Model for Frequency-Dependence of Elastic Wave Velocities in Porous Rocks

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ABSTRACT

A model is proposed for the frequency dependence of elastic wave velocities in porous rocks, using the spheroidal geometry for the pores. The model is based on the assumption that the rock contains a distribution of "closable" cracks having small aspect ratios, and one family of "non-closable" pores. At a given wave frequency, some pores obey the Gassmann equation, and others are isolated, with a critical aspect ratio demarcating the two families that depends on frequency and fluid viscosity. An effective medium model is used to add the compliances of the individual pores, so as to yield effective moduli. The model also allows for calculation of "intrinsic" seismic attenuation by applying the Kramers-Kronig relations to the velocities. By considering the crack closure process, the model is capable of describing the frequency dispersion of both the compressional and shear velocities at each pressure. The predictions, for some sandstones datasets taken from the literature, show that P and S-wave velocities generally increase in a relatively similar manner with frequency, and that dispersion of both velocities rapidly decreases with pressure. Attenuation values are consistent with typical values found in the literature.

1 Introduction

It is now generally acknowledged that a significant velocity dispersion is observed between the low-frequency "Gassmann" regime of poroelasticity, which occurs at seismic frequencies, and the high-frequency regime, which occurs at ultrasonic frequencies, such as in laboratory testing of rocks (Winkler, 1986). Dispersion is caused by the ability of the pore fluid to move from pore to pore at the passing of a wave: in the Gassmann or "undrained" regime, pores are in local equilibrium, whereas in the high-frequency of "isolated" regime, the fluid is completely trapped in each pore and the induced pore pressure is different from pore to pore. It is well known that frequency dispersion is greatly dependent on the pore geometry, and more precisely on the presence of open cracks in a rock (Winkler, 1986; Jones, 1986).

Various mechanisms, comprehensive reviews of which have been notably given by Jones (1986) and Bourbié *et al.* (1987), have been proposed to account for the role of the pore fluid on velocity dispersion. The effect of viscous relaxation of the fluid present in a single crack under shear (Walsh, 1969; O'Connell and Budiansky, 1977; Cleary, 1978) becomes significant only at very high frequencies, around GHz in rocks. The same remark holds for Biot's equations of dynamic poroelasticity theory (Biot, 1956b), for which dispersion is accounted for by inertial coupling between fluid and solid phases. As noted by Bourbié *et al.* (1987), the dependence of Biot's characteristic frequency on both rock permeability and fluid viscosity is the inverse of the one actually observed in experiments. Another reason why Biot's model is not completely satisfying for rocks is that the wavelengths involved for obtaining significant inertial effects are *smaller* than the largest pore size, thus violating Biot's initial assumption of local equilibrium of variables on a *mesoscopic* Representative Elementary Volume (REV). In fact, owing to the same assumption, the theory of Biot did not consider the existence of local fluid flows and pressure gradients, from pore to pore, generated by the passing of a wave at "intermediate to high" frequencies.

The *local flow mechanism*, often referred as "squirt-flow" in the literature (Mavko and Nur, 1975), is now regarded as the main mechanism responsible for velocity dispersion between sonic and ultrasonic frequencies. Mavko and Nur (1975) and O'Connell and Budiansky (1977) propose that such dispersion is directly related to the distribution of pore shapes. More precisely, fluid should be squirted from thin cracks into the stiff pores, as pore pressures induced at the passing of a wave are higher in the compliant porosity. A certain number of so-called "squirt-flow models" have been proposed to explain the viscoelastic behaviour of saturated rocks, which are often based on a distribution of pore geometries (or in different terms, of relaxation times) (O'Connell and Budiansky, 1977; Mavko and Nur, 1979; Palmer and Traviola, 1980; Jones, 1986); or, alternatively, on the concept of dual porosity (Mavko and Jizba, 1991; Dvorkin and Nur, 1993; Pride *et al.*, 2004; Gurevich *et al.*, 2010). However, such models are not entirely satisfying, as they are only phenomenological, or dependent on diffusive transport laws, since they are based on restrictive assumptions of viscous flow at the pore scale.

