

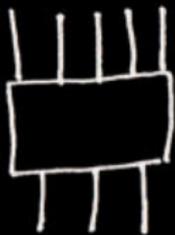
Une construction arborescente des progaphes

Christophe Cordero

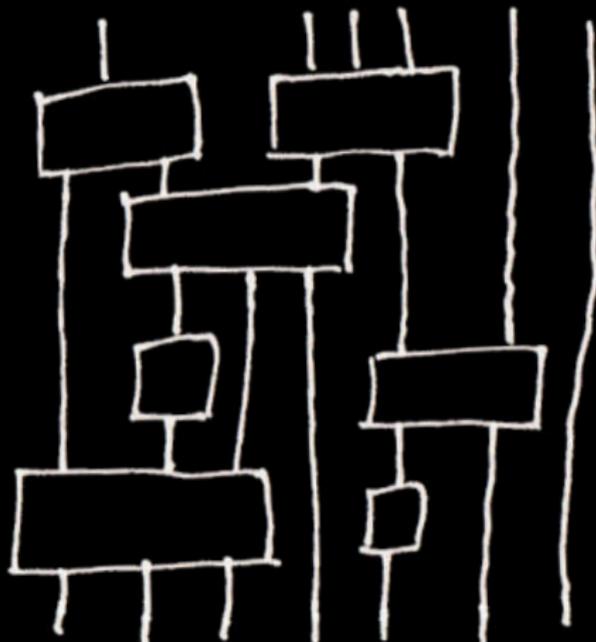
Université Paris-Est Marne-la-Vallée
Laboratoire d'Informatique Gaspard-Monge

22 février 2018

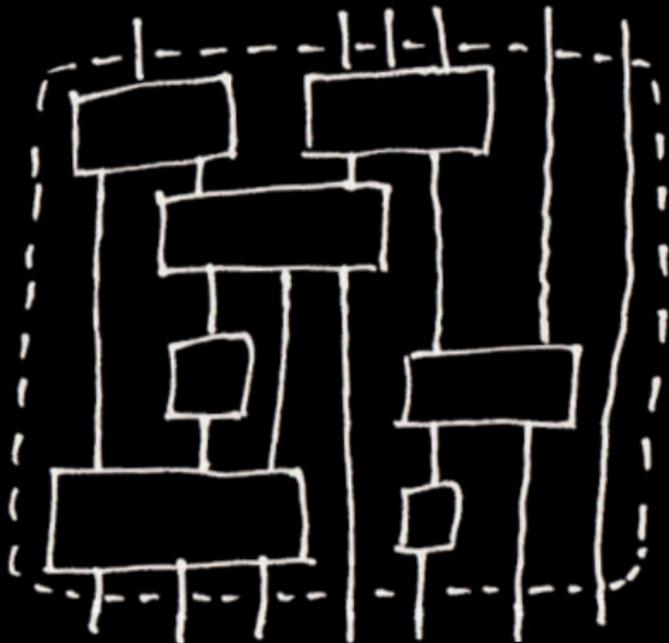
Prographe : générateur



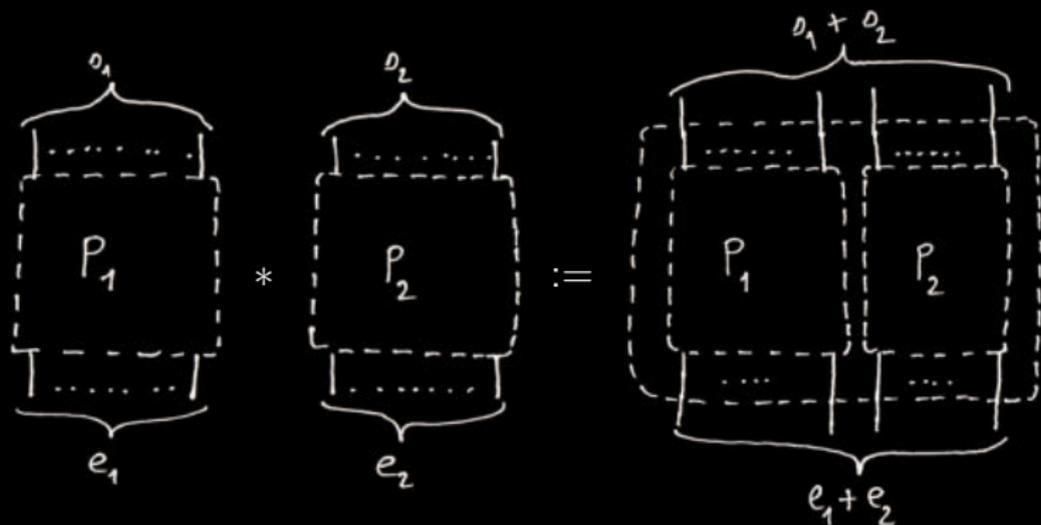
Prographe : assemblage



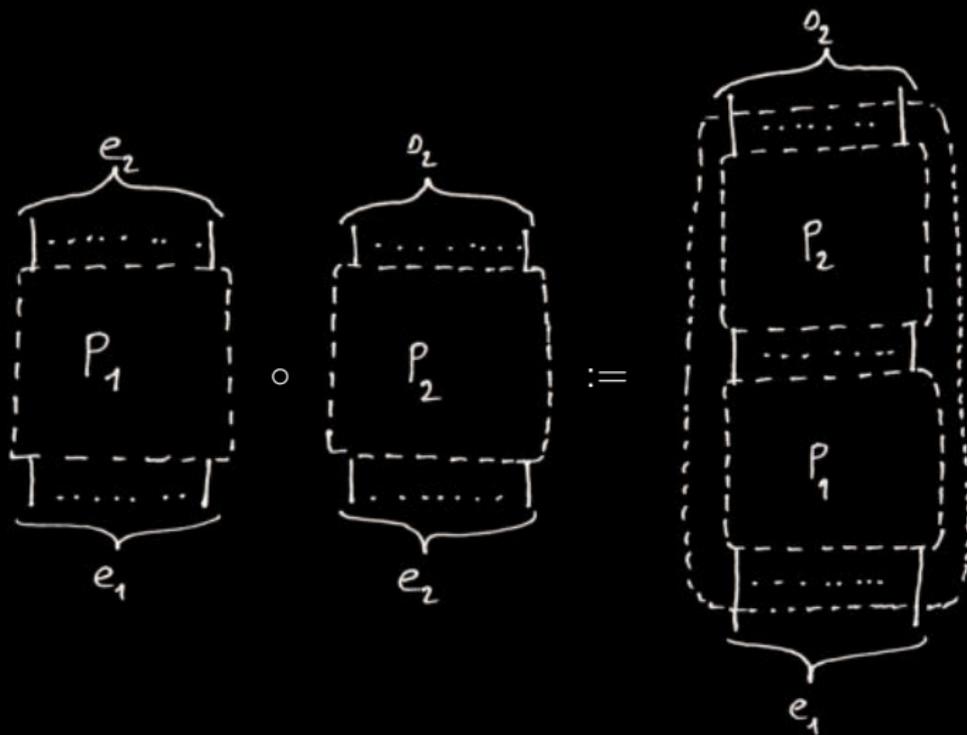
Prographe : assemblage



Opérations d'assemblages : *



Opérations d'assemblages : \circ



Prographes particuliers : forêts

Générateurs à une entrée :

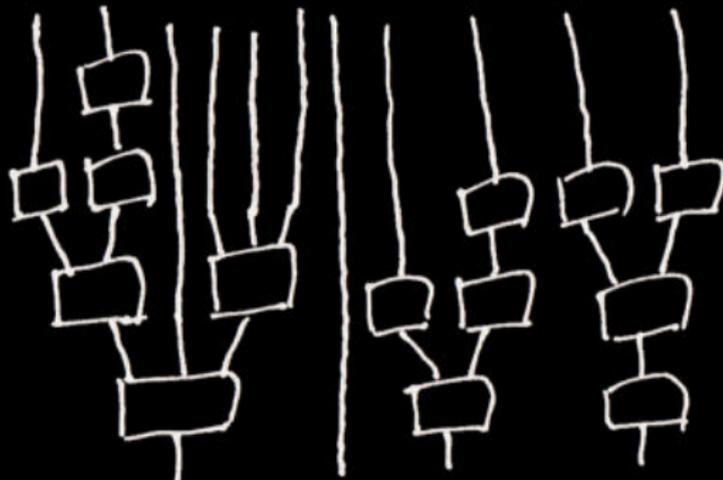


Prographes particuliers : forêts

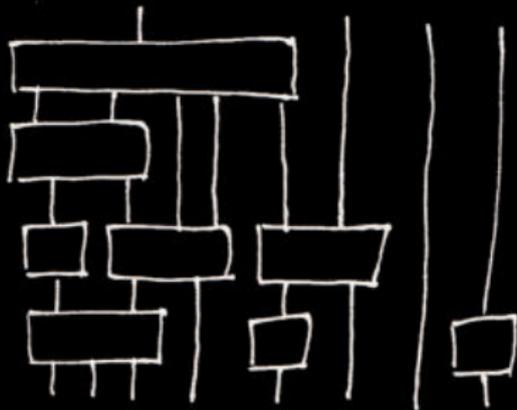
Générateurs à une entrée :



Forêt :

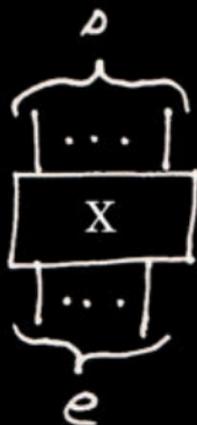


Question : progaphes > forêts ?



