$\boxed{\text{EX 19}}$ GUÉDEL

2) $f(x) = e^{x^2}\int_0^x e^{-t^2}\,dt$

Or f est C^1 car composée de fonction C^1 sur $\mathbb{R}$.

alors $f'(x) = 2xe^{x}\int_0^x e^{-t^2}dt + e^{x^2}e^{-x^2}$

$= 2x f(x) + 1.$

$\Rightarrow \boxed{f'(x) = 2xf(x) - 1 = 0 \quad (1)}$

Soit $f(x) = \sum_{n=0}^{\infty} a_n x^n$

$f'(x) = \sum_{n=1}^{\infty} n a_n x^{n-1}.$

$\Rightarrow \sum_{n=1}^{\infty} n a_n x^{n-1} - 2\sum_{n=0}^{+\infty} a_n x^{n+1} - 1 = 0$

dans(1)

$\Rightarrow \sum_{n=0}^{\infty} (n+1)a_{n+1}x^n - 2\sum_{n=1}^{\infty} a_{n-1}x^n - 1 = 0$

$\Rightarrow (n+1)a_{n+1} - 2a_{n-1} = 0$ par unicité du DSE.

$\Rightarrow \boxed{a_{n+1} = \frac{2a_{n-1}}{n+1}}$ $\forall n > 1.$

or $a_0 = f(0) = 0$ et $a_1 = f'(0) = 1.$

alors $\boxed{a_{2p} = 0}$ $\underline{\forall p \in \mathbb{N}}$ par récurrence immédiate car a_0 est nul.

et $a_{2p+1} = \frac{2a_{2p-1}}{2p+1}$ $\Rightarrow \frac{2^p}{2p+1 \times 2p-1 \times \dots \times 3} = \frac{2^p \times 2p \times 2(p-1) \times \dots \times 2}{(2p+1)!}$

$= \frac{2^p \times 2^p p!}{(2p+1)!}$

$\Rightarrow$

Ex 19

```
> restart;
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```
> A := x^n + 2·x^m + 1;
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$$A := x^n + 2\,x^m + 1 \tag{1}$$

```
> P := (x − 1)·(x − 2)·(x − 3)·(x − 4);
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$$P := (x-1)\,(x-2)\,(x-3)\,(x-4) \tag{2}$$

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> R := a·x^3 + b·x^2 + c·x + d;
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$$R := a\,x^3 + b\,x^2 + c\,x + d \tag{3}$$

```
> A = P·Q + R;
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$$x^n + 2\,x^m + 1 = (x-1)\,(x-2)\,(x-3)\,(x-4)\,Q + a\,x^3 + b\,x^2 + c\,x + d \tag{4}$$

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>
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```
> solve({subs(x = 1, A) = subs(x = 1, P)·Q + subs(x = 1, R), subs(x = 2, A) = subs(x = 2, P)·Q
    + subs(x = 2, R), subs(x = 3, A) = subs(x = 3, P)·Q + subs(x = 3, R), subs(x = 4, A)
    = subs(x = 4, P)·Q + subs(x = 4, R)}, [a, b, c, d]);
```

$$\left[\left[a = -\frac{1}{2} + 2^{-1+n} + 2^m - \frac{1}{2}\,3^n - 3^m + \frac{1}{6}\,4^n + \frac{1}{3}\,4^m,\ b = -4\,2^n - 8\,2^m + \frac{9}{2} + \frac{7}{2}\,3^n + 7\,3^m - 4^n - 2\,4^m,\ c = -7\,3^n - 14\,3^m - 13 + \frac{11}{6}\,4^n + \frac{11}{3}\,4^m + 19\,2^{-1+n} + 19\,2^m,\ d = -4^n - 2\,4^m + 13 + 8\,3^m - 6\,2^n - 12\,2^m + 4\,3^n\right]\right] \tag{5}$$

$$> a := -\frac{1}{2} + 2^{-1+n} + 2^m - \frac{1}{2}\,3^n - 3^m + \frac{1}{6}\,4^n + \frac{1}{3}\,4^m :$$

$$b := -4\,2^n - 8\,2^m + \frac{9}{2} + \frac{7}{2}\,3^n + 7\,3^m - 4^n - 2\,4^m :$$

$$c := -7\,3^n - 14\,3^m - 13 + \frac{11}{6}\,4^n + \frac{11}{3}\,4^m + 19\,2^{-1+n} + 19\,2^m :$$

$$d := -4^n - 2\,4^m + 13 + 8\,3^m - 6\,2^n - 12\,2^m + 4\,3^n :$$

```
> R;
```

$$\left(-\frac{1}{2} + 2^{-1+n} + 2^m - \frac{1}{2}\,3^n - 3^m + \frac{1}{6}\,4^n + \frac{1}{3}\,4^m\right) x^3 + \left(-4\,2^n - 8\,2^m + \frac{9}{2} + \frac{7}{2}\,3^n + 7\,3^m - 4^n - 2\,4^m\right) x^2 + \left(-7\,3^n - 14\,3^m - 13 + \frac{11}{6}\,4^n + \frac{11}{3}\,4^m + 19\,2^{-1+n} + 19\,2^m\right) x - 4^n - 2\,4^m + 13 + 8\,3^m - 6\,2^n - 12\,2^m + 4\,3^n \tag{6}$$

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> subs(n = 100, m = 43, R);
```

$$\begin{aligned}&267823007376240690496627676391976054279021031181686720068252\,x^3\\&- 1606938044257186454219400052687605745709143348905463618820675\,x^2\\&+ 2946053081137874529181806423320252507774511057863672705234050\,x\\&- 1606938044256928765459034047024622816344388740139895806481623\end{aligned} \tag{7}$$

```
> rem(x^100 + 2·x^43 + 1, (x − 1)·(x − 2)·(x − 3)·(x − 4), x);
```

$$\begin{aligned}&267823007376240690496627676391976054279021031181686720068252\,x^3\\&- 1606938044257186454219400052687605745709143348905463618820675\,x^2\\&+ 2946053081137874529181806423320252507774511057863672705234050\,x\end{aligned} \tag{8}$$

$$- 1606938044256928765459034047024622816344388740139895806481623$$

$$> \ is\big(subs(n=100, m=43, R) = rem\big(x^{100} + 2\cdot x^{43} + 1,\ (x-1)\cdot(x-2)\cdot(x-3)\cdot(x-4), x\big)\big);$$

$$true \qquad \textbf{(9)}$$