MAT 2348 — midterm exam

No calculators allowed.

Numeric answers need to be justified. You can use the result of a previous question in an answer even you did not answer it. ***More difficult questions are marked with a *, do what you know how to do first.*** **Don't Panic.**

A. Remind briefly the formulas for arrangements, combinations and arrangements with types. *(here, exceptionally, you do not need to justify)*

B. We consider the equation $x_1 + x_2 + x_3 = 12$ with x_1, x_2, x_3 positive integers.

1. How many solutions are there?
2. How many solutions such that x_1, x_2, x_3 are even are there?
3. How many solutions such that $x_1, x_2, x_3 \geq 1$ are there?

*4. How many solutions such that $x_1, x_2, x_3 \leq 5$ are there?

C.

1. What is the coefficient of x^5y^6 in the development of $(x+y)^{11}$? Of $(2x+3y)^{11}$?
2. What is the value of $\sum_{k=0}^{n} \binom{n}{k}(-2)^k$?

D. We consider a $n \times n$ board with black and white squares. A *line* in the board is either a row or a column. Show that at least two lines have the same number of black squares.

E.

1. Show that $\binom{2n+2}{n+1} = \frac{(2n+2)(2n+1)}{(n+1)^2}\binom{2n}{n}$.
2. We have a function on positive integers f such that $f(0) = 1$ and $f(n+1) = f(n) + 2$ for all n. Prove by induction that for all n, $f(n) = 2n+1$

*3. Prove by induction that for any $c \geq 1$, $\sum_{k=c}^{n} \binom{k}{c} = \binom{n+1}{c+1}$. *(hint: fix c and do the induction on n)*

***F.** We have a set E with n elements.

1. Explain why there are as many triples (A, B, C) of *pairwise disjoint* subsets of E such that $A \cup B \cup C = E$ and functions from E to the set $\{1, 2, 3\}$.
2. Deduce how many such triples there are.
3. How many pairs (A, B) of (not necessarily disjoint) subsets of E such that $A \cup B = E$ are there?