Construction arborescente (morphisme) : générateur

Générateur

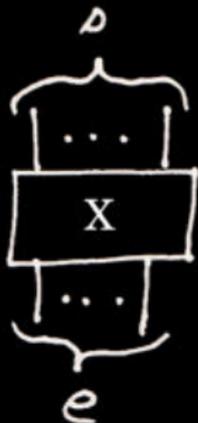


Construction arborescente (morphisme) : générateur

Générateur

\leftrightarrow

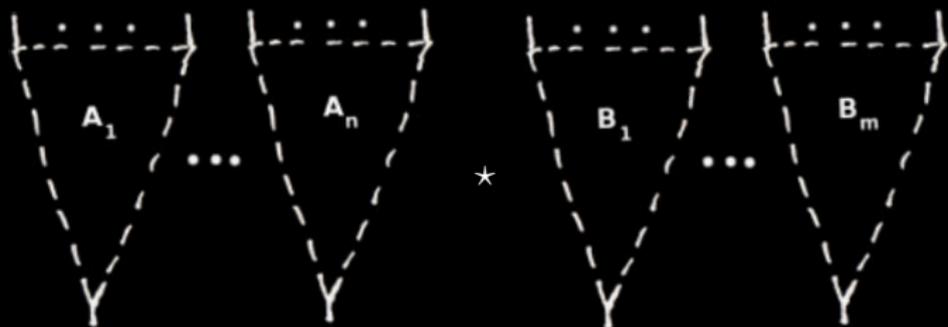
forêt



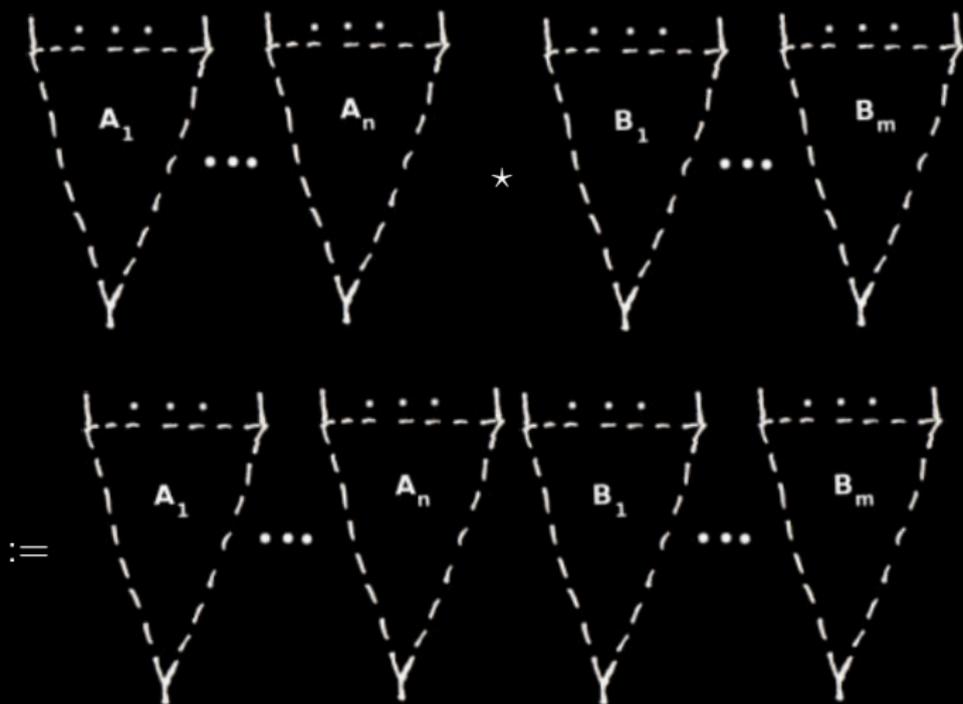
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Construction arborescente (morphisme) : l'opération *



Construction arborescente (morphisme) : l'opération $*$

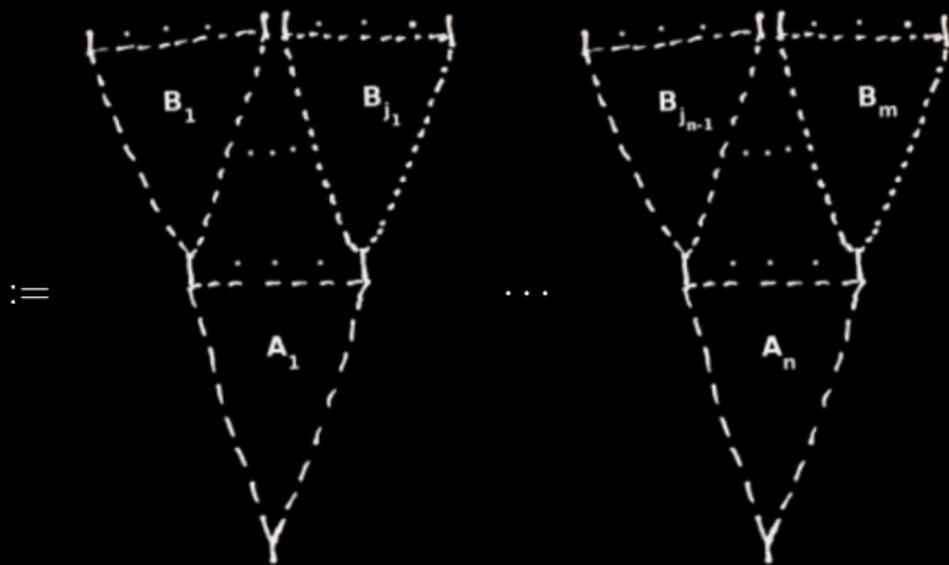


Construction arborescente (morphisme) : l'opération \circ

$$(A_1, \dots, A_n) \bullet (B_1, \dots, B_m)$$

Construction arborescente (morphisme) : l'opération \circ

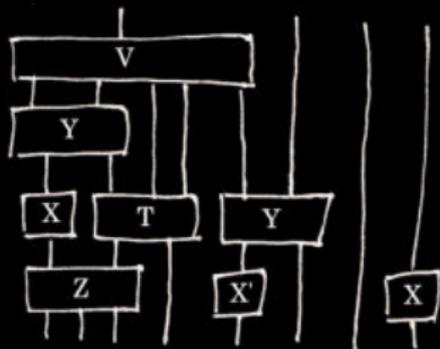
$$(A_1, \dots, A_n) \bullet (B_1, \dots, B_m)$$



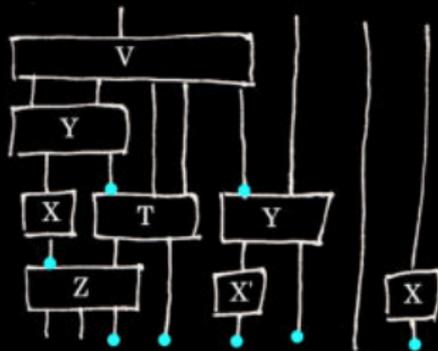
Exemple de forêt invalide



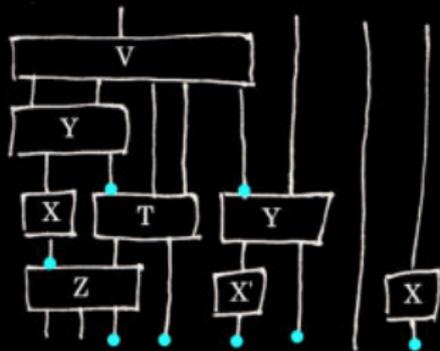
Algorithme : prographe \rightarrow forêt



Algorithme : prographe \rightarrow forêt

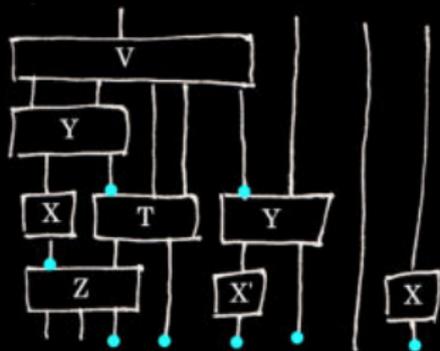


Algorithme : prographe \rightarrow forêt

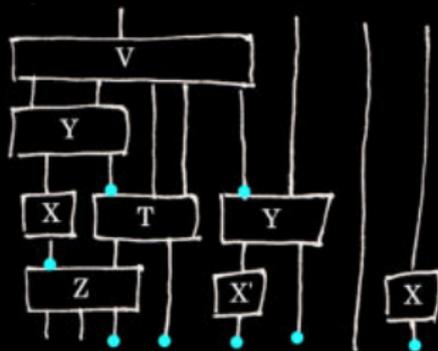


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Algorithme : prographe \rightarrow forêt

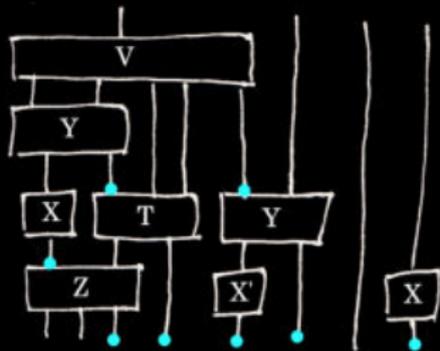


Algorithme : prographe \rightarrow forêt



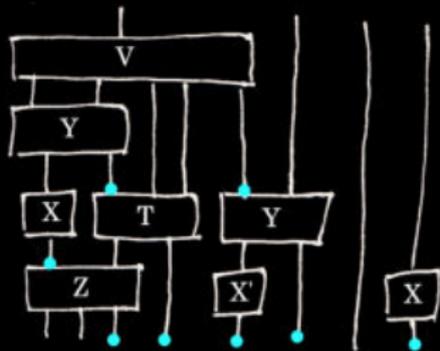
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Algorithme : prographe \rightarrow forêt



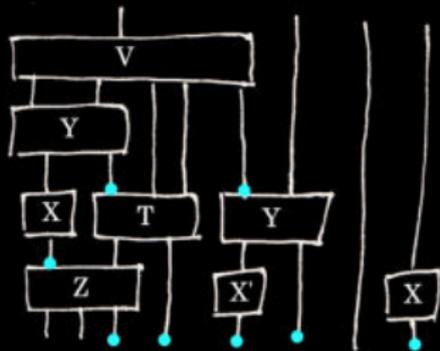
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Algorithme : prographe \rightarrow forêt



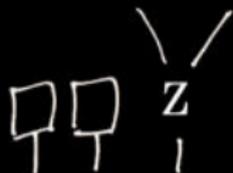
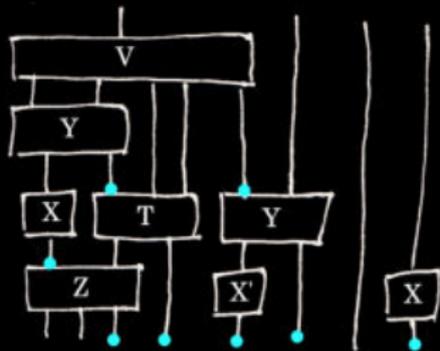
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Algorithme : prographe \rightarrow forêt

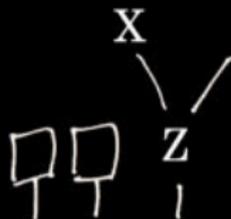
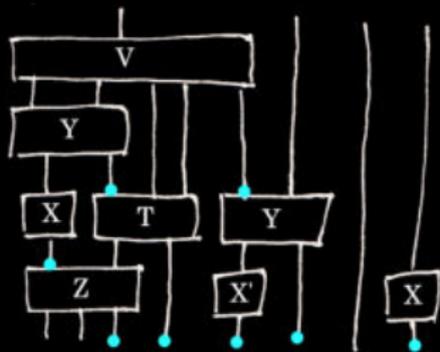


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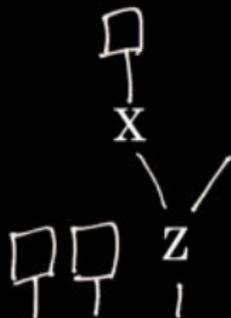
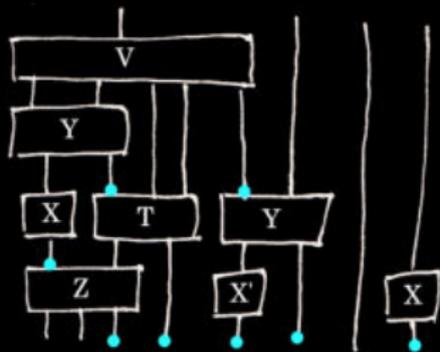
Algorithme : prographe \rightarrow forêt



