Rupture energetics in crustal rock from laboratory-scale seismic tomography

Franciscus M. Aben\textsuperscript{1}, Nicolas Brantut\textsuperscript{1}, Thomas M. Mitchell\textsuperscript{1}, and Emmanuel C. David\textsuperscript{1}

The energy released during earthquake rupture is partly radiated as seismic waves, and mostly dissipated by frictional heating on the fault interface and by off-fault fracturing of surrounding host rock. Quantification of these individual components is crucial to understanding the physics of rupture. We use a quasi-static rock fracture experiment combined with a novel seismic tomography method to quantify the contribution of off-fault fracturing to the energy budget of a rupture, and find that this contribution is around 3\% of the total energy budget, and 10\% of the fracture energy $G_F$. The off-fault dissipated energy changes the physical properties of the rock at the early stages of rupture, illustrated by the 50\% drop in elastic modulus of the rock near the fault, and thus is expected to greatly influence later stages of rupture and slip. These constraints are a unique benchmark for calibration of dynamic rupture models.

1. Introduction

Strain energy released during earthquakes is partly radiated as seismic waves that cause ground shaking, and mostly dissipated by frictional heating on the fault interface and by fracturing of the rocks surrounding the fault. The latter energy sink, and a component of frictional heating, constitute the fracture energy ($G_F$, sometimes referred to as rupture energy) that dictates the dynamics of rupture propagation \cite{Rice1980}. Quantification of these individual components is crucial to understanding the physics of rupture, to better understand the feedback between rupture and slip processes, and to improve ground motion predictions.

Fracture energy is the work associated with the breakdown of the rock strength towards its residual frictional strength. $G_F$ is a collective term for several dissipative processes in the breakdown zone around the rupture tip, both on and off fault \cite{KanamoriRivera1982, FaATSO}. These dissipative processes may include shear heating, plastic yielding, on- and off-fault creation of fractures (surface energy), and grain comminution. A measure of $G_F$ can be inferred from earthquake data \cite{Tinti2005}, and from laboratory mechanical \cite{Wong1982, Wong1986, Nielsen2016} and acoustic data \cite{Lockner1991}, but such data do not provide a breakdown of the individual components of $G_F$. Cumulated surface energy measured on fault rocks \cite{Wilson2005, Chester2005, Rockwell2009, Faulkner2011, Savage2011} provide on- and off-fault components of $G_F$, but these estimates are measured on rocks that have recorded numerous earthquakes and deformation episodes and therefore do not represent a single earthquake, nor can they constrain energy dissipation into heat. Cumulated surface energy obtained from microstructural studies on off-fault damage in laboratory samples \cite{Moore1995}; $\text{Zang et al.}, 2000$ are only static snapshots of the dynamic breakdown process. To establish the off-fault energy dissipation component $G_{off}$, one possibility is to use the change in off-fault elastic properties caused by off-fault deformation. Such changes must be measured \textit{in situ} during rupture, ideally under realistic crustal conditions (i.e., at elevated pressure and temperature). This may be done by the acquisition of active seismic surveys during laboratory rupture experiments \cite{Lockner1977}. Yet, the size and geometry of the off-fault damage zone and the actual local wavespeeds therein remain unconstrained because of the lack of spatial resolution of conventional laboratory ultrasonic measurements.

Here, we combine stress, strain, and acoustic emission (AE) and ultrasonic velocity measurements obtained \textit{in situ} during a laboratory rock fracture experiment. From this, we determine the time-resolved 3D seismic velocity structure of a growing fault zone that provides the size and geometry of the off-fault elastic properties. Taken together, our measurements allows us to estimate the partitioning of $G_F$ into off-fault ($G_{off}$) and on-fault energy dissipation.

2. Method

We performed a triaxial rupture experiment on a 40 mm diameter, 100 mm length sample of dry Lanlhenil granite (Brittany, France). The sample was placed into a rubber jacket equipped with 16 piezoelectric P-wave transducers, and two pairs of axial-radial strain gauges (Fig. 1). Acoustic signals were recorded by a digital oscilloscope at a 50 MHz sampling frequency, after being amplified to 40 dB. Active ultrasonic velocity surveys were performed every 5 minutes by sending a 1 MHz pulse at a voltage of 250 V to one transducer, while the other transducers recorded the resulting waveforms. During one survey, all 16 transducers were used as a source, and the results of six pulses for each source transducer were stacked to enhance the signal-to-noise ratio. In between the acoustic velocity surveys, the waveforms of acoustic emissions were recorded provided that a signal amplitude of 250 mV was surpassed on at least two transducers. All waveforms, both of active acoustic velocity surveys and AEs, consisted of 4096 datapoints (82 $\mu$s length).