The objective of this paper is to develop a simple model accounting for velocity dispersion, which is based on a spheroidal pore model (Eshelby, 1957; David and Zimmerman, 2011a), and can explain the frequency-dependence of saturated velocities between the Gassmann and high-frequency regimes. It should be recalled that in the Gassmann regime, saturated velocities can be calculated from the drained velocities by using the Gassmann equations (Gassmann, 1951); drained velocities are assumed to be the same as *dry* velocities, which can be calculated using effective medium theories, if the distribution of pores shapes is known. In the other limiting regime, at high frequency, saturated velocities can also be calculated by effective medium theories, considering as such theories implicitly assume that fluid-saturated pores are completely isolated with regards to fluid flow. It is assumed here that the rock contains an exponential distribution of crack aspect ratios, and one family of non-closable pores, as such a type of pore aspect ratio distribution has been shown to successfully invert dry velocities, as well as to predict saturated ultrasonic velocities, on many sandstones (David and Zimmerman, 2012). At a given wave frequency, some pores are isolated, and the other pores obey a Gassmann-type assumption of locally-equilibrated pore pressure. For a given pore fluid, the critical aspect ratio that demarcates the two families of pores is related to the frequency, following the relation given by O'Connell and Budiansky (1977) for the critical frequency of the local-flow mechanism. In this way, the model proposed here, which is only dependent on a simple distribution of pore shapes, differs from previous models, as it remains within the philosophy of "effective medium modelling", and is not based on any viscoelastic analysis. The model is developed here in two versions, using either the Differential scheme (Zimmerman, 1984; David and Zimmerman, 2011b), or the Mori-Tanaka scheme (Mori and Tanaka, 1973).

Seismic attenuation is of great importance in geophysics, due to its extreme sensitivity to the presence of pore fluids in rocks (Jones, 1986). The attenuation of interest here is the "intrinsic" attenuation, which is related to the viscoelastic behaviour of fluid-saturated rocks, as opposed to the "geometric" or "apparent" attenuation, which is caused by scattering or interference effects as the wave passes through a rock mass. The model therefore also allows for calculation the intrinsic attenuation by applying the Kramers-Kronig relations to velocities, if they are considered to be related to the "real part" of the viscoelastic moduli.

2 Model derivation

Critical frequency for local flow mechanism

Consider a rock whose aspect ratio distribution is known, for instance, by inverting the pressure dependence of dry velocities measured in the laboratory (David and Zimmerman, 2012). The rock contains an exponential distribution of crack aspect ratios $c(\alpha)$ (see, for instance, Figure 2), and one family of stiff pores of aspect ratio α^{hp}. The non-closable pores account for the total porosity, ϕ, which is also assumed to be available from experiments, as well as the rock density, and the elastic moduli of the minerals, (K_0, G_0). At a given frequency f, the critical aspect ratio α_{c} that distinguishes between the pores that follow a Gassmann-type behaviour of local pore pressure equilibrium, and the pores that behave as individually undrained, is given by $f = \zeta \frac{K_0}{\eta} \alpha_{\mathrm{c}}^3$, so,

$$\alpha_{\mathrm{c}} = \left(\frac{f\eta}{\zeta K_0} \right)^{1/3}. \tag{1}$$

Note the introduction of the additional dimensionless coefficient ζ. Indeed, as noted by O'Connell and Budiansky (1977), the critical frequency for local fluid flow is estimated from simplified diffusive models at the pore scale, which use very idealised pore geometries. The various estimates proposed in the literature give different values of ζ

(Mavko and Nur, 1975; O'Connell and Budiansky, 1977; LeRavalec *et al.*, 1996). It is also unclear from the expressions given in the literature whether the *representative elastic modulus* used in equation 1 should actually be the solid's modulus, an effective modulus (*e.g.*, of the surrounding material containing other pores), or even the bulk modulus of the fluid (Cleary, 1978). For these reasons, ζ is taken to be an adjustable parameter of the model. For simplicity, ζ is also assumed to be *independent* of pressure. A first-order pressure dependence on relation (1) could be alternatively taken into account by assuming that the representative bulk modulus is not the solid's bulk modulus, but the effective modulus of the rock, which is also pressure-dependent. Nevertheless, the question of whether the relation above should be treated as pressure-dependent is out of the scope of the present paper, as constraining the simplest model is already difficult due to the lack of experimental data.