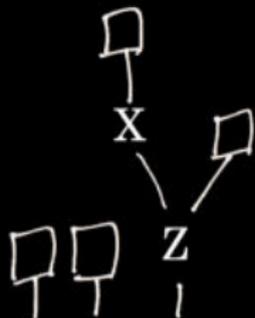
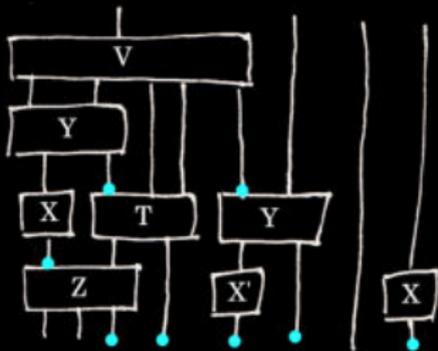
Algorithme : prographe \rightarrow forêt



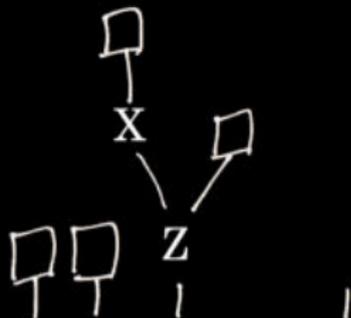
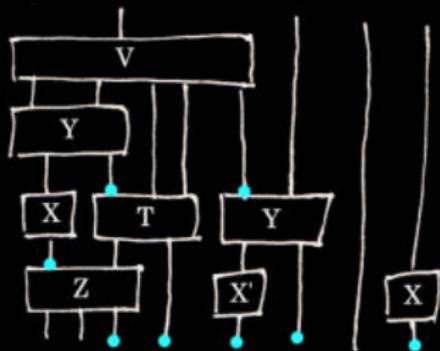
Algorithme : prographe \rightarrow forêt



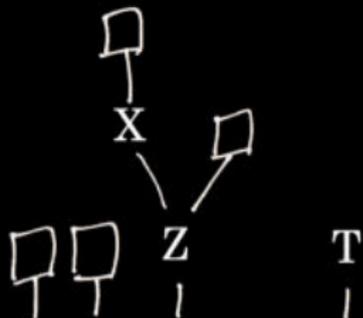
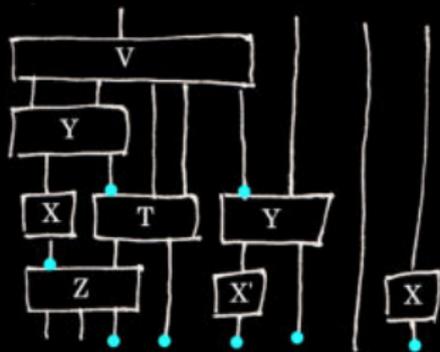
Algorithme : prographe \rightarrow forêt



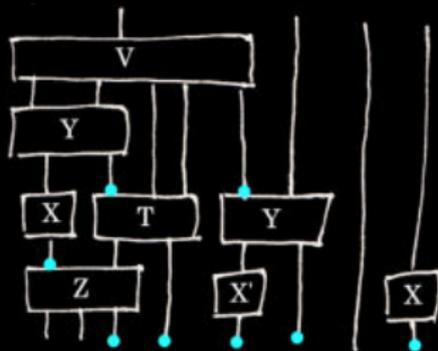
Algorithme : prographe \rightarrow forêt



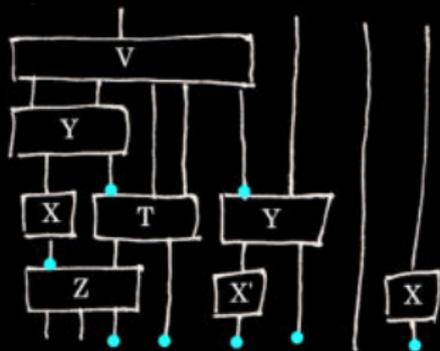
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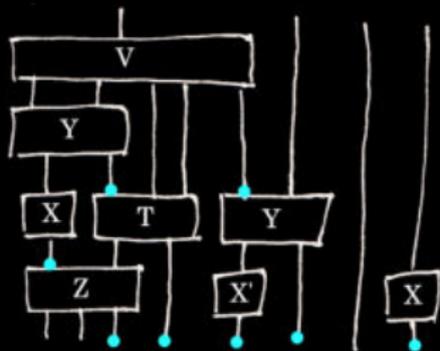
Algorithme : prographe \rightarrow forêt



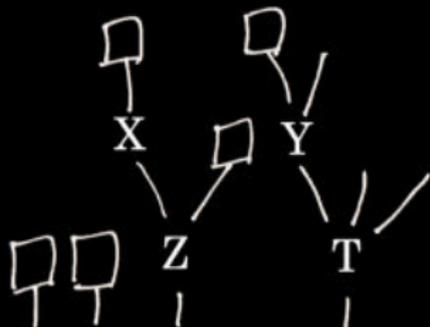
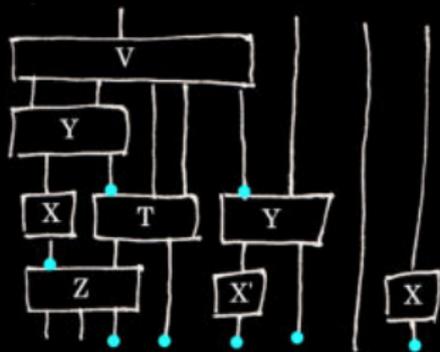
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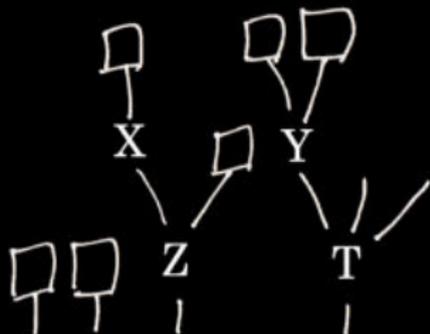
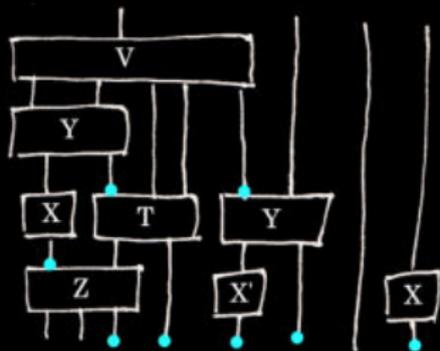
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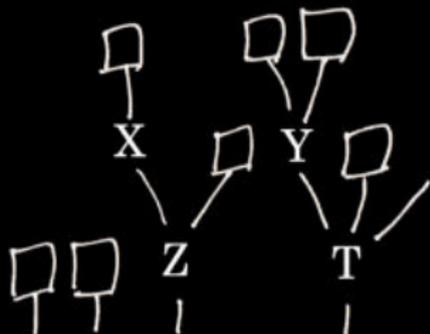
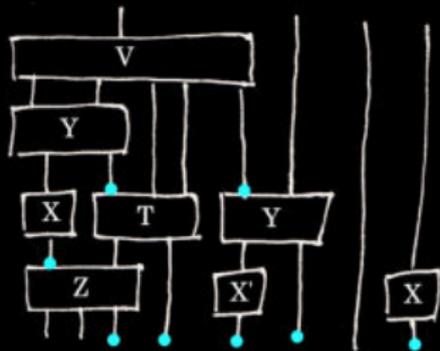
Algorithme : prographe \rightarrow forêt