The jacketed sample was placed into a triaxial deformation rig and pressurised to 100 MPa confining pressure. Axial stress was measured by a load cell, axial shortening was measured by a pair of Linear Variable Differential Transducers (LVDTs). The axial deformation was then applied by a piston that moved with a strain rate of $10^{-3}$ s$^{-1}$ for the elastic portion of the stress-strain curve, and $10^{-6}$ s$^{-1}$ for the remainder of the experiment. The axial shortening rate was controlled in such a way as to hamper the dynamic propagation of shear rupture, using a technique similar to that of \textit{Lockner et al.} [1991]: The AE-rate was monitored visually and, when the AE-rate showed acceleration (about 8 hits or more per second, recorded on at least two channels), the direction of movement of the piston was reversed to reduce the load. More than 100 of such load reductions were performed. The overall fracture propagation across the sample occurred over a time interval of around 16 hours.

From the ultrasonic dataset, we computed the AE source locations together with the evolution of the seismic velocity structure within the sample by using the 3D seismic tomography code \textit{FaATSO}, specifically designed for laboratory rock deformation experiments \cite{Brantut2018}. 12000 of the highest quality AE events...
Before the peak stress and the onset faulting, we observe an overall decrease in $V_p$ from around 6 km/s down to 5 km/s (Fig. 2a). Then, rupture starts at the bottom of the sample and propagates upwards (Fig. S1, Movie S1). A low velocity zone develops parallel to the rupture plane and migrates along with the growing fault (delineated by the AE source locations, Fig. 2b,c and Fig. S1). Velocities in the localised zones are as low as 4.6 km/s – a 25% drop relative to the areas outside of the fault zone where $V_p$ remains nearly constant (Fig. 2c). This corresponds to a drop of around 50% in P-wave modulus. We interpret this low velocity zone as the fault damage zone, which is generated ahead and along the propagating rupture tip. In the wake of the rupture tip the damage zone width decreases slightly by several millimeters. There is a widespread partial recovery of $V_p$ throughout the damage zone (the minimum value rising from 4.6 km/s to about 4.7 km/s) at the onset of the frictional sliding stage (Fig. 2d). The $V_p$ anisotropy at the peak stress is 13% (i.e., vertical $V_p$ is 13% higher than horizontal $V_p$, Fig. S3), and increases during fault growth up to 20% in two large zones adjacent to the fault. In the wake of the passing fault tip, the change in velocity anisotropy is much less than the change in velocity (Fig. 1). The anisotropy decreases again as the axial stress is reduced during the frictional sliding stage.

The robustness of the tomographic inversion results is demonstrated by the mean posterior velocity and 500 individual posterior solutions along a fault-perpendicular transect (Fig. 3a-d). The individual solutions (grey curves) show how the $V_p$ along the transect evolves from a near-constant value (Fig. 3a) to a localised reduction in $V_p$ as the rupture tip approaches (Fig. 3b), which is amplified once the rupture tip has passed the transect (Fig. 3c). Again, the $P$-wave velocity increases slightly at the frictional sliding stage (Fig. 3d). A single solution tomography slice (Fig. 3e) shows the same features as the (interpolated) mean solution (Fig. 2).

