How to deal with them?

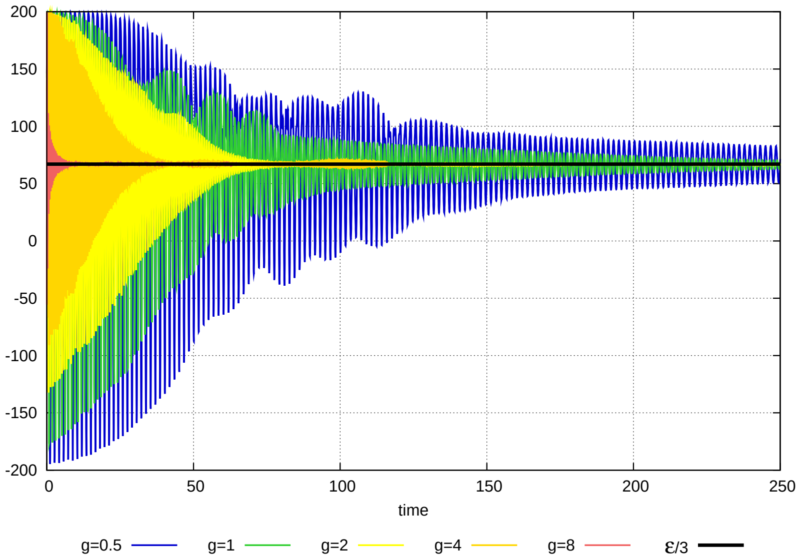
- At the initial time $\tau = \tau_0$, take:

$$\vec{E}_0(\tau_0, \vec{x}) = \vec{\mathcal{E}}_0(\tau_0, \vec{x}) + \int_{\vec{k}} c_{\vec{k}} \vec{e}_{\vec{k}}(\tau_0, \vec{x})$$

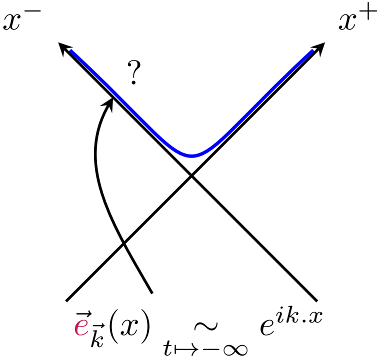
  where $c_{\vec{k}}$ are random coefficients: $\left\langle c_{\vec{k}} c_{\vec{k}'} \right\rangle \sim \delta_{\vec{k}\vec{k}'}$

- Solve the **Classical** equation of motion $D_\mu F^{\mu\nu} = J^\nu$
- Compute $\left\langle \vec{E}^2(\tau, \vec{x}) \right\rangle$, where $\langle\rangle$ is the average on the $c_{\vec{k}}$ (Monte-Carlo)
- One can show that this resums all the fastest growing terms at each order, leading to a result that remain bounded when $\tau \to \infty$ [GELIS, LAPPI, VENUGOPALAN (2008)]

This gives: LO+NLO+Subset of higer orders


200
150
100
50
0
-50
-100
-150
-200
0
50
100
150
200
250
time
g=0.5
g=1
g=2
g=4
g=8
ε/3

- Need to know $\vec{e}_{\vec{k}}(\tau_0, \vec{x})$ at the time τ_0 we start the numerical simulation
- For practical reasons, we must start in the forward light cone $(\tau_0 > 0)$



This can be done analytically [TE,GELIS 1307:1765]

Result

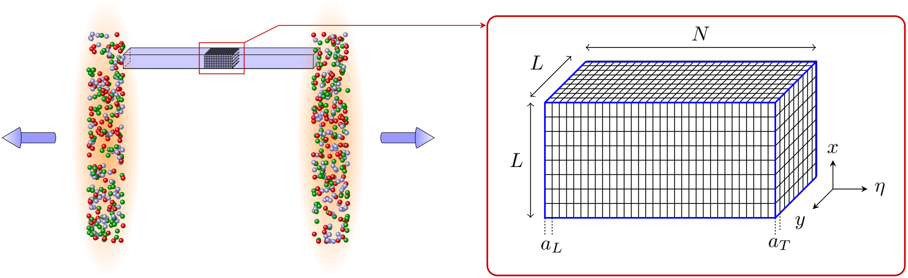
$$e^{i}_{\nu\vec{k}_\perp} = i\nu\left(F^{i,-} - F^{i,+}\right) \qquad e^{\eta}_{\nu\vec{k}_\perp}(x) = \mathcal{D}^{i}\left(F^{i,-} - F^{i,+}\right)$$