In order to distinguish between the "Gassmann-type" and "isolated-type" pores, equation (1) should be interpreted in the correct way. A given frequency f corresponds to the critical frequency for local fluid flow to occur in a pore of aspect ratio α_c. Such a pore will be isolated at any frequency greater than f. In other words, all pores having aspect ratios greater than α_c would have critical frequencies greater than f, therefore such pores have a "Gassmann-type" behaviour at frequency f. In summary, at given frequency f, pores of lower aspect ratio $(\alpha < \alpha_c)$ behave as "isolated" pores, whereas pores of higher aspect ratios $(\alpha > \alpha_c)$ behave as "Gassmann-type" pores.

Calculation procedure

A diagram of the procedure used to calculate the saturated "effective" elastic moduli as function of frequency (starting from the elastic moduli of the minerals) is schematised in Figure 1. Saturated moduli are in turn converted into saturated velocities, using the appropriate saturated rock density. Note that the elastic moduli (K, G) denote dry effective moduli, and barred elastic moduli $(\bar{K}, \bar{G})$ denote saturated effective moduli. Four different situations are possible, depending on where the value of the critical aspect ratio, α_c, lies in the pore aspect ratio distribution: in situation 1, which occurs in the low-frequency limit, all pores are "Gassmann-type"; in the high-frequency limit (situation 4), all the pores behave as "isolated"(α_c is greater than the aspect ratio of the stiff pores); situation 3 occurs if the value α_c lies precisely *in between* the highest aspect ratio of closable cracks and the aspect ratio of the stiff pores; finally, in situation 2, non-closable pores are again "Gassmann-type", but α_c now separates a family of "Gassmann cracks" (having higher aspect ratios) and one family of "isolated cracks". Properly speaking, not four but *three* main types of situations must exist, as the distinction made between situation 2 and 3 is only an artefact due to the original assumption that stiff pores are only represented by one average aspect ratio. The calculation procedure described in Figure 1 is applicable for any effective medium theory, whose use only differ in the specific way pores are incrementally added. The simulations pre-