### 3. Results

Before the peak stress and the onset faulting, we observe an overall decrease in $V_p$ from around 6 km/s down to 5 km/s (Fig. 2a). Then, rupture starts at the bottom of the sample and propagates upwards (Fig. S1, Movie S1). A low velocity zone develops parallel to the rupture plane and migrates along with the growing fault (delineated by the AE source locations, Fig. 2b,c and Fig. S1). Velocities in the localised zones are as low as 4.6 km/s – a 25% drop relative to the areas outside of the fault zone where $V_p$ remains nearly constant (Fig. 2c). This corresponds to a drop of around 50% in P-wave modulus. We interpret this low velocity zone as the fault damage zone, which is generated ahead and along the propagating rupture tip. In the wake of the rupture tip the damage zone width decreases slightly by several millimeters. There is a widespread partial recovery of $V_p$ throughout the damage zone (the minimum value rising from 4.6 km/s to about 4.7 km/s) at the onset of the frictional sliding stage (Fig. 2d). The $V_p$ anisotropy at the peak stress is 13% (i.e., vertical $V_p$ is 13% higher than horizontal $V_p$, Fig. S3), and increases during fault growth up to 20% in two large zones adjacent to the fault. In the wake of the passing fault tip, the change in velocity anisotropy is much less than the change in velocity (Fig. 1). The anisotropy decreases again as the axial stress is reduced during the frictional sliding stage.

The robustness of the tomographic inversion results is demonstrated by the mean posterior velocity and 500 individual posterior solutions along a fault-perpendicular transect (Fig. 3a-d). The individual solutions (grey curves) show how the $V_p$ along the transect evolves from a near-constant value (Fig. 3a) to a localised reduction in $V_p$ as the rupture tip approaches (Fig. 3b), which is amplified once the rupture tip has passed the transect (Fig. 3c). Again, the $P$-wave velocity increases slightly at the frictional sliding stage (Fig. 3d). A single solution tomography slice (Fig. 3e) shows the same features as the (interpolated) mean solution (Fig. 2).

### 4. Rupture energetics

Now, we can determine the fracture energy $G_c$, the total dissipated energy, and the off-fault dissipated energy $G_{off}$. $G_c$ and the total dissipated energy are computed from the shear stress vs. fault slip record [Wong, 1982] up to the slip-weakening distance $d_0$ (Fig. 4). We assume that all axial shortening is caused by slip along the fault from localisation onward. Axial shortening is corrected for elastic strain by using the intact elastic moduli for the rock. $G_c$ is $2.7 \times 10^4$ J m$^{-2}$, similar to previous experimental results ($1.3 - 2.9 \times 10^4$ J m$^{-2}$) [Wong, 1982, 1986; Lockner et al., 1991].

Next, we estimate the off-fault dissipated energy $G_{off}$ during the slip-weakening stage. $G_{off}$ is given by the change in stored elastic strain energy (i.e. elastic softening) around the fault interface. These elastic compliance changes are caused by off-fault dissipative processes, of which microcracking is dominant. Strain derived...
Figure 2: Tomographic slices of the horizontal $V_p$ normalised to the initial velocity. The slices run through the centre of the sample, perpendicular to the rupture. The four slices represent time intervals (a) during localization of deformation, (b, c) during two stages of rupture propagation, and (d) during frictional sliding of the fault. The corresponding parts of the stress-strain curve are indicated on the right. $\delta_0$ indicates the slip-weakening distance. All AE source locations up to the time interval are projected onto the slice, illustrating the rupture propagation. The AE source locations are within 2.5 mm distance perpendicular to the slide, and were determined using the 3D seismic velocity model. The seismic velocities are smoothed to a 1 mm resolution. See movie S1 for the complete evolving seismic structure.

Figure 3: Posterior solutions and $V_p$ evolution with respect to the rupture tip position (a-d): The $V_p$ of 500 individual posterior solutions (gray curves) and the $V_p$ of the mean posterior solution (blue curve) along a fault-perpendicular transect, for the time intervals shown in Fig. 2. (e): An individual posterior solution (spatial resolution 5 mm) shows a similar velocity structure as the mean solutions in Fig. 2c. The black line indicates the location of the transect in (a-d) and the squares indicate points $p_1$-$p_3$ in (f). The circles show the progression of the rupture tip through the centre of the sample. (f): $V_p$ evolution as a function of distance from the rupture tip (circles in (e)) for three points at varying distance perpendicular to the fault plane.
served (points flaws. Closer to the fault interface such full recovery is not ob-
the stiffness reduction and the full recovery that follows is caused 
around a passing rupture tip (Fig. 3e and f). This matches qualitatively with the predictions of
from the fault interface after passing of the rupture front (point 
for each time interval,
poral evolution of the seismic velocity structure is used to obtain 
from mechanical data includes slip along the fault and thus can-
10%. slip-weakening distance, the relative energy contribution is around
weakened sliding distance (width of 10 mm, since
Earthquake fault slip is linearly proportional to fault length (Cowie and Scholz, 1992). Earthquake fault slip \( \delta \) increases linearly with fault length (Scholz, 1982). From this it follows that damage zone width scales linearly with fault slip \( \delta \) [Savage and Brodsky, 2011; Faulkner et al., 2011].