with

$$F^{i,+}_{k}(x) \sim e^{i\nu\eta}\,\mathcal{U}_1^\dagger(\vec{x}_\perp)\int_{\vec{p}_\perp} e^{i\vec{p}_\perp\cdot\vec{x}_\perp}\,\widetilde{\mathcal{U}}_1(\vec{p}_\perp+\vec{k}_\perp)\left(\frac{p_\perp^2\tau}{2k_\perp}\right)^{i\nu}\left[\delta^{ij} - \frac{2p^i_\perp p^j_\perp}{p_\perp^2}\right]\epsilon^{j}_{k\lambda}\,.$$

- $\mathcal{U}_1^\dagger$ depends on the color source J^+ of the first nucleus
- Analogous formula for $F^{i,-}$.

Gauge potential $A^\mu \rightarrow$ link variables (exact gauge invariance on the lattice)

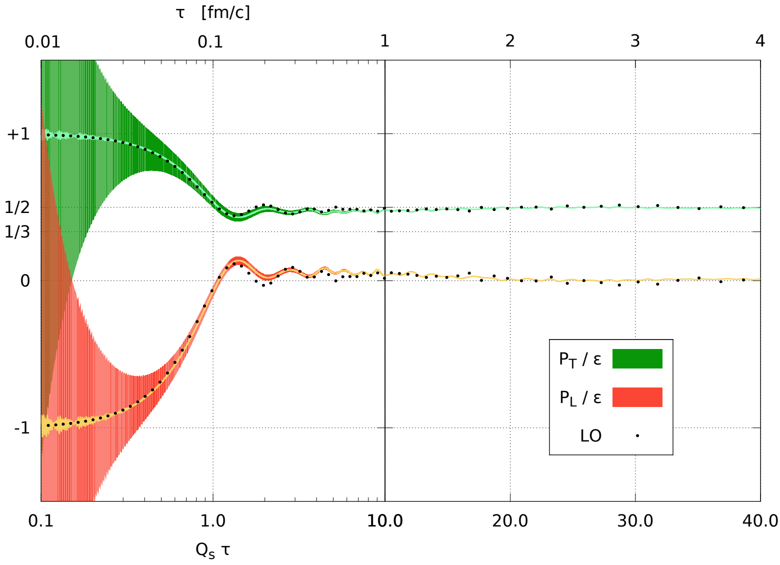


Numerical parameters

- Transverse lattice size $L = 64$, transverse lattice spacing $Q_s a_T = 1$
- Longitudinal lattice size $N = 128$, longitudinal lattice spacing $a_L = 0.016$
- Number of configurations for the Monte-Carlo $N_{\text{conf}} = 200$ to 2000
- Initial time $Q_s \tau_0 = 0.01$

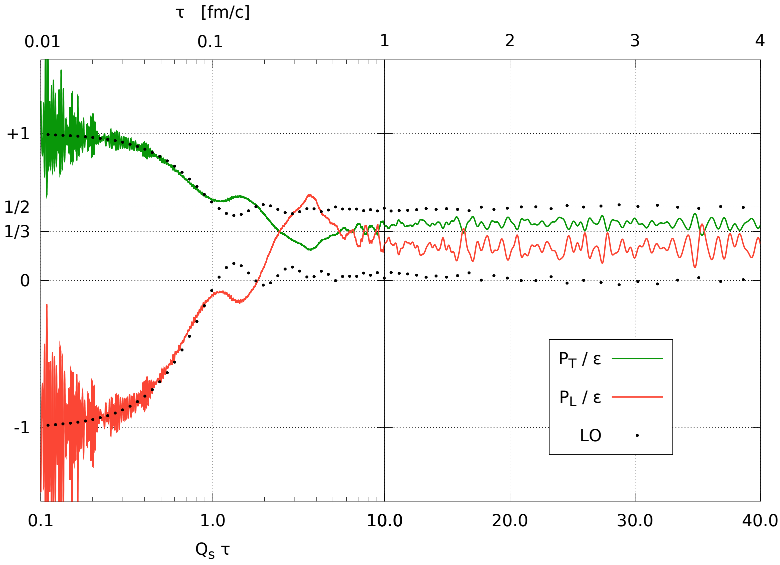
NUMERICAL RESULTS [TE,GELIS 1307:2214]

