# MAT 2348 — exercises #6

## A Pidgeonhole principle

1. Explain why if we take a group of $13$ people, at least two of them must be born the same month.
2. Show that if we pick $11$ numbers from $\{1, 2, \ldots, 30\}$ there must be $x, y$ among them such that $|\frac{x}{3} - \frac{y}{3}| \leq 1$.
3. Consider an equilateral triangle of side length $1$. How many points at most can we place in the triangle without having necessarily two of them at a distance less than $\frac{1}{3}$?
4. Same question but in the case points can be placed only on the sides of the triangle.

## B Inclusion-exclusion

1. How many numbers in $\{1, 2, \ldots, 50\}$ are there that are either multiples of $2$, $3$ and $7$
2. Same question with multiples of $2$, $3$ and $6$.
3. How many rearrangements of the letters GAUSSIAN are there such that we have G in first position or A in second position?
4. Same question with G in first position or A in first position.

## C Stirling numbers

We prove the Stirling numbers count surjections without induction, using the I-E principle instead. Let us fix a set $A$ of size $n$ and a set $B$ of size $m$.

1. Recall how many functions there are from a set of size $n$ to a set of size $m$.
2. For any $b \in B$, how many functions are there that such that for all $x$, $f(x) \neq b$? We write $S_b$ this set of functions.
3. For any $b_1, \ldots, b_k \in B$, what is the value of $|S_{b_1} \cap S_{b_2} \cap \cdots \cap S_{b_k}|$?
4. Applying the I-E principle, give a formula for the number of functions that are not surjections *(hint: a non-surjection misses at least one $b \in B$)*
5. Conclude: the Stirling number corresponds to the number of surjections.

**Grimaldi's exercises 5.5:** 3, 5, 7, 9, 20.
**Grimaldi's exercises 8.1:** 2, 5, 6, 8, 16.