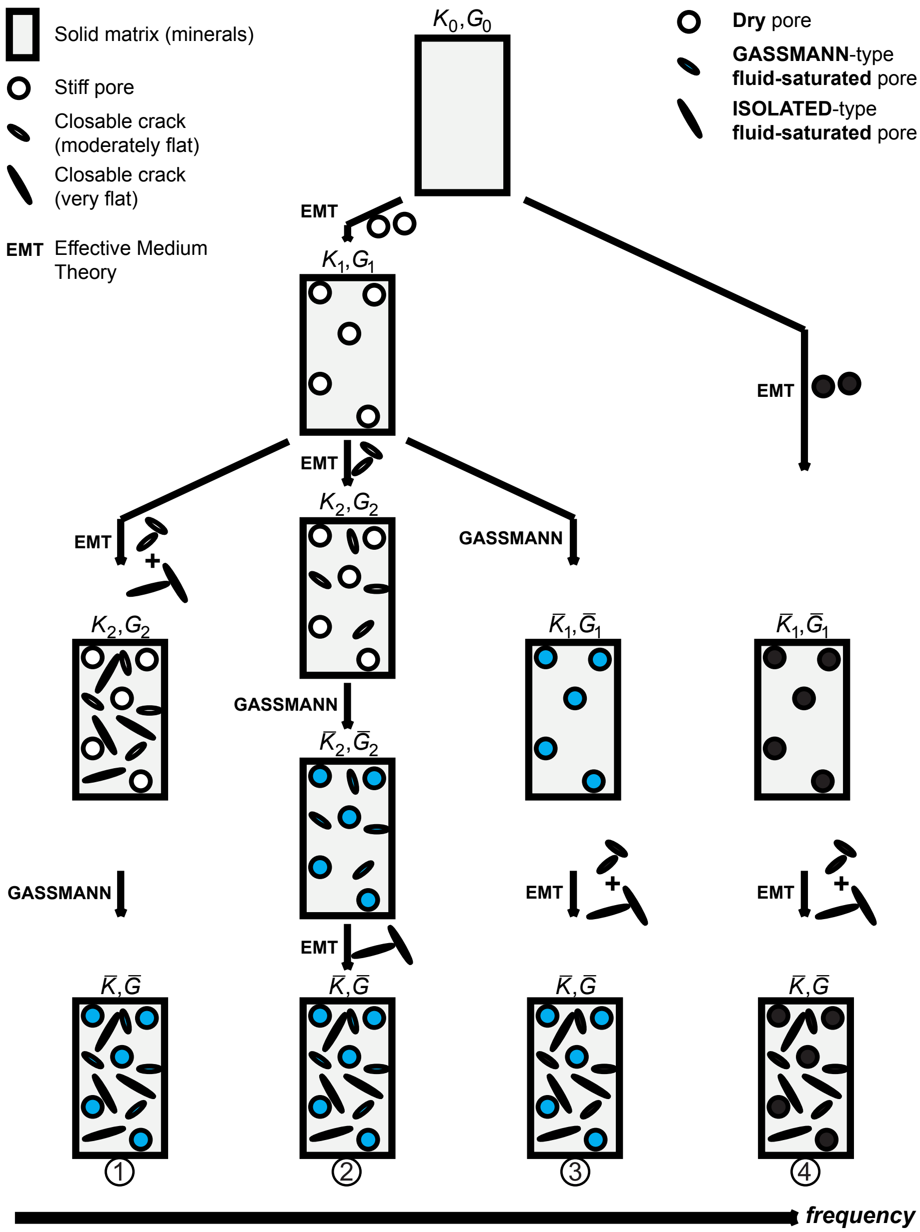

Figure 1. Theoretical process for obtaining saturated effective moduli as function of frequency.

sented in this paper have been obtained using the Mori-Tanaka and the Differential schemes.

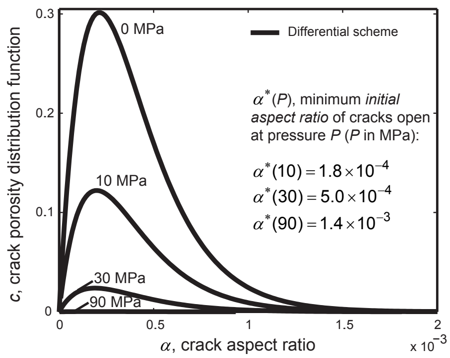

Figure 2. Crack porosity distribution function, $c(\alpha)$ at different pressures P, according to the Differential scheme (4$\%$ porosity Fontainebleau sandstone, after David (2012)).

Pressure dependence

One of the main interests of the present model, which is purely based on the pore aspect ratio distribution used as an input, is that frequency dependence of elastic moduli can be predicted at *any pressure*. This is simply achieved if the *zero pressure* distribution function, $c(\alpha)$, is "updated" at a given pressure P, by considering the crack closure process (Walsh, 1965; David and Zimmerman, 2012). The effect of increasing pressure on the crack porosity distribution function is illustrated in Figure 2.

Calculation of attenuation

If the elastic velocity (or elastic modulus) is now considered as the *real* part $M_{\mathrm{R}}(\omega)$ of a "viscoelastic" complex modulus $M(\omega) = M_{\mathrm{R}}(\omega) + iM_{\mathrm{I}}(\omega)$, where $\omega = 2\pi f$ denotes the angular frequency, the attenuation $Q^{-1}(\omega)$ is simply given by $Q^{-1}(\omega) = \frac{M_{\mathrm{I}}(\omega)}{M_{\mathrm{R}}(\omega)}$ (Bourbie *et al.*, 1987). The imaginary part $M_{\mathrm{I}}(\omega)$ can be obtained from one of the reciprocal Kramers-Kronig relations: $M_{\mathrm{I}}(\omega) = \frac{2\omega}{\pi}\mathrm{P}\int_0^{+\infty}\left[\frac{M_{\mathrm{R}}(\omega')}{\omega'^2-\omega^2}\right]\mathrm{d}\omega'$, where the symbol P denotes the Cauchy's principal value of the integral.