A first order approximation of \( w \) is established by analysing the spatial extent of permanent damage in our data. Nearly elastic behaviour (i.e., full recovery of \( V_p \)) is observed at some distance from the fault interface after passing of the rupture front (point \( p_3 \), Fig. 3c and f). This matches qualitatively with the predictions of a stress field around a passing rupture tip [Freund, 1990], whereby the stiffness reduction and the full recovery that follows is caused by the rupture tip stress field that dilates or contracts pre-existing flaws. Closer to the fault interface such full recovery is not ob-

5. Discussion

The constraints on rupture energetics were obtained during quasi-static rupture propagation and are representative of the nucleation phase of an earthquake. Once the rupture velocity has accelerated towards the critical wave speed of the rock (equal to the Rayleigh wave speed in in-plane conditions, or to the shear wave speed in anti-plane conditions), the stress field around the rupture tip is distorted relative to that of a quasi-static rupture tip [Fren-
und, 1979]. Such a distortion is more likely to increase the size of the off-fault regions where fracturing might occur [Poljakov et al., 2002; Rice et al., 2005]. In addition, the transient rupture tip stress field imposes high strain rates on the off-fault rock volume, which results in increased fragmentation [Grady, 1982; Bhat et al., 2012]. \( G_{\text{off}} \) is thus expected to increase with increasing rupture velocity, changing the ratio \( G_{\text{off}}/G_c \) over total energy dissipation. Therefore, the quasi-static ratio of 3% estimated from our experimental data is a lower bound for dynamic ruptures, but provides a unique calibration benchmark for dynamic rupture models that allow for off-fault damage [Xu et al., 2015; Thomas and Bhat, 2018]. Such models predict a maximum drop in \( V_p \) of around 30% [Xu et al., 2015; Thomas and Bhat, 2018], which is consistent with the maximum drop of 25% observed here.

This maximum drop in \( V_p \) of 25% during experimental rupture is of a similar order to geophysical observations on coseismic \( V_p \) reduction near recently ruptured faults [Cochran et al., 2009; Al-
laam and Ben-Zion, 2012; Froment et al., 2014]. However, these velocity reductions were the product of multiple ruptures with a higher rupture velocity than our experimental rupture, and they also reflect the post-seismic state rather than the co-seismic state that we document here – thus without the transient reduction of elastic properties. Geophysical observations of seismic velocity reductions caused by single rupture events are of the order of 20–45%, but are restricted to S-wave velocity only [Karabulut and Bouchon, 2007; Xu et al., 2009]. Rather than directly comparing the absolute values observed here with those measured on faults, our constraints on rupture energetics of laboratory-sized samples can be upscaled to larger faults by relying on scaling relations established by other studies.

First, studies along exhumed faults suggest that damage zone width increases linearly with total fault displacement [Savage and Brodsky, 2011; Faulkner et al., 2011], and total fault displacement \( D \) is linearly proportional to fault length [Cowie and Scholz, 1992]. Earthquake fault slip \( \delta \) increases linearly with fault length [Scholz, 1982]. Second, seismological estimates and theoretical predictions indicate that fracture energy \( G_c \) scales with fault slip \( \delta \) with an exponent \( \lambda \) [Viesca and Garagash, 2015; Brantut and Viesca, 2017]. By adopting aforementioned linear scaling of \( G_{\text{off}} \) with \( \delta \), we ob-
tain \( G_{\text{off}}/G_c = \delta^{1-\lambda} \). The exponent \( \lambda \approx 2 \) for small earthquakes
ABEN ET AL.: RUPTURE ENERGETICS IN CRUSTAL ROCK