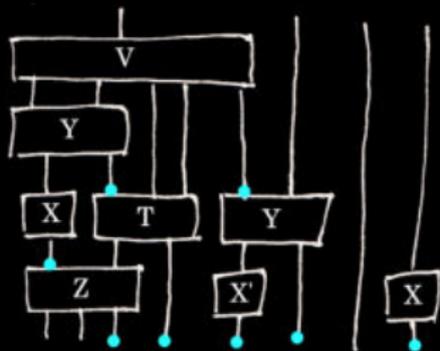
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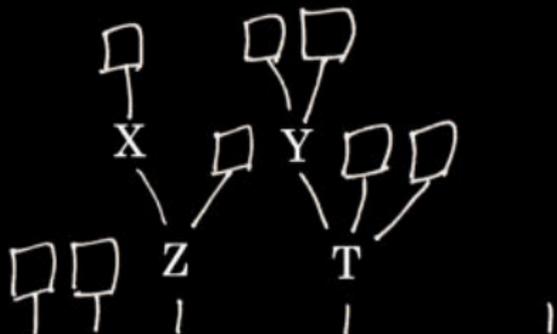
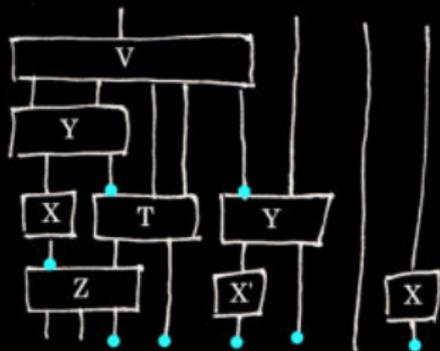
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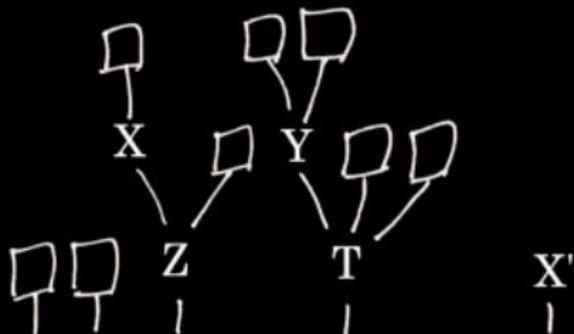
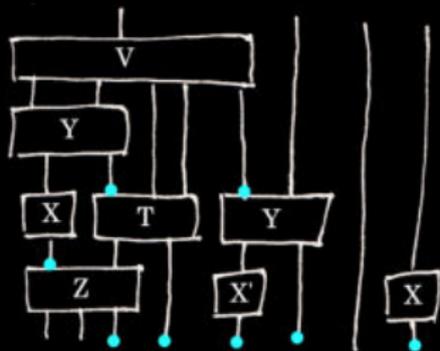
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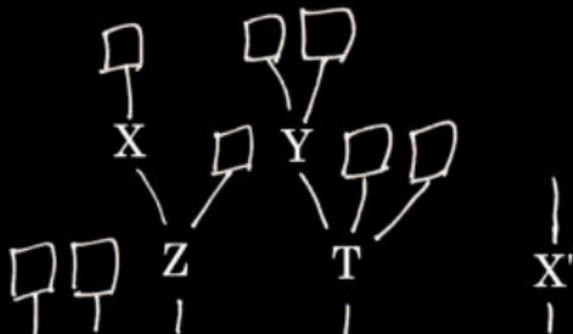
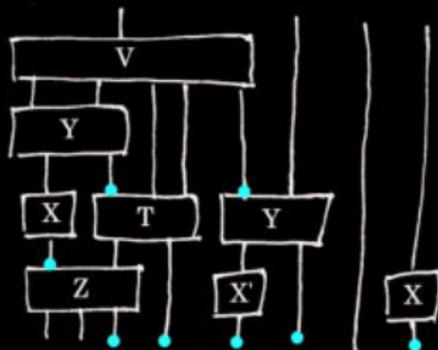
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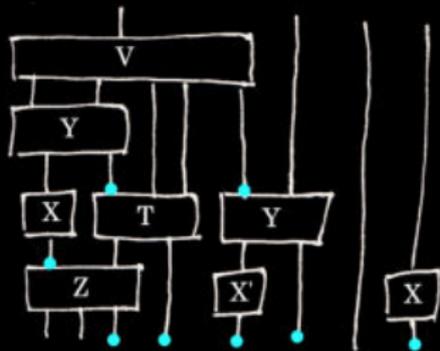
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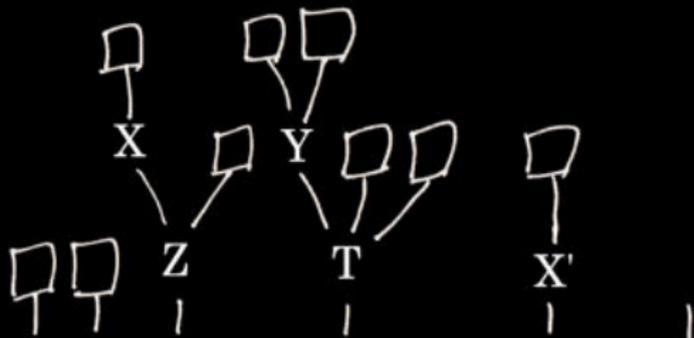
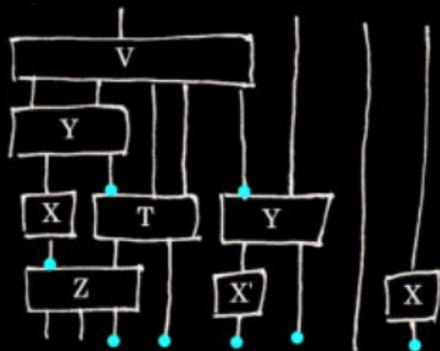
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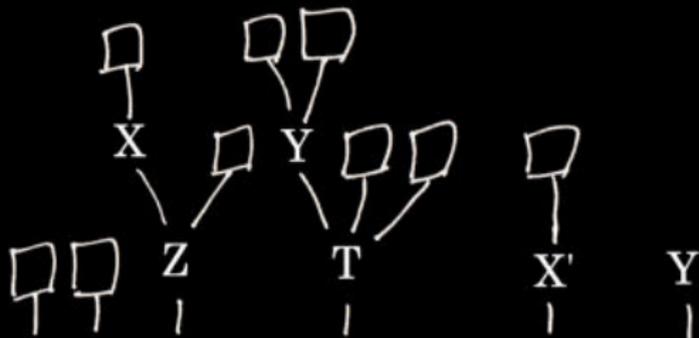
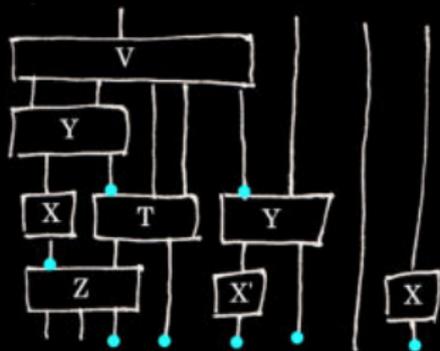
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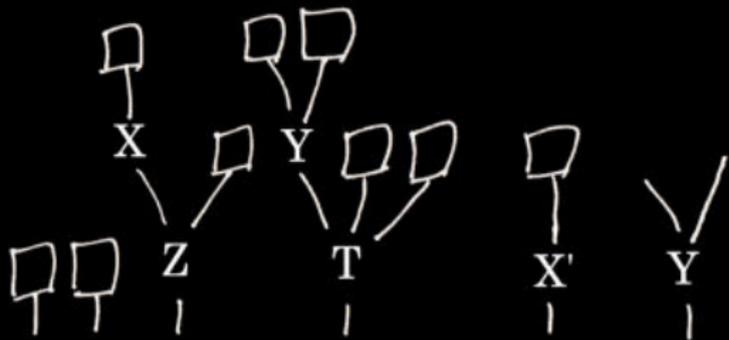
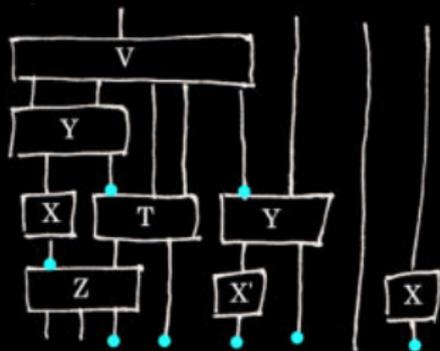
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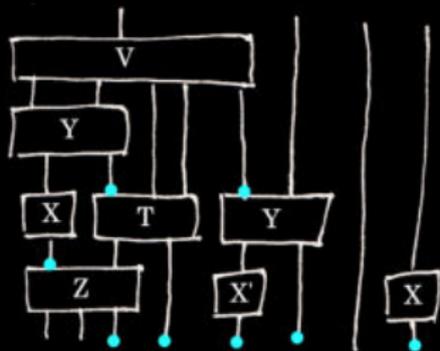
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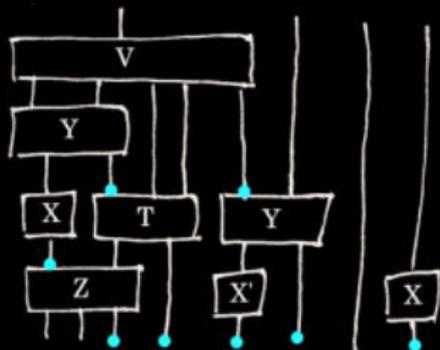
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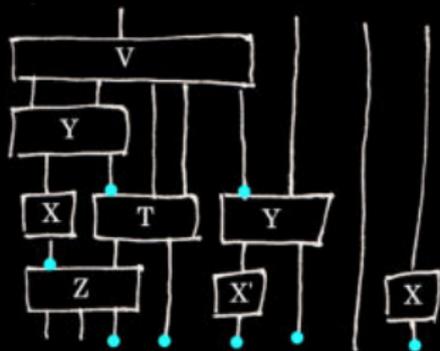
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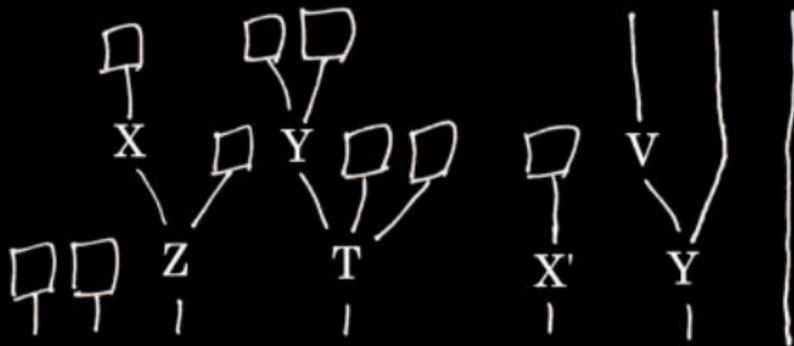
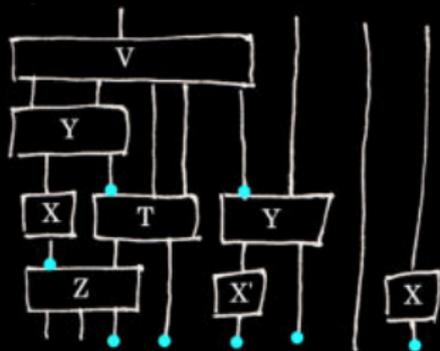
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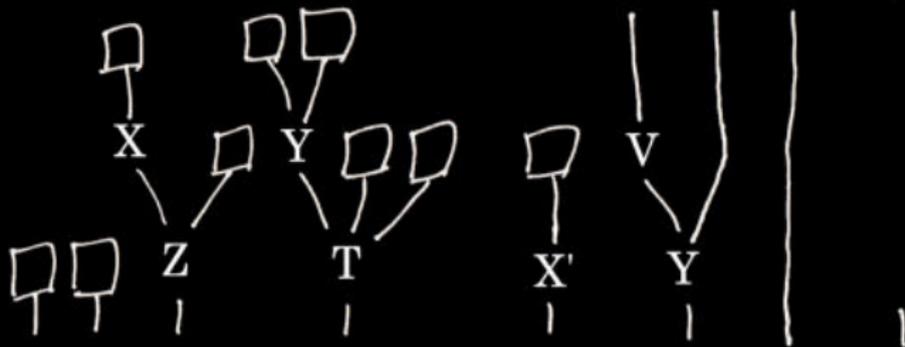
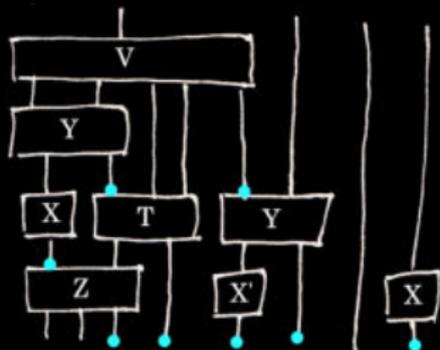
Algorithme : prographe \rightarrow forêt



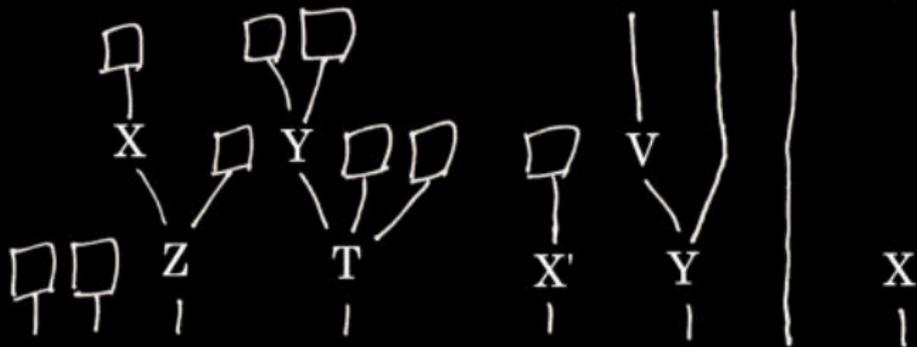
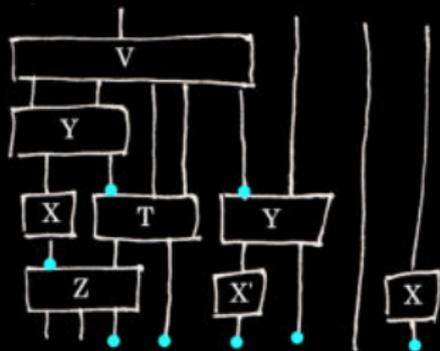
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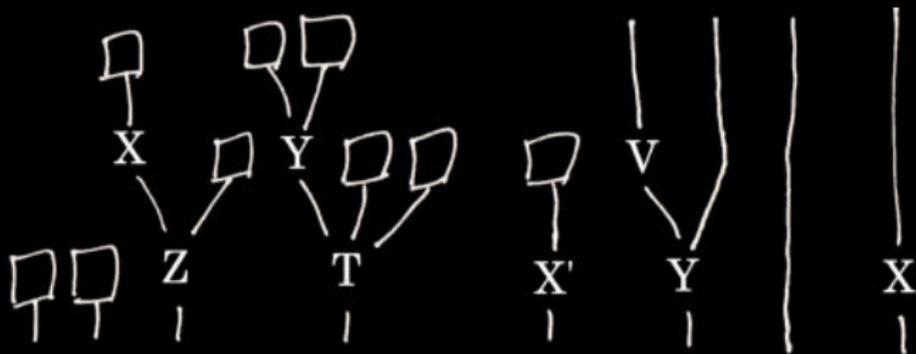
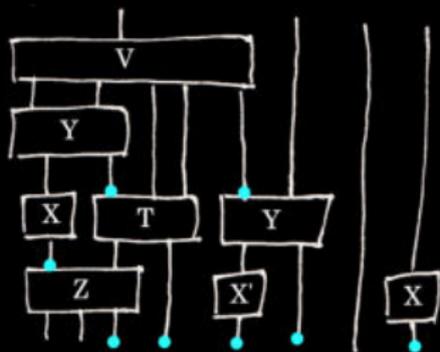
Algorithme : prographe \rightarrow forêt



