MAT 2348 — exercises #7

A   Rook polynomials

Just practice on a few examples the decomposition of boards to make sure you understood well.
(you can look at Grimaldi 8.4/8.5: 4,5 — or make your own examples)

B   Generating functions

1. Give the generating functions for the following sequences:

   - The sequence: $0, 0, 1, 1, 1, \ldots$
   - The sequence of powers of 2: $1, 2, 2^2, 2^3, \ldots$
   - The sequence of cubes: $1^3, 2^3, 3^3, \ldots$
   - The sequence: $1, -1, 1, -1, 1, -1, \ldots$

2. Consider the equation on positive integers $x_1 + x_2 + x_3 = 16$. Give generating functions such that the coefficient of 15 is the number of solutions to the equation, given the following constraints:

   - All numbers are even.
   - x_1 is odd.
   - $1 \le x_1$, $3 \le x_2$, $4 \le x_3$.
   - $x_1, x_2, x_3 \le 5$.

   Give the solution to the problem using these generating functions.

3. What are the sequences generated by the following functions:

   - $f(x) = (1 + 2x)^n$
   - $g(x) = \frac{1}{1+x^2}$
   - $h(x) = \frac{3x}{(1-x)^2}$
   - $k(x) = e^x$

Grimaldi's exercises 9.1: 1, 4.
Grimaldi's exercises 9.2: 1,2,5,9,13.