3 Results

The example of a water-saturated 4$\%$ Fontainebleau sandstone (see David (2012); David and Zimmerman (2012)) is taken as the input of the model in all the subsequent simulations. Predictions for the frequency dependence of P and S-wave velocities are shown, respectively, in Figures 3a and 3b, at increasing differential pressures ($(P_{\mathrm{c}} - P_{\mathrm{p}})$, where $(P_{\mathrm{c}}, P_{\mathrm{p}})$ are the confining and pore pressures, respectively), taking $\zeta = 1$ (see equation (1)), and according to the Mori-Tanaka and Differential schemes. By assuming a distribution of "relaxation frequencies", the model predicts that P and S-wave velocities increase in a very similar manner with frequency, between the seismic range ($\sim$ Hz) and the ultrasonic range ($\sim$ MHz). Note that the influence of

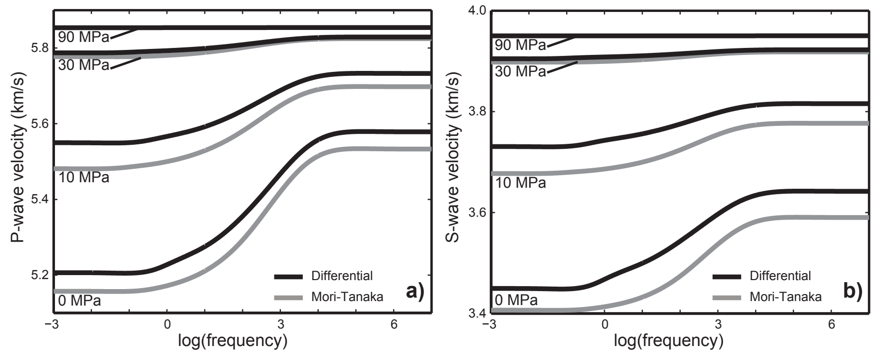

Figure 3. Model predictions for a) the compressional seismic velocity; b) the shear seismic velocity; for water-saturated 4$\%$ Fontainebleau sandstone (see David (2012)), as functions of the wave frequency, at increasing differential pressures $(P_{\mathrm{c}} - P_{\mathrm{p}})$. Results are shown for the Mori-Tanaka and Differential schemes, taking $\zeta = 1$ (see equation (1)).

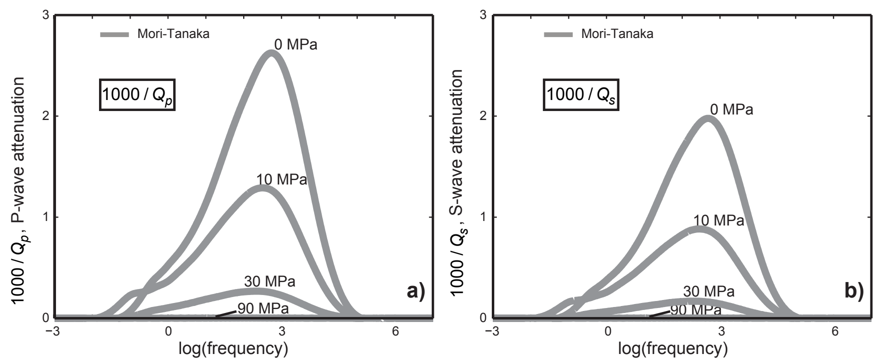

Figure 4. Model predictions for a) attenuation of the compressional seismic velocity; b) attenuation of the shear seismic velocity; for water-saturated 4$\%$ Fontainebleau sandstone (see David (2012)), as functions of the wave frequency, at increasing differential pressures $(P_{\mathrm{c}} - P_{\mathrm{p}})$. Results are shown for the Mori-Tanaka scheme, taking $\zeta = 1$ (see equation (1)).

the parameter ζ is trivial, and would simply result in a shift in frequency of the curves, as expected. The same effect would be obtained if the viscosity of the fluid was varied (see equation (1)). From their experimental results, Jones and Nur (1983) suggested that, indeed, acoustic properties of rocks are actually dependent on the *product* of fluid viscosity and frequency, rather than the frequency itself.

