MAT 2348 — exercise sheet #4

A Functions

We consider the sets of integers $A = \{1,2,3\}$, $B = \{2,3,4,6\}$, $C = \{2,3\}$ and the functions $f : A \to B$ such that $f(x) = x+1$, $g : B \to C$ such that $g(2) = g(4) = 2$ and $g(3) = g(6) = 3$.

1. Compute $g \circ f$.
2. Are f and g injections? Surjections? Bijections?
3. Does there exist a bijection between any two of the sets A, B, C?
4. For which couples of sets (X, Y) do we have an injection from X to Y?

B Bijections, injections, surjections

1. Show that if $f \circ g$ is injective, then g is injective.
2. Show that if $f \circ g$ is surjective, then f is surjective.
3. Show that if f is bijective and $f \circ g = \mathrm{Id}$ then $g \circ f = \mathrm{Id}$. *(unicity of the inverse of a bijection)*

C Powerset

1. Let $A = \{1,2,3\}$, $B = \{3,4\}$. Compute $\mathcal{P}(A)$, $\mathcal{P}(B)$, and if you feel brave $\mathcal{P}(A \cup B)$.
2. Show that in general, *i.e.* for any A and B, we have a surjective function
$$f : \mathcal{P}(A) \times \mathcal{P}(B) \to \mathcal{P}(A \cup B)$$
3. Show that in case $A \cap B = \varnothing$ then this function is a bijection, *i.e.* we have
$$\mathcal{P}(A) \times \mathcal{P}(B) \simeq \mathcal{P}(A \cup B)$$

Grimaldi's exercises 3.1: 5, 6, 9, 13, 18, 27.
Grimaldi's exercises 3.2: 1, 3, 4, 7, 13.