MAT 2348 — exercise sheet #3

A Counting

1. How many three-letter words are there are not just three times the same letter?
   (like AAA, BBB, etc.)
2. We have lottery tickets in a box, numbered from 1 to n. We pick one, write it down, and then put it back. If we do this k times, how many possible sequence can we write?

B Arrangements and combinations

1. How many shuffle of the letters of MISSISSAUGA are there?
2. How many of them contain the word GAUSS? *(hint: think of "GAUSS" as one big letter)*
3. How many of them *do not* contain the word GAUSS?
4. Show that $\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$ by a combinatorial argument. *(hint: a set with $2n$ elements can be seen as the union of two sets of n elements)*

C Binomial theorem

1. What is the coefficient of x^5y^2 in the development of $(2x+3y)^7$?
2. Show that $n2^{n-1} = \sum_{k=1}^{n} k\binom{n}{k}$ using the binomial theorem.
   (hint: compute the derivative of $(x+1)^n$ and that of its development)

Grimaldi's exercises: same as #2.