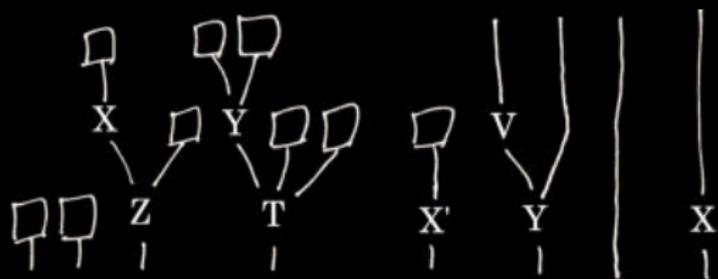
Algorithme : prographe \rightarrow forêt



Algorithme : prographe \rightarrow forêt

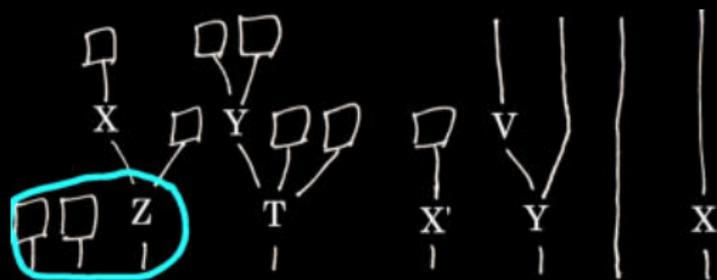


Algorithme : forêt \rightarrow prographe



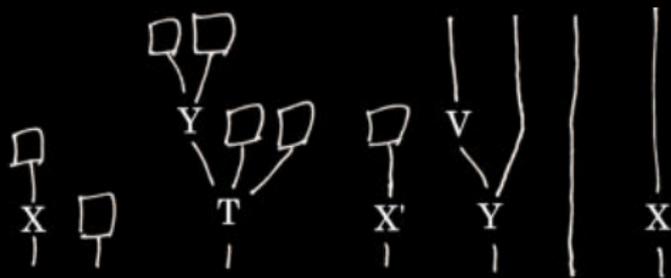
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Algorithme : forêt \rightarrow prographe

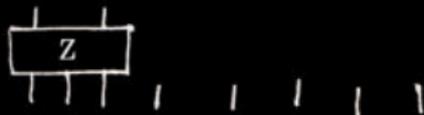
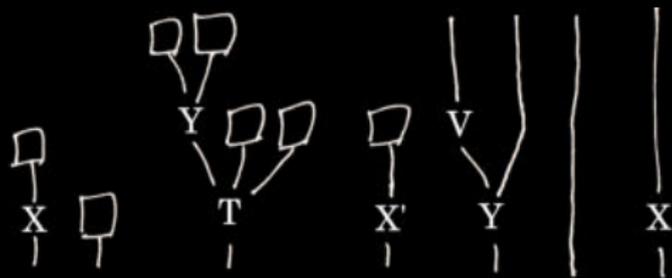