An important result is that significant dispersion is observed for *both* P and S-

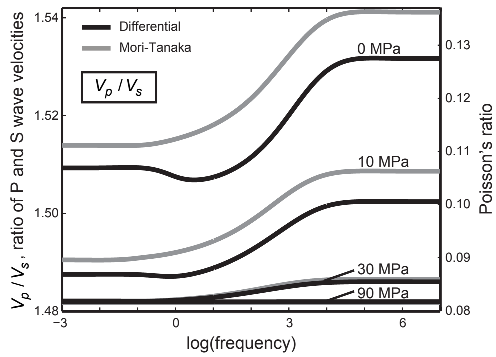

Figure 5. Model predictions for the ratio of compressional and shear seismic velocities, $V_\mathrm{p}/V_\mathrm{s}$, for water-saturated 4$\%$ Fontainebleau sandstone (see David (2012)), as a function of the wave frequency, at increasing differential pressures $(P_\mathrm{c} - P_\mathrm{p})$. Results are shown for the Mori-Tanaka and Differential schemes, taking $\zeta = 1$ (see equation (1)).

wave velocities; although simulations for the bulk and shear modulus are not shown here, bulk and shear modulus increase by 20% and 10% with frequency at "zero pressure", respectively. No velocity dispersion is observed at high pressures, when the rock is only left with the stiff pores. This result was expected, as the fundamental assumption of the model is that velocity dispersion is caused by the presence of cracks, and more precisely by the difference of local pore pressures between compliant cracks and stiff pores. The same conclusions are reached for the ratio of P and S wave velocities, $V_\mathrm{p}/V_\mathrm{s}$ (Figure 5), which is related to Poisson's ratio, ν, as $\frac{V_\mathrm{p}^2}{V_\mathrm{s}^2} = \frac{2(1-\nu)}{1-2\nu}$, which is a monotonically increasing function of ν. Poisson's ratio, which is in turn a monotonically increasing function of the *ratio* of bulk to shear moduli, is found to increase monotonically with frequency (at low pressures); however, the sense of evolution of the ratio $V_\mathrm{p}/V_\mathrm{s}$ (or, equivalently, of Poisson's ratio) with pressure (Figure 5) is again not entirely trivial. For instance, it was shown in David (2012) and Brantut *et al.* (2012) that the addition of saturated cracks in a solid could in some cases result in a decrease in Poisson's ratio.

The results obtained for the attenuation of P and S-wave velocities (Figure 4) are consistent with the typical values of attenuation in sandstones found in the literature, such as those obtained from resonant bar experiments by Winkler *et al.* (1979) and Murphy (1982). The relatively small values of attenuation $(1000/Q < 3)$ seem to be only case-dependent, as values of $1000/Q$ around 10 were obtained, for instance, when the model was applied to Vosges sandstone (see David (2012) after Fortin *et al.* (2007)). P-wave velocity is more attenuated than S-wave velocity (Figure 4). The same result is obtained for bulk modulus relative to shear modulus, although attenuation curves for bulk and shear moduli are not shown here.

REFERENCES

Biot, M. A. (1956b). “Theory of propagation of elastic waves in a fluid-saturated porous solid–II: Higher frequency range.” *J. Acoust. Soc. Am*, 28, 179–191.

Bourbié, T., Coussy, O. , and Zinszner, B. (1987). *Acoustics of Porous Media,* Ed. Technip, Paris.

Brantut, N., Schubnel, A., David, E. C., Héripré, E., Guéguen, Y. (2012). “Dehydration-induced damage and deformation in gypsum.” *J. Geophys. Res.*, 117, B03205.

Cleary, M. P. (1978). “Elastic and dynamic response regimes of fluid-impregnated solids with diverse microstructures.” *Int. J. Sol. Struct.*, 14, 795–819.

David, E. C. (2012). *The Effect of Stress, Pore Fluid and Pore Structure on Elastic Wave Velocities in Sandstones,* Ph.D. thesis, Imperial College London.