References


Acknowledgments. We thank La Générale du Grant for their generosity in providing Lanlöhén granite. This work was supported by the UK Natural Environment Research Council, grants NE/K009656/1 to NB and NE/M004716/1 to TM and NB. FMA, NB, ECD performed the experiment. FMA processed raw data, FMA and NB performed analysis. NB created animations and AE relocation codes. FMA and NB wrote the paper. All authors discussed the results. Faatso and the AE relocation code found at https://github.com/abrantut/laatso.git. All data needed to evaluate the conclusions in the paper are present in the paper and/or the Supplementary Materials. Additional data related to this paper may be found at the NGDC repository of the British Geological Survey.
Savage, H., and E. Brodsky, Collateral damage: Evolution with displace-

Rockwell, T., M. Sisk, G. Girty, O. Dor, N. Wechsler, and Y. Ben-Zion, 

Rice, J. R., The mechanics of earthquake rupture, in 

Poliakov, A., R. Dmowska, and J. Rice, Dynamic shear rupture interactions 

Ohnaka, M., A constitutive scaling law and a unified comprehension for 

Moore, D., and D. Lockner, The role of microcracking in shear-fracture 

Lachenbruch, A., Frictional heating, fluid pressure, and the resistance to 

Lockner, D., J. Byerlee, V. Kuksenko, A. Ponomarev, and A. Sidorin, 


Lockner, D. A., J. B. Walsh, and J. D. Byerlee, Changes in seismic velocity 

Lockner, D., J. Byerlee, V. Kuksenko, A. Ponomarev, and A. Sidorin, 

Lachenbruch, A., Frictional heating, fluid pressure, and the resistance to 

6


age zones, 


earthquake physics, 

1980. 

Chemical and physical characteristics of pulverized tejon lookout gran-

1980. 

of Physics E. Fermi, Italian Physical Society/North Holland Publ. Co., 

MOORE ET AL.: RUPTURE ENERGETICS IN CRUSTAL ROCK 

Bye et al., Scaling in natural and laboratory earthquakes, 


propagation in granite, 

17 


process zone in granite, 


Poliakov, A., R. Dmowska, and J. Rice, Dynamic shear rupture interactions 


Rice, J., C. Sammis, and R. Parsons, Off-Fault Secondary Failure Initiated 


Rockwell, T., M. Sisk, G. Girty, O. Dor, N. Wechsler, and Y. Ben-Zion, 


F.M. Aben (f.aben@ucl.ac.uk)
Supporting Information for ”Rupture energetics in crustal rock from laboratory-scale seismic tomography”

Franciscus M. Aben  Nicolas Brantut  Thomas M. Mitchell
Emmanuel C. David
Department of Earth Sciences, University College London
London, UK
July 23, 2019

Contents of this file

1. Text S1 to S2
2. Figures S1 to S5

Additional Supporting Information (Files uploaded separately)

1. Captions for Movie S1

Introduction The supporting information consists of the methodology used to determine the position of the rupture tip through time (text S1), and the detailed equations used to determine the fracture energy $G_c$ and the elastic compliances from the tomography (text S2). We present supporting figures that show the anisotropy evolution during rupture (Figure S3) and the post-mortem microstructures (Figure S4). Figure S5 shows a tomographic slice of a ’failed’ quasi-static rupture experiment performed at the same conditions, and reveals similar features to the presented experiment in the main text. Last, we have included movie S1, which shows the tomography result for each time interval and highlights the propagating rupture (Movie S1).
Text S1. Determination of the rupture tip position

Several studies have obtained 2D[7, 2, 4] and 3D[6] ultrasonic tomography data on deforming rock under laboratory conditions, using active ultrasonic data only. The quasi-Newton inversion algorithm FaATSO[1] used in this study extents laboratory-scale tomography by: i) Using passive source data (acoustic emissions) in addition to active sources to increase ray coverage in the sample volume, ii) computing ultrasonic ray paths rather than assuming straight ones, and iii) the results are obtained under high pressure conditions without compromising the sample setup.

For each time interval, the position of the rupture front was determined from the AE events that occurred within the specified time interval. The AE source locations were relocated by using the 3D seismic velocity structure. A convex hull was then defined around the volume of AE source locations, excluding clusters of three AEs or less that were separated by more than 5 mm from the other AEs. Next, the vertices and edges of this seismically active volume were projected onto a surface with a 45° strike. The dip of this surface (relative to the horizontal) was determined by a linear fit through the AE locations. From the resulting 2D-projection we determined the seismically active portion of the fault plane, the progressive growth of the fault, and the fault angle over time (Fig. S1).