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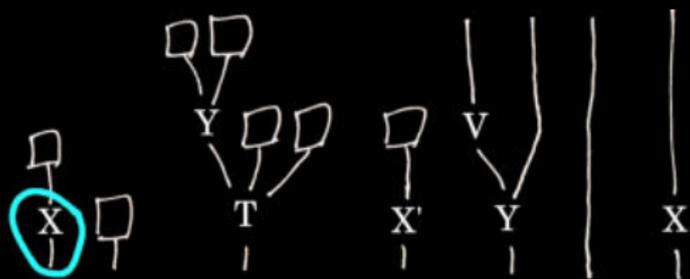
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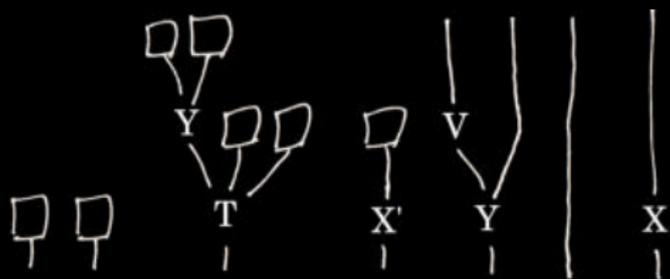
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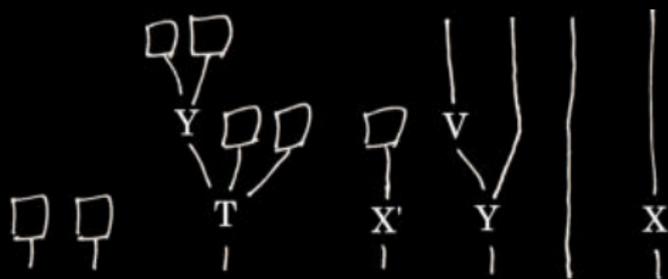
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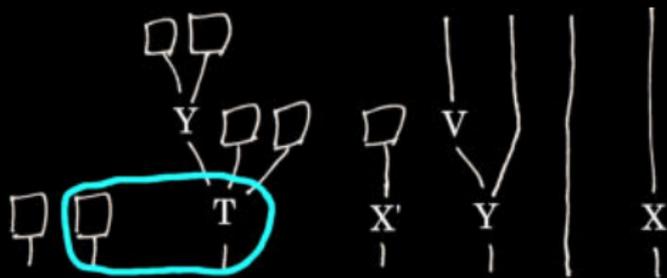
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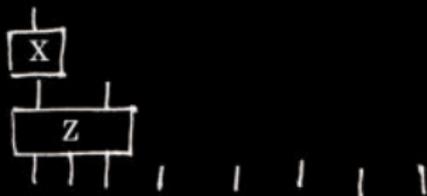
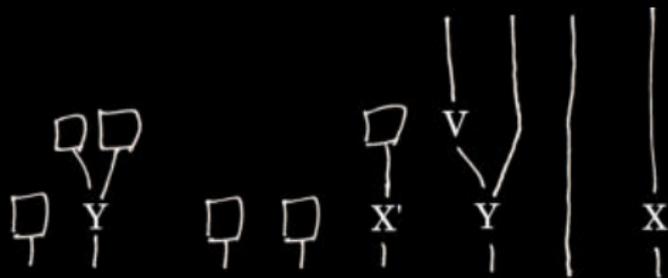
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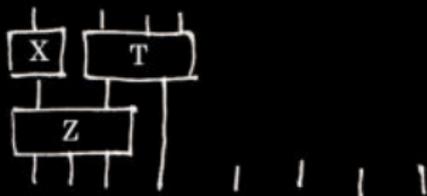
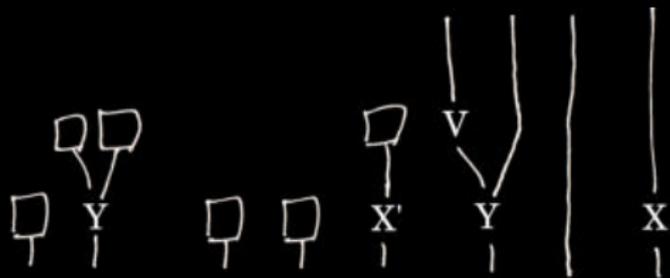
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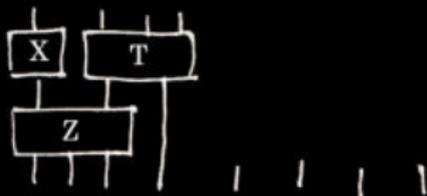
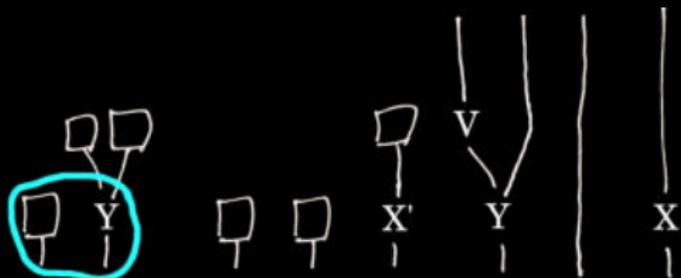
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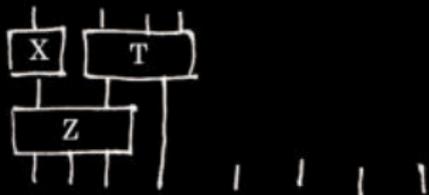
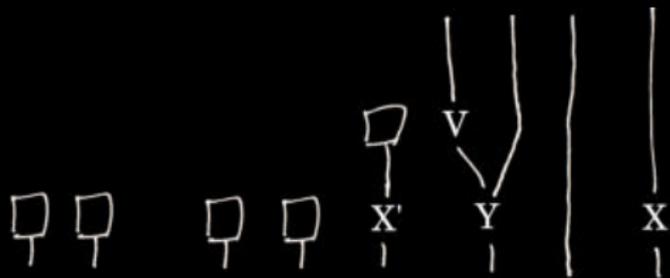
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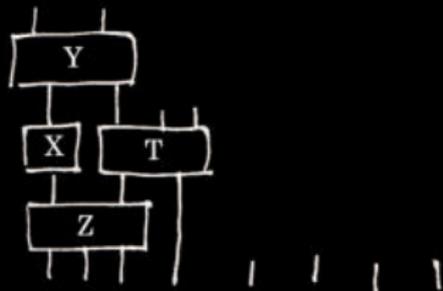
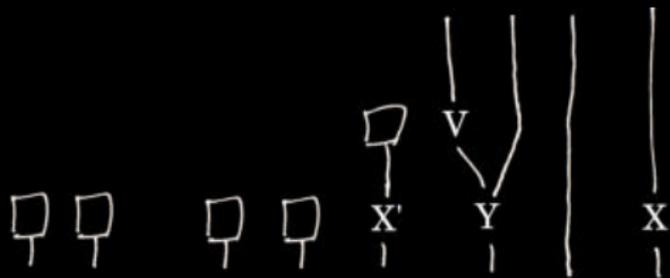
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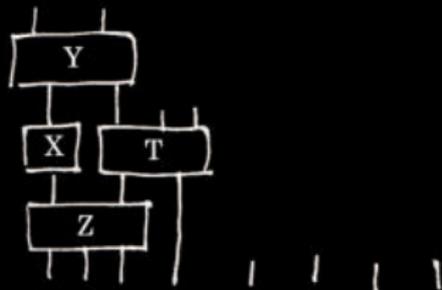
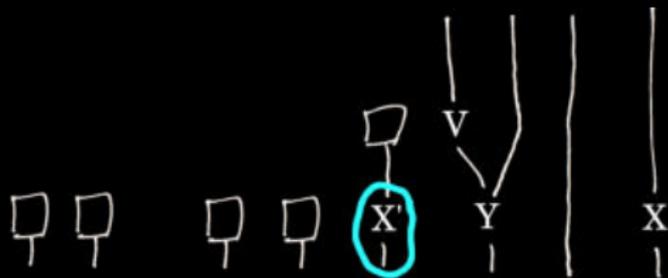
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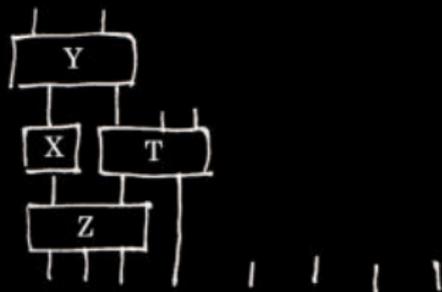
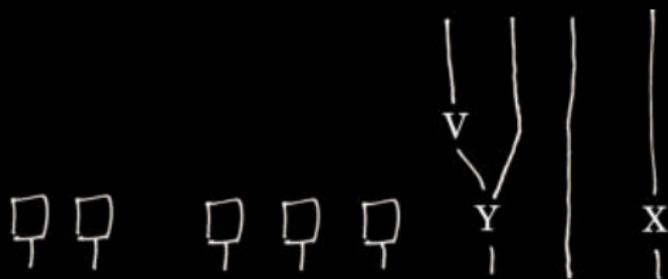
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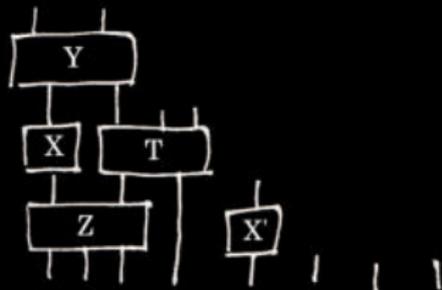
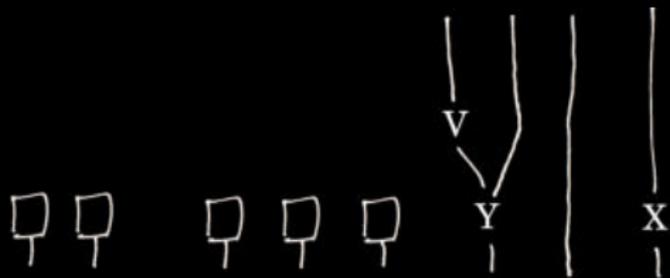
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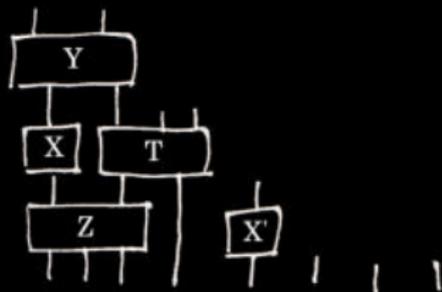
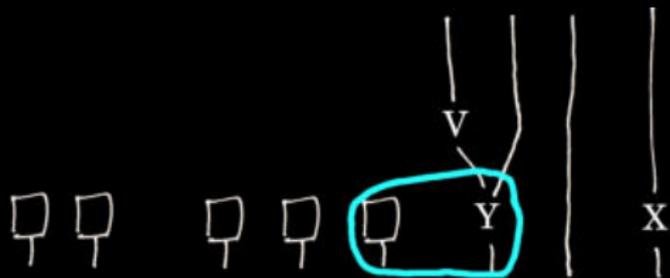
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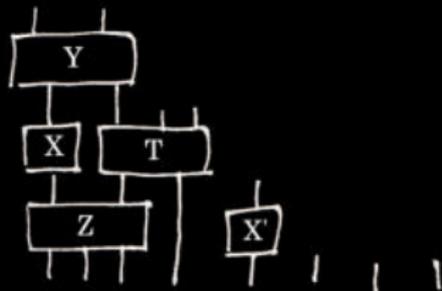
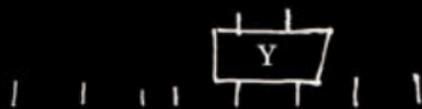
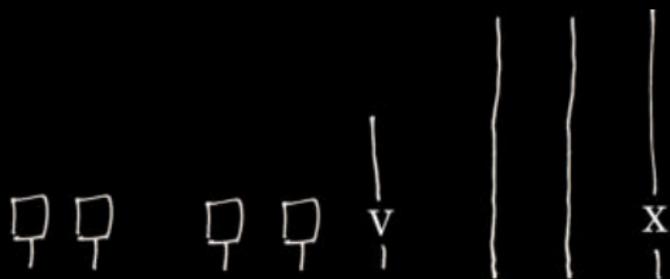
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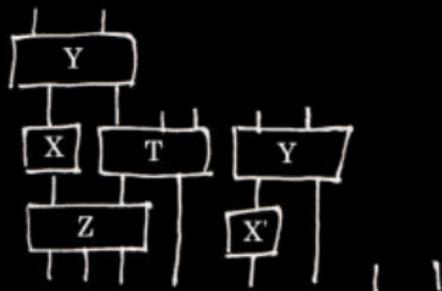
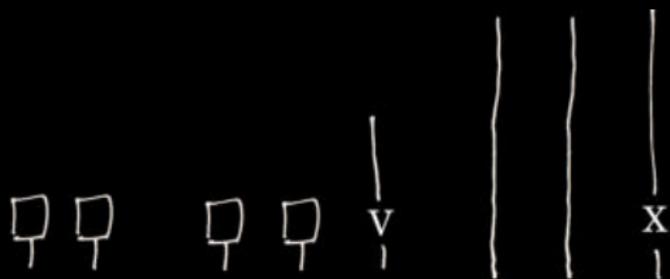
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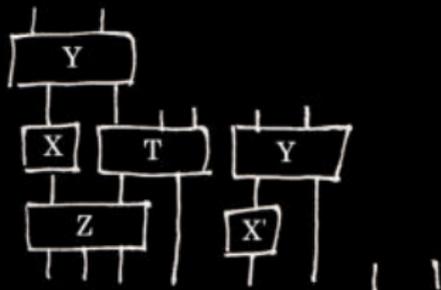
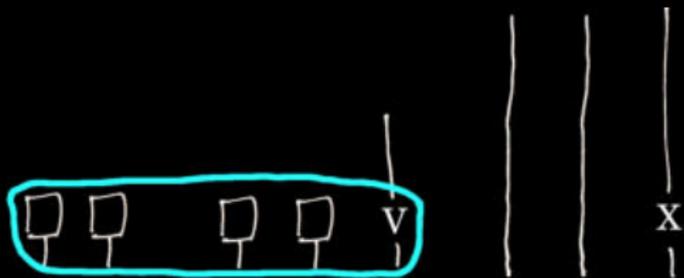
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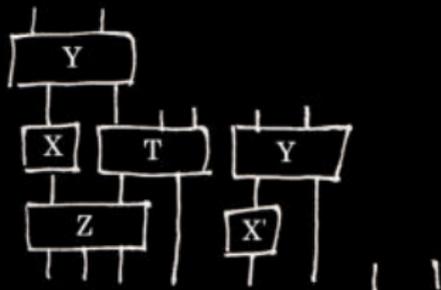
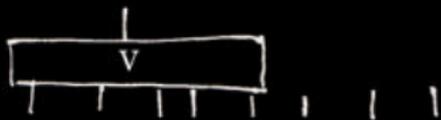
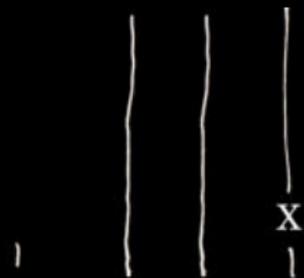
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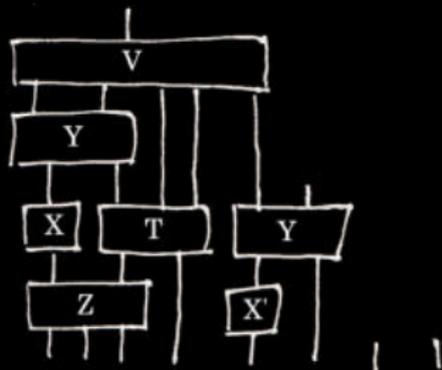
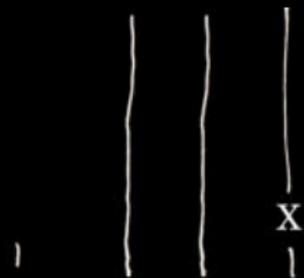
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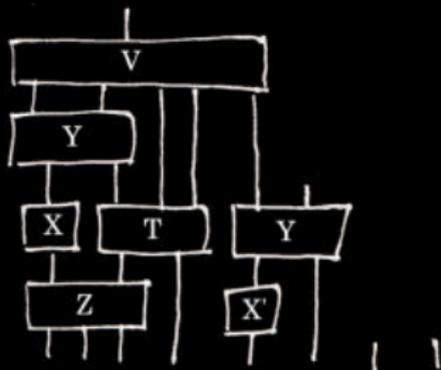
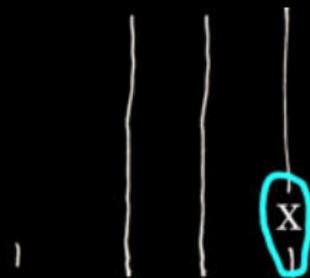
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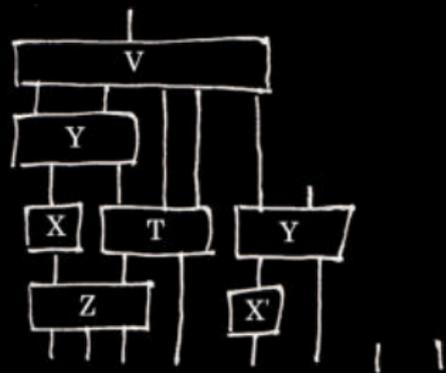
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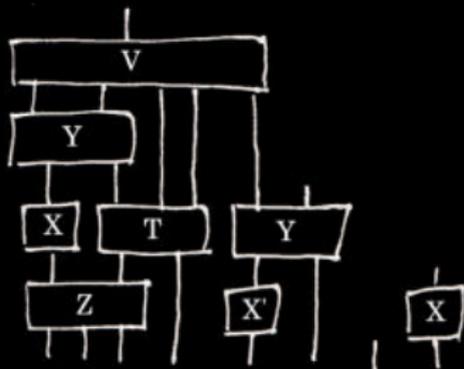