$\alpha_s = 8\,10^{-4}$ $(g = 0.1)$



NUMERICAL RESULTS [TE,GELIS 1307:2214]

$$\alpha_s = 2\,10^{-2}\ (g = 0.5)$$



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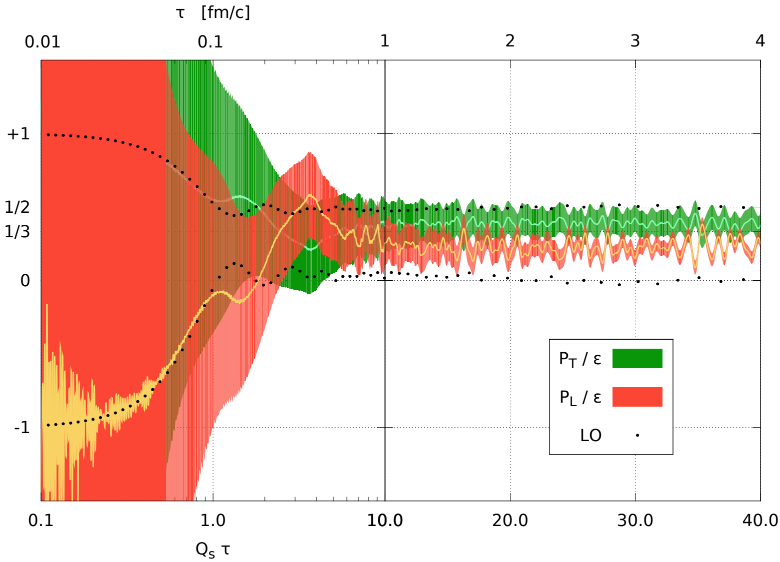


Assuming simple first order viscous hydrodynamics

$$\epsilon \approx \underbrace{\epsilon_0 \tau^{-\frac{4}{3}}}_{\text{Ideal hydro}} - \underbrace{2\eta_0 \tau^{-2}}_{\text{first order correction}}$$

we can compute the dimensionless ratio ($\eta = \eta_0 \tau^{-1}$)

$$\eta \epsilon^{-\frac{3}{4}} \lesssim 1$$

In contrast, perturbation theory at LO gives $\eta \epsilon^{-\frac{3}{4}} \sim 300$.

If the system is closed from being thermal

$$\epsilon^{-\frac{3}{4}} \sim s \Longrightarrow \frac{\eta}{s} \text{ Not far from } \frac{1}{4\pi}$$

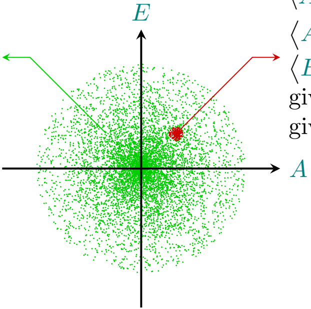
CONCLUSION

- Correct NLO spectrum from first principles
- Fixed anisotropy for $g = 0.5$ at $\tau \sim 1 fm/c$
- No need for strong coupling to get isotropization
- Compatible with viscous hydrodynamical expansion
- Assuming simple first order viscous hydrodynamics

$$\eta \epsilon^{-\frac{3}{4}} \lesssim 1$$

Backup: completely incoherent initial fields vs CGC

$\langle A \rangle \sim 0$, $\langle E \rangle \sim 0$
$\langle A^2 \rangle - \langle A \rangle^2 \sim \frac{Q_s^2}{g^2}$
$\langle E^2 \rangle - \langle E \rangle^2 \sim \frac{Q_s^4}{g^2}$
May give correct answer at LO
Not correct at NLO



$\langle A \rangle \sim \frac{Q_s}{g}$, $\langle E \rangle \sim \frac{Q_s^2}{g}$
$\langle A^2 \rangle - \langle A \rangle^2 \sim Q_s^2$
$\langle E^2 \rangle - \langle E \rangle^2 \sim Q_s^4$
give correct answer at LO
give correct answer at NLO