The rupture starts at a 45° degree dip (Fig. S1a) and progressively increases towards a final dip of around 63°. The goodness of fit for the angle (Fig. S1b) gives some insight into the scatter of the AE source locations (i.e. a lower goodness of fit indicates more scatter): it increases as the fault grows, and is highest for the time intervals during frictional sliding. This means that the AE source locations tend to be more concentrated on the fault plane at later stages of rupture, while being more diffuse during the fault growth stage. The seismically active fault surface area stays relatively constant throughout the rupture (Fig. S1c), although time intervals containing more AE events typically show a larger active surface area. There is a significant increase in seismically active area for the time intervals during frictional sliding. Hence, the whole fault surface is seismically active, while during rupture propagation only the zone in the wake of the rupture front is active. For the first stages of rupture, the rupture front progresses mostly on one side of the sample (Fig. S1d). Subsequently, rupture front progression is limited the other side of the sample. The fault angle increases during this rebalancing stage (Fig. S1e). From time interval 15 onward, the rupture front progresses across the whole width of the sample (Fig. S1e).

The final fault plane does not show the increase in angle inferred from the AE source locations. In the area where the fault plane angle was initially low (45°), the fault has been overprinted by the 60° fault surface. This is apparent from the AE source locations during the late rupture stage and the frictional sliding stage.
A similar shift to the final fault plane is visible in the seismic velocity tomography as the early stage low velocity zone migrates upwards in the later stages of rupture. Also, post-mortem microstructures (Fig. S4) show only one well-defined high angle fault plane ($60^\circ$). Thus, the early fault plane is a region of diffuse microcracking at a $45^\circ$ angle. Hence, very little slip is accumulated in this early stage and a relatively larger amount of energy is dissipated into off-fault microcracking relative to the later stages of rupture (Fig. 3 main text).

Figure S1: Active fault surface and rupture tip over time (a): Fault angle for each time interval. (b): The $R^2$ value for determination of the fault angle. (c): Seismically active surface area for each time interval. (a)-(c): The two dashed vertical lines represent the onset of rupture (left line) and the onset of frictional sliding (right line). (d): Rupture front progressing through time. Some of the time intervals are indicated on the right. View perpendicular to fault surface. (e): Rupture front progressing through time, view parallel to fault surface.
Text S2. Calculation of fracture energy and off-fault energy dissipation

Fracture energy

The total fracture energy $G_c$ spend on propagating the fault was obtained following the approach of Wong. First, the unloading loops were removed from the stress and strain data so that only mechanical data associated to loading remained. The shear stress on the fault plane was calculated from the axial stress, assuming a fault dip of $60^\circ$. The axial shortening was corrected for elastic strain by using the intact elastic moduli of the rock. The onset of localisation was set at the point where the linear relation between inelastic axial shortening versus radial strain broke down, which roughly correlates to the peak stress. We assume that from the localisation onward all permanent axial shortening measured by the LVDTs is due to slip along the fault. This allows us to calculate the slip history of the fault plane at a $60^\circ$ angle from the axial strain data. $G_c$ is equal to the area bounded by the shear stress - fault slip curve, down to the frictional strength of the fault at 155 MPa shear stress. $G_c$ equals about $2.7 \times 10^4$ Jm$^{-2}$.

Off-fault energy dissipation

Next, we calculated the amount of energy needed to soften the rock in the damage zone. Here we assume that the increase in elastic compliance is caused by the presence of cracks. The energy required to increase the elastic compliance, equal the off-fault energy dissipation $G_{off}$, is the change in elastic stored strain energy:

$$G_{off} = \frac{1}{2} w \Delta \varepsilon_{ij} \sigma_{ij}, \quad (1)$$

where $\sigma_{ij}$ are the macroscopic stress components, $\Delta \varepsilon_{ij}$ is the variation of elastic strain, and $w$ the width of the damage zone. The summation convention for repeated indices is used here. The variation of elastic strain is derived from the variation of the compliance tensor, $\Delta S_{ijkl}$, as:

$$\Delta \varepsilon_{ij} = \Delta S_{ijkl} \sigma_{kl}, \quad (2)$$

so that (1) becomes:

$$G_{off} = \frac{1}{2} w \sigma_{ij} \Delta S_{ijkl} \sigma_{kl} = \frac{1}{2} w \sigma_{ij} \Delta C_{ijkl}^{-1} \sigma_{kl}, \quad (3)$$

where $\Delta C = \Delta S^{-1}$ denotes the stiffness tensor.

