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Exercice 11.

$$\begin{cases} f(x,y) = \dfrac{x^2 y}{x^2+y^2} & \text{pour } (x,y) \neq 0 \\ f(0,0) = 0 \end{cases}$$

$$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \end{cases}$$

$$f(x,y) = \frac{\rho^3 \cos^2\theta \sin\theta}{\rho^2} = \rho(1-\sin^2\theta)\sin\theta = \rho(\sin\theta - \sin^3\theta).$$

$$\lim_{\rho \to 0} f(x,y) = 0 = f(0,0)$$

donc f est continue en 0.

et f est continue sur $\mathbb{R}^2 \setminus \{(0,0)\}$

donc f est continue sur $\mathbb{R}^2$.

f est dérivable sur $\mathbb{R}^2 - \{0\}$

$$\frac{\partial f}{\partial x}(x,y) = \frac{2xy(x^2+y^2) - yx^2(2x)}{(x^2+y^2)^2} = \frac{2xy^3}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial x}(x,x) = \frac{1}{2}.$$

$x \mapsto f(x,0)$ est dérivable $\forall x \neq 0$ et $\dfrac{\partial f}{\partial x}(x,0) = 0$.

si $x \mapsto \dfrac{\partial f}{\partial x}(x,0)$ est continue alors $\dfrac{\partial f}{\partial x}(0,0) = 0 \neq \dfrac{1}{2}$

donc $x \mapsto \dfrac{\partial f}{\partial x}(x,0)$ n'est pas continue en 0.

donc f n'est pas $\mathcal{C}^1$ sur $\mathbb{R}^2$

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{2xy^3}{(x^2+y^2)^2}\right)$$

$$= \frac{2x(3y^2)(x^2+y^2)^2 - 2xy^3(2(x^2+y^2)2y)}{(x^2+y^2)^4}$$

$$= \frac{6xy^2(x^2+y^2) - 8xy^4}{(x^2+y^2)^3}$$

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{y^2(6x^3 - 2xy^2)}{(x^2+y^2)^3}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{x^2(x^2+y^2) - x^2y \cdot 2y}{(x^2+y^2)^2}\right) = \frac{\partial}{\partial x}\left(\frac{x^2(x^2-y^2)}{(x^2+y^2)^2}\right)$$

$$= \frac{(x^2+y^2)^2(4x^3-2y^2x) - x^2(x^2-y^2)4x(x^2+y^2)}{(x^2+y^2)^4}$$

$$= \frac{(x^2+y^2)(4x^3-2y^2x) - 4x^3(x^2-y^2)}{(x^2+y^2)^3}$$

$$= \frac{y^2(6x^3-2y^2x)}{(x^2+y^2)^3}$$

$$\Rightarrow \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

Maple.

```
> restart; with(linalg);
> M = matrix([[a, b, c], [-b, a, b], [-c, -b, a]]);
> MM = multiply(M, M);
> I3 = diag(1, 1, 1);
> X = matadd(MM, -I3);
> eq: = {seq(seq(X[i, j] = 0 , j = 1..3), i = 1..3)};
> Sol : = solve(eq);
```

$\Rightarrow (a = 1,\ b = c = 0)$ ou $(a = -1,\ b = c = 0)$