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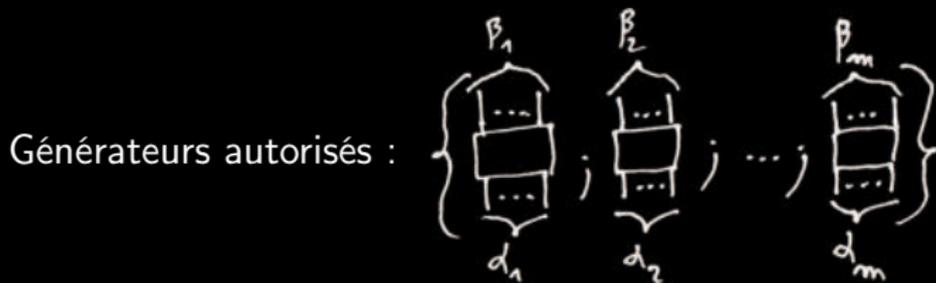
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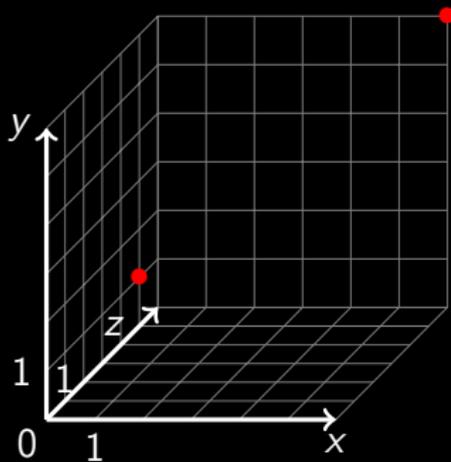
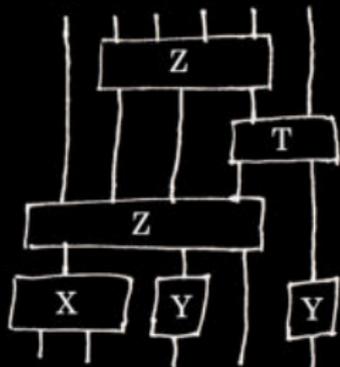
Applications : dénombrer les prographes



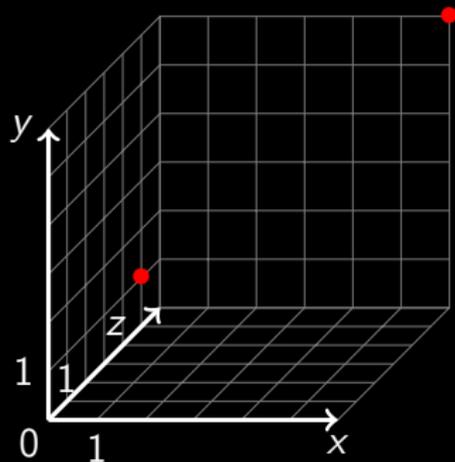
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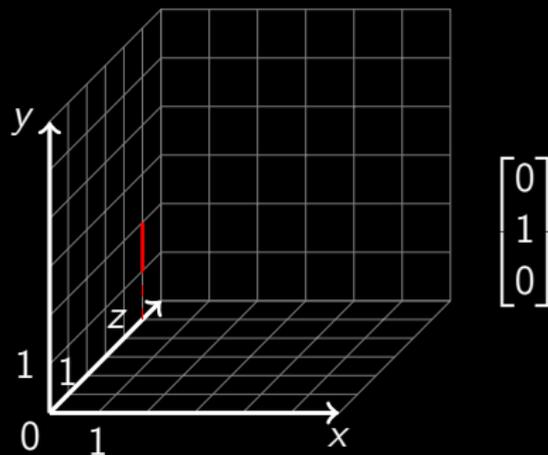
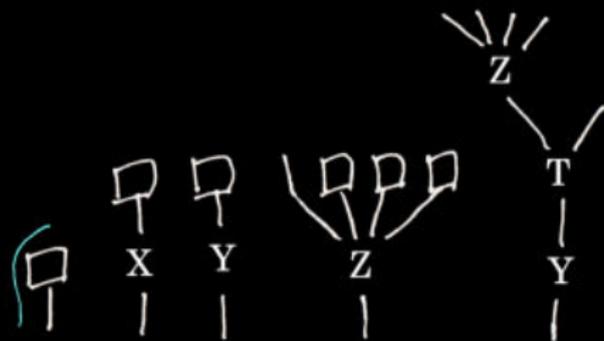
Bijection : progaphes \rightarrow des chemins de \mathbb{Z}^3



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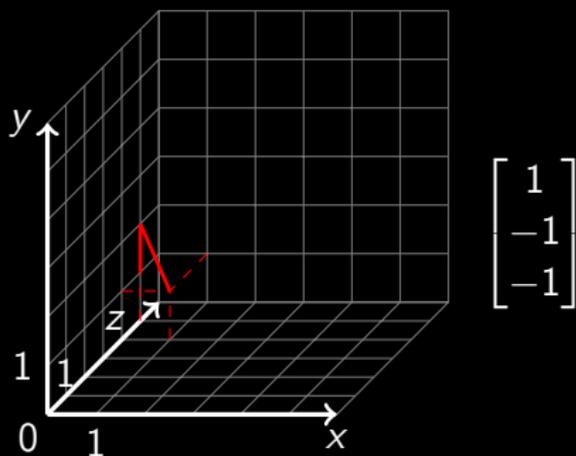
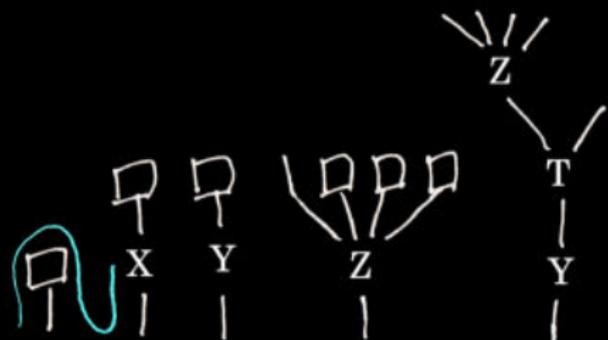


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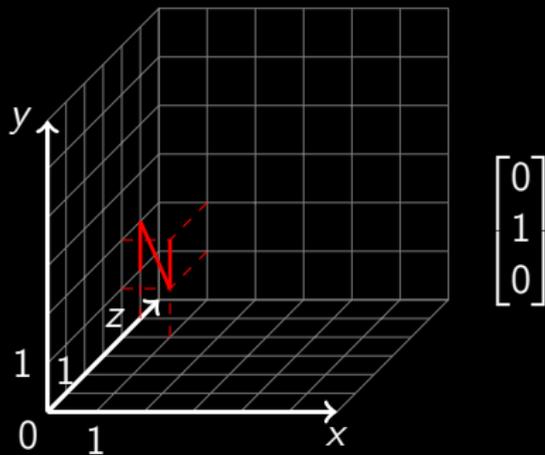
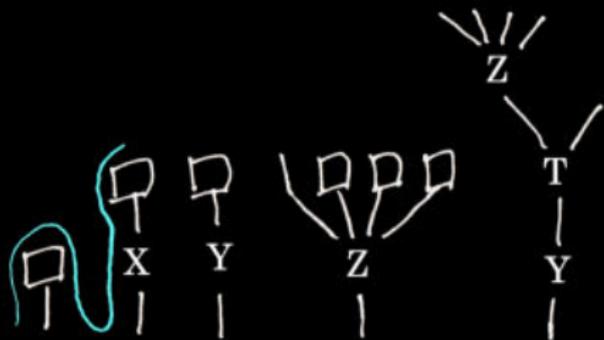
$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

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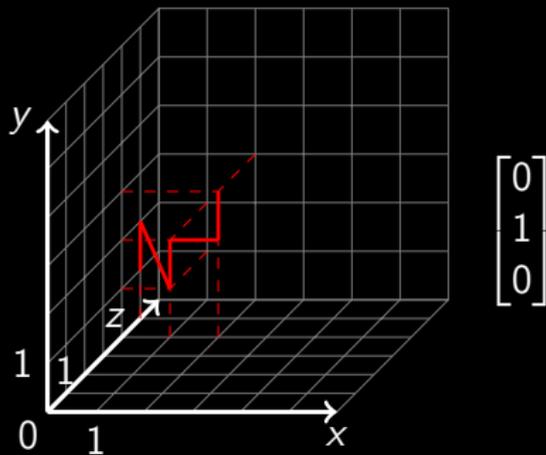
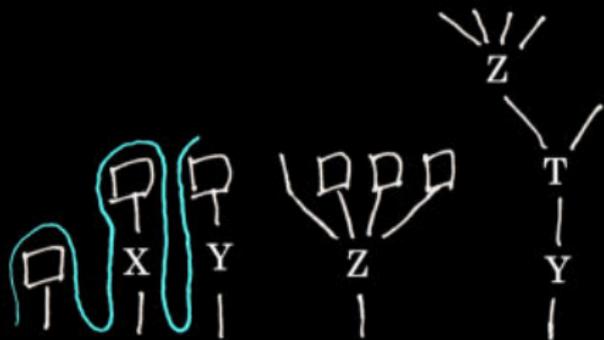


$$\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

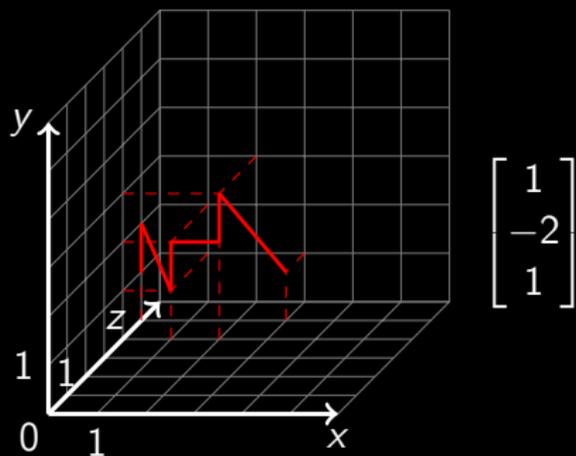
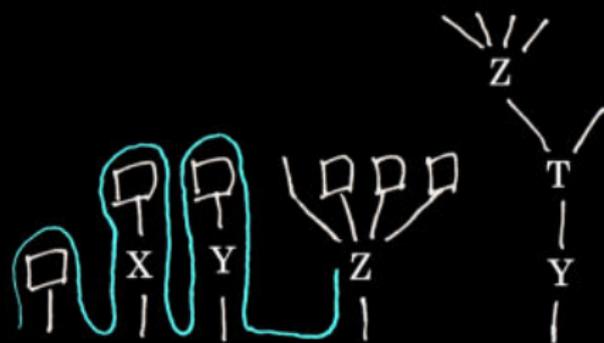
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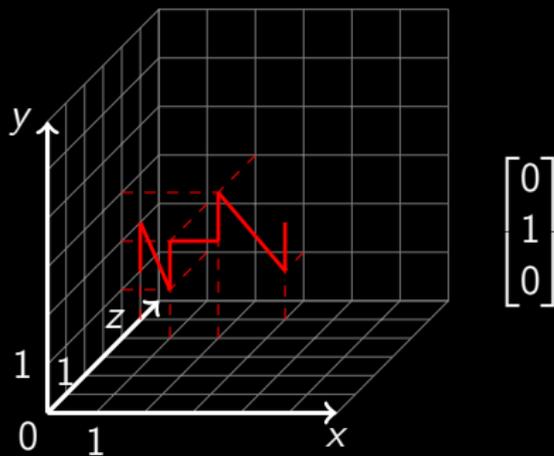
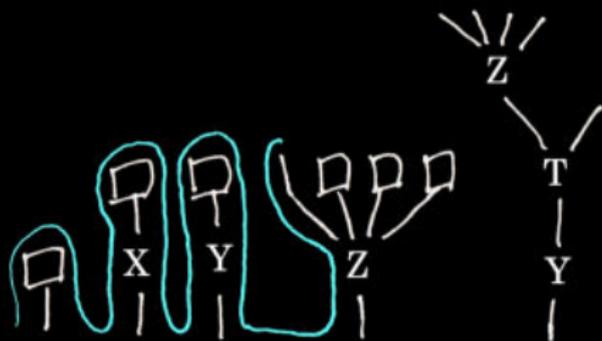


