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exe 18) 1) Maple

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> with(linalg)
> A := matrix(2,2,[-1,-2,-2,1]);
```

$$A := \begin{pmatrix} -1 & -2 \\ -2 & +1 \end{pmatrix}$$

```
> J := jordan(A);
```

$$J := \begin{pmatrix} \sqrt{5} & 0 \\ 0 & -\sqrt{5} \end{pmatrix}$$

```
> B := matrix(2,2,[0,a,-b,0]);
```

$$B := \begin{pmatrix} 0 & a \\ -b & 0 \end{pmatrix}$$

```
> K := jordan(B)
```

$$K := \begin{pmatrix} \sqrt{-ab} & 0 \\ 0 & -\sqrt{ab} \end{pmatrix}$$

$$\Rightarrow B = \begin{pmatrix} 0 & x \\ +5/x & 0 \end{pmatrix}$$

Or A est symétrique et B est orthogonalement semblable à A, donc B doit aussi être symétrique : x= $\sqrt{5}$

2) $\forall n \in \mathbb{N} : U_n = \int_0^{\pi/4} \tan^n t \, dt$

a) $U_{n+1} - U_n = \int_0^{\pi/4} \tan^{n+1} t \, dt - \int_0^{\pi/4} \tan^n t \, dt = \int_0^{\pi/4} \underbrace{\underbrace{\tan^n t}_{\geq 0} \underbrace{(\tan t - 1)}_{\leq 0}}_{\leq 0} dt$

d'où $(U_n) \searrow$

b) $U_{n+2} + U_n = \int_0^{\pi/4} \tan^n t \underbrace{(\tan^2 t + 1)}_{\tan'(t)} dt = \left[\frac{1}{n+1} \tan^{n+1} t\right]_0^{\pi/4}$

$= \frac{1}{n+1} = U_{n+2} + U_n$

c) $(U_n) \searrow$ et $\geq 0 \Rightarrow (U_n)$ cv

on a $\begin{cases} U_{n+2} + U_n = \frac{1}{n+1} \\ U_n + U_{n-2} = \frac{1}{n-1} \end{cases}$

$2U_n \geq U_{n+2} + U_n = \frac{1}{n+1}$

$2U_n \leq U_n + U_{n-2} = \frac{1}{n-1}$

$$\Rightarrow \quad \frac{1}{n+1} \leq 2U_n \leq \frac{1}{n-1}$$

$$\underbrace{\frac{n}{n+1}}_{\substack{\longrightarrow 1 \\ n \to +\infty}} \leq \underbrace{U_n 2n}_{\longrightarrow 1} \leq \underbrace{\frac{n}{n-1}}_{\substack{\longrightarrow 1 \\ n \to +\infty}} \quad \text{car } n \geq 0$$

$$\Rightarrow U_n \sim \frac{1}{2n}$$