To apply equation (3) we simplified the results of the seismic tomography by defining a zone of width $w$ around the fault where elastic properties are allowed
Figure S2: Setup to calculate the off-fault dissipated energy. The low velocity zone around the fault obtained from the seismic inversion is simplified into a damage zone with reduced elastic properties surrounded by host rock with constant elastic properties of the intact rock.

to change (Fig. S2). The surrounding host rock remains intact ($S_{ijkl}^0$ components are constants). The largest stress component is $\sigma_{33}$, $\sigma_{11}$ is equal to the confining pressure. Although elastic heterogeneities cause rotation of stress within the damage zone[3], we set the stress state within the damage zone equal to the far-field applied $\sigma_{11}$ and $\sigma_{33}$ for simplicity.

The measured strain during the experiment is the sum of elastic strain (or strain induced by microcracks) and inelastic slip along the fault. As the inelastic slip is an undesired component in the strain tensor, $S_{ijkl}$ components are calculated from the seismic velocities in the damage zone. Based on microstructural observations, we assume that the reduction of the seismic velocities and the stiffness in the damage zone are caused by the formation of microfractures aligned predominantly with the largest stress component (Fig. S4). Hence, we assume that no microfractures are aligned perpendicular to the highest stress component, and that the damage zone is transversely isotropic. The microfracture density and orientations can be quantified as a crack density tensor $\alpha$ [5]. Tensor $\alpha$ is only valid when we assume
no interaction between fractures; i.e., the cracks are randomly positioned [5].

Three components of the stiffness tensor $C_{ijkl}$ can be calculated directly from the horizontal velocity $V_p^h$ and vertical velocity $V_p^v$ of the damage zone:

$$C_{1111} = C_{2222} = \rho (V_p^h)^2 \quad C_{3333} = \rho (V_p^v)^2,$$

where $\rho$ is the density of the granite. The other components of $C_{ijkl}$ are obtained by relating the components of the crack density tensor $\alpha_{ij}$ (the indices indicate the direction to the normal of the cracks) to $V_p^h$ and $V_p^v$. Following the assumptions above, $\alpha_{33} = 0$. From equations (22) and (23) in Sayers and Kachanov [5], it follows that $C_{1111}$ is a function of $\alpha_{11}$:

$$C_{1111} = \frac{1}{2} \left[ \frac{1}{S_{1111}^0 - S_{1122}^0 + \alpha_{11}} + \frac{S_{1111}^0}{(S_{1111}^0)^2 + S_{1111}^0 S_{1122}^0 + S_{1111}^0 \alpha_{11} - 2(S_{1122}^0)^2} \right],$$

with

$$S_{1111}^0 = \frac{1}{E_0} \quad \text{and} \quad S_{1122}^0 = -\frac{v_0}{E_0},$$

where $E_0$ and $v_0$ are the Young’s modulus and Poisson’s ratio of the intact rock, respectively. $\alpha_{11}$ is thus obtained by inverting equation (5) as:

$$\alpha_{11} = -\frac{4C_{1111} \left[ (S_{1111}^0)^2 - (S_{1122}^0)^2 \right] + \sqrt{D}}{4C_{1111} S_{1111}^0},$$

with

$$D = \left( -4C_{1111} \left[ (S_{1111}^0)^2 - (S_{1122}^0)^2 \right] \right)^2 - 8C_{1111} S_{1111}^0 \left( 2C_{1111} \left[ (S_{1111}^0)^3 + 2(S_{1122}^0)^3 - 3S_{1111}^0 (S_{1122}^0)^2 \right] - 2(S_{1111}^0)^2 + 2(S_{1122}^0)^2 \right).$$

