MASTER IMALIS - ENS PSL

Training in Mathematics and Statistics

Exercises

September 2020

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Planning

Exercises - Lecture 1

Exercise 1: Calculate the derivatives of the following functions:
1. $\forall x \in \mathbb{R},\ f(x) = e^{3x} + 2x - 6$
2. $\forall x \in \mathbb{R}_+^*,\ g(x) = \ln(3x + 4)$
3. $\forall x \in \mathbb{R},\ h(x) = 2xe^{-x}$
4. $\forall x \in \mathbb{E}$ (to define), $i(x) = \sqrt{3 - 2x}$
5. $\forall (x, y) \in \mathbb{R}^2,\ j(x, y) = x^3y + e^{xy^2}$

Exercise 2: Operations on matrices:

$$1.\ \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} \quad 2.\ \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad 3.\ \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$$

$$4.\ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix} \quad 5.\ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^3 \quad 6.\ \begin{pmatrix} 1 & 2 \\ 3 & -2 \end{pmatrix}^2 \quad 7.\ \begin{pmatrix} 1 & 2 \\ 0 & -2 \end{pmatrix}^3$$

Exercise 3: Calculate the determinant associated with the following matrices:

$$1.\ A = \begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix} \quad 2.\ B = \begin{pmatrix} 1 & -1 \\ 0 & 5 \end{pmatrix} \quad 3.\ C = \begin{pmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \\ 1 & -1 & 2 \end{pmatrix}$$

Exercise 4: $\forall x \in E, f(x) = \dfrac{e^x - 1}{e^x + 1}$,
1. Determine (E), the domain of definition of f , and demonstrate that f is an odd function.
2. Study the variations of f (increasing, decreasing, ...).

Exercise 5: Solve the following differential equations:
1. $y' - 3y = 1$ and $y(1) = -2$.
2. $3y' - y = x + 2$ with solution(s) verifying $x \longrightarrow ax + b$.

Exercise 6: Discrete probabilities. Let's consider 32 cards (8 spades, 8 hearts, 8 diamonds, and 8 clubs):
1. We randomly distribute one card from a game with 32 cards. What is the probability of having one king?
2. We randomly distribute five cards from a game with 32 cards. What is the probability of having four kings?
3. We randomly distribute five cards from a game with 32 cards. What is the probability of having only red cards?
4. We randomly distribute five cards from a game with 32 cards. What is the probability of having 2 diamonds and 3 hearts?

5. We randomly distribute five cards from a game with 32 cards. What is the probability of having at least one card of each color (at least 1 spade, 1 heart, 1 diamond, and 1 club)?
6. We randomly distribute five cards from a game with 32 cards. What is the probability of having at least one king?
7. We randomly distribute five cards from a game with 32 cards. What is the probability of having two king and 3 hearts?