David, E. C., and Zimmerman, R. W. (2011a). “Compressibility and shear compliance of spheroidal pores: Exact derivation via the Eshelby tensor, and asymptotic expressions in limiting cases.” *Int. J. Sol. Struct.*, 48, 680–686.

David, E. C., and Zimmerman, R. W. (2011b). “Elastic moduli of solids containing spheroidal pores.” *Int. J. Eng. Sci.*, 49, 544–560.

David, E. C., and Zimmerman, R. W. (2012). “Pore structure model for elastic wave velocities in fluid-saturated sandstones.” *J. Geophys. Res*, 177, B07210.

Dvorkin, J., and Nur, A. (1993). “Dynamic poroelasticity: A unified model with the squirt and the Biot mechanisms.” *Geophysics*, 58, 524–533.

Eshelby, J. D. (1957). “The determination of the elastic field of an ellipsoidal inclusion, and related problems.” *Proc. Roy. Soc. Lond. A*, 241, 376–396.

Fortin, J., Guéguen, Y., and Schubnel, A. (2007). “Effect of pore collapse and grain crushing on ultrasonic velocities and V_p/V_s.” *J. Geophys. Res.*, 112, B08207.

Gassmann, F. (1951). “Uber die Elasticität Poröser Medien [On the elasticity of porous media].” *Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich*, 96, 1–23.

Gurevich, B., Makarynska, D., de Paula, O. B., and Pervukhina, M. (2010). “A simple model for squirt-flow dispersion and attenuation in fluid-saturated granular rocks.” *Geophysics*, 75, N109–N120.

Jones, T. D. (1986). “Pore fluids and frequency-dependent wave propagation in rocks.” *Geophysics*, 51, 1939–1953.

Jones, T. D., and Nur, A. (1983). “Velocity and attenuation in sandstone at elevated temperatures and pressures.” *Geophys. Res. Lett.*, 10, 140–143.

LeRavalec, M., Guéguen, Y., and Chelidze, T. (1996). “Elastic wave velocities in partially saturated rocks: Saturation hysteresis.” *J. Geophys. Res.*, 101, 837–844.

Mavko, G., and Jizba, D. (1991). “Estimating grain-scale fluid effects on velocity dispersion in rocks.” *Geophysics*, 56, 1940–1949.

Mavko, G., and Nur, A. (1975). “Melt squirt in the asthenosphere.” *J. Geophys. Res.*, 80, 1444-1448.

Mavko, G., and Nur, A. (1979). “Wave attenuation in partially saturated rocks.” *Geophysics*, 44, 161–178.

Mori, T., and Tanaka, K. (1973). “Average stress in matrix and average elastic energy of materials with misfitting inclusions.” *Acta Mater.*, 21, 571–574.

Murphy, W. F. (1982). “Effects of partial water saturation on attenuation in Massilon sandstone and Vycor porous glass.” *J. Acoust. Soc. Am.*, 71, 1458–1468.

O’Connell, R. J., and Budiansky, B. (1977). “Viscoelastic properties of fluid-saturated cracked solids.” *J. Geophys. Res.*, 82, 5719–5735.

Palmer, I. D., and Traviola, M. L. (1980). “Attenuation by squirt flow in undersaturated gas sands.” *Geophysics*, 45, 1780–1792.

Pride, S. R., Berryman, J. G., and Harris, J. M. (2004). “Seismic attenuation due to wave-induced flow.” *J. Geophys. Res.*, 109, B01201.

Walsh, J. B. (1965). “The effect of cracks on the compressibility of rocks.” *J. Geophys. Res.*, 70, 381–389.

Walsh, J. B. (1969). “New analysis of attenuation in partially melted rock.” *J. Geophys. Res.*, 74, 4333–4337.

Winkler, K. (1986). “Estimates of velocity dispersion between seismic and ultrasonic frequencies.” *Geophysics*, 51, 183–189.

Winkler, K., Nur, A., and Gladwin, M. (1979). “Friction and seismic attenuation in rocks.” *Nature*, 277, 528–531.

Zimmerman, R. W. (1984). “Elastic moduli of a solid with spherical pores: New self-consistent method.” *Int. J. Rock Mech. Min. Sci.*, 21, 339–343.