Alternatively, $\alpha_{11}$ can be obtained from $C_{3333}$. However, $\alpha_{11}$ has a greater effect on $V_p^h$, and can thus be established more accurately through $C_{1111}$. With $\alpha_{11}$ known and by using equations (22)-(27) in Sayers and Kachanov [5], the remaining components $C_{ijkl}$ are obtained. The independent components of the compliance tensor $S_{ijkl}$ are given by:

$$S_{1111} = \frac{1}{F} \left( C_{1111} C_{3333} - C_{1133}^2 \right),$$

$$S_{1122} = \frac{1}{F} \left( C_{1133}^2 - C_{1122} C_{3333} \right),$$

$$S_{1133} = \frac{1}{F} \left( C_{1133} (C_{1111} - C_{1122}) \right),$$

where $F = 1 - C_{1111} C_{3333}$. The remaining components are obtained by symmetry.
\[ S_{3333} = \frac{1}{F} (C_{1111}^2 - C_{1122}^2), \quad (12) \]
\[ S_{4444} = C_{4444}, \quad (13) \]
\[ S_{6666} = \frac{2}{C_{1111} - C_{1122}}, \quad (14) \]

with
\[ F = (C_{1111} - C_{1122}) \left[ (C_{1111} + C_{1122})C_{3333} - 2C_{1133}^2 \right]. \quad (15) \]

To obtain all the components of \( S_{ijkl} \), we took the minimum \( V_{ph} \) from the seismic tomography at each time interval, with the corresponding \( V_{vph} \) obtained from the \( V_{ph} \) and anisotropy, and applied them to equations (4)-(15). Equation (3) was then used to obtain the off-fault dissipated energy \( G_{off} \) for each successive time interval. Shear stress components were not computed and were set to zero, so that equation (3) becomes:

\[ G_{off} = \frac{1}{2} w \bar{\sigma}_{33} (2 \bar{\sigma}_{11} \Delta S_{1133} + \bar{\sigma}_{33} S_{3333}) + w \bar{\sigma}_{11} (\bar{\sigma}_11 (S_{1122} + S_{1111}) + \bar{\sigma}_{33} S_{1133}). \quad (16) \]

The cumulative off-fault dissipated energy is the sum of the energies obtained for the individual time intervals.
Figure S3: P-wave anisotropy structure. (a)-(d) represent the same time intervals as shown in Fig. 1. P-wave anisotropy ranges from 13% initially up to 20% during the propagation of the rupture. The anisotropy decreases when the fault has formed across the sample.
Figure S4: Post-mortem microstructures. (a): Image of a polished section through the center of the sample, perpendicular to the fault. Darker zones around the fault are caused by epoxy impregnating the rock, thus providing a first order estimate of fracture damage. (b): Tomographic slice D from Figure 1 shows that the low velocity zone is at roughly the same location as the dark zones impregnated by epoxy in (a). (c): Microphotographs showing that a zone of microfracture damage extends several millimeters from the fault. Most microfractures are oriented (sub-) vertical. The images were taken with transmitted light and cross polarizers at 45° angle.
Figure S5: Tomographic slice of the horizontal $P$-wave velocity normalised by the initial velocity, obtained on a sample of Lanhélin granite subjected to quasi-static rupture at 100 MPa confining pressure (i.e. similar conditions to the experiment presented in the main text). The slice runs through the centre of the sample, perpendicular to the rupture. The rupture became unstable and propagated dynamically around 520 MPa differential stress (see stress-strain curve on the right). The tomographic slice corresponds to the stress-strain interval highlighted by the asterisk, and thus reveals the post-failure tomographic structure. AE source locations up to the time interval are projected onto the slice, illustrating the extent of quasi-static rupture propagation. The AE source locations are within 2.5 mm distance perpendicular to the slide, and were determined using the 3D seismic velocity model. The seismic velocities are smoothed to a 1 mm resolution. A nascent conjugate failure plane is highlighted on the tomographic slice, this failure plane did not accumulate any slip and did not develop to a full fault interface. The spatial extent of the low velocity zone and the $P$-wave drop in the low velocity zone of this ‘failed’ quasi-static rupture experiment are of similar of magnitude to the post-rupture $P$-wave structure obtained on the quasi-static rupture shown in Fig. 1d).
**Movie S1.** Movie of the evolution of the seismic velocities during the controlled rupture experiment. The interval on the stress-strain curve represented by the tomographic slice is highlighted in black. The tomographic slice is oriented similar to the images in Figure 1 in the main text.

**References**