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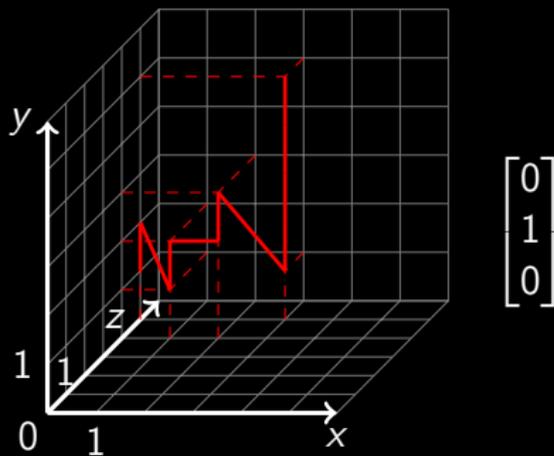
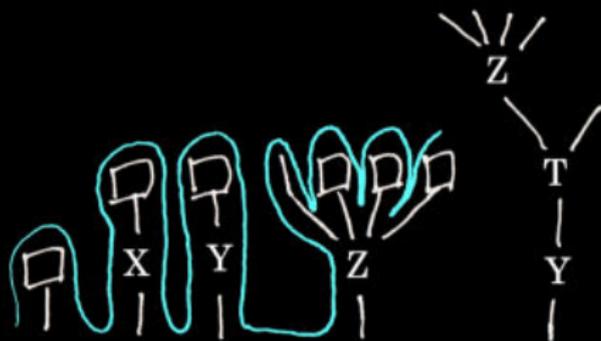


$$\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

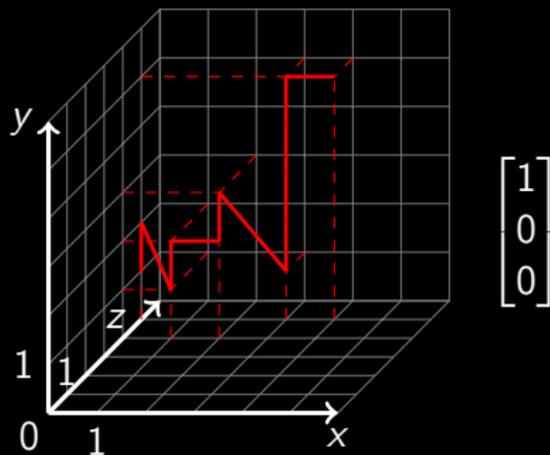
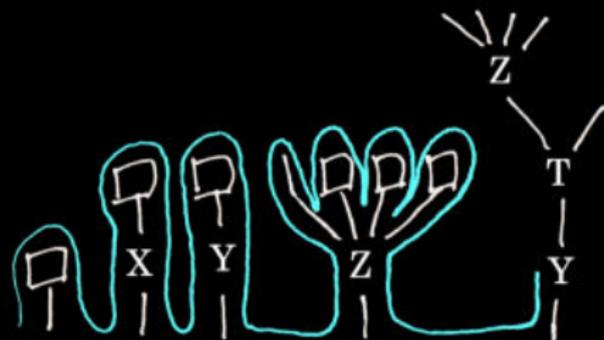
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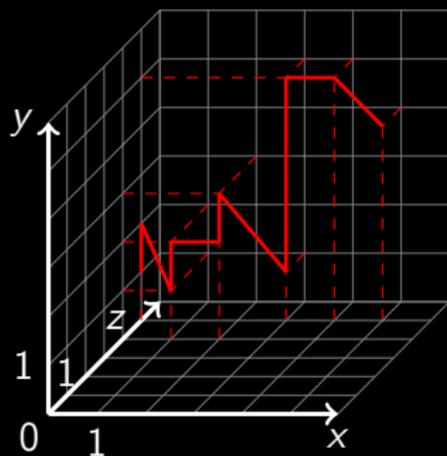
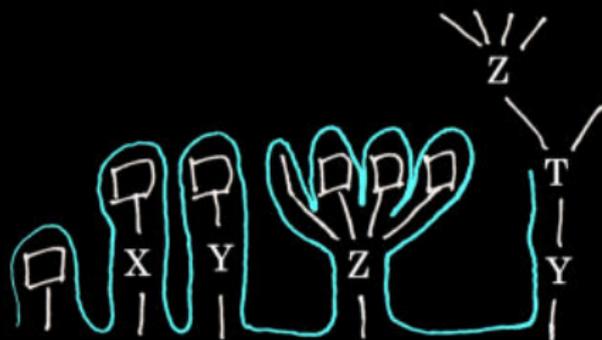
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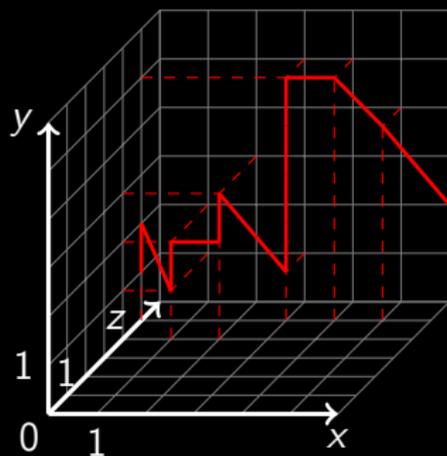
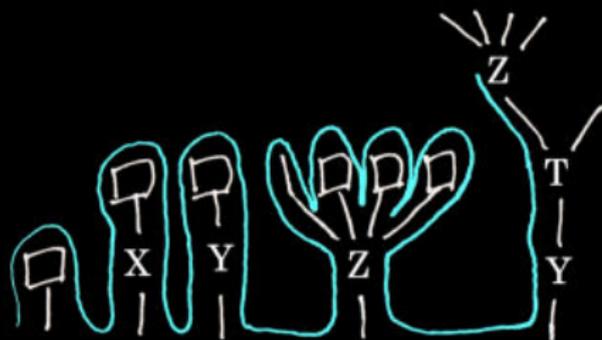


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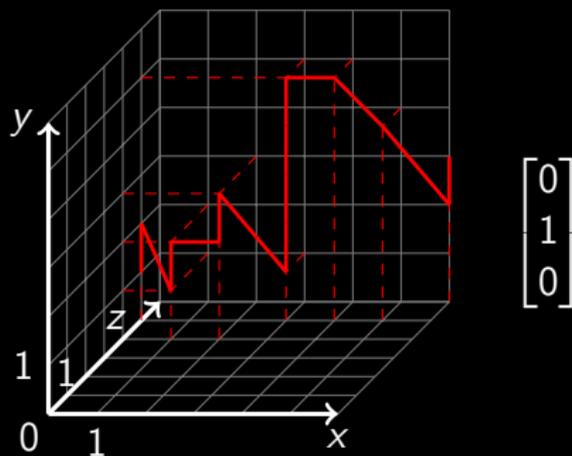
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Solution : formule de récurrence

$a_{n,s}$ vérifie la récurrence :

$$\left\{ \begin{array}{l} 1 \quad \text{si } n = 0 \text{ et } s = e \\ \sum_{\ell=1}^n (-1)^{\ell+1} \sum_{c_1+\dots+c_m=\ell} \binom{\ell}{c_1, \dots, c_m} \binom{s+\ell-\sum_{i=1}^m c_i \beta_i}{\ell} a_{n-\ell, s-\sum_{i=1}^m c_i(\beta_i-\alpha_i)} \\ 0 \quad \text{sinon.} \end{array} \right. \quad \text{si } n \geq 0 \text{ et } s > 0$$

PRO (structure algébrique) : $(\mathcal{P}, *, \circ, \mathbf{1})$

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Deux opérations associatives :

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$$\text{Unité : } x \in \mathcal{P}(e, s), \mathbf{1}^e \circ x = x \circ \mathbf{1}^s = x \text{ où } \mathbf{1}^s = \underbrace{\mathbf{1} * \dots * \mathbf{1}}_s$$

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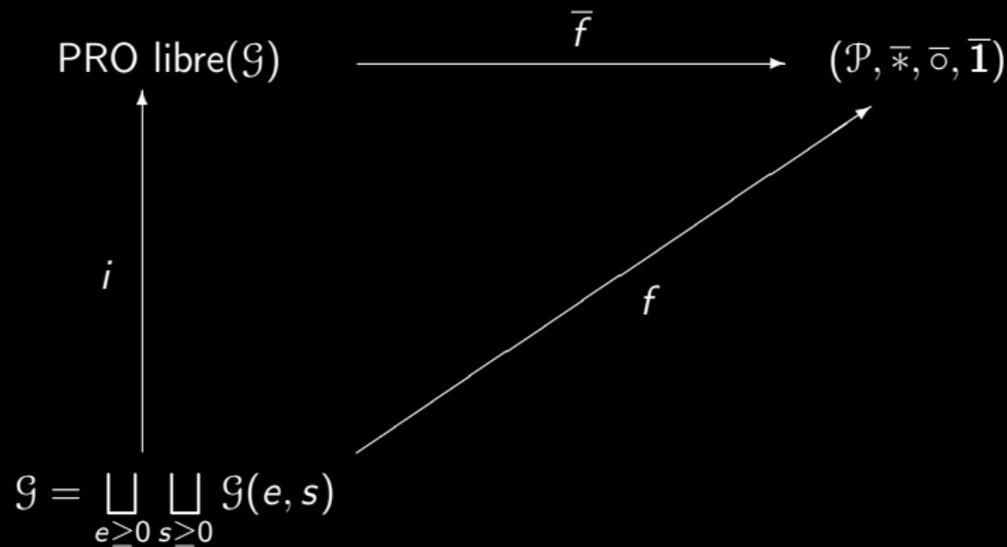
$$(5, 0, 11) + (4, 4, 1) = (9, 4, 12)$$

Relation non axiomatique :

$$(x_1, \dots, x_n) + (y_1, \dots, y_n) = (y_1, \dots, y_n) + (x_1, \dots, x_n)$$

PRO-graphe \in PRO libre

Propriété universelle :



Une construction arborescente des progaphes

Christophe Cordero

Université Paris-Est Marne-la-Vallée
Laboratoire d'Informatique Gaspard-Monge

22 février 2018