MAT 2348 — exercise sheet #2

A Counting

1. How many solutions to $x_1 + x_2 + x_3 + x_4 + x_5 = 4$ are there? (the x_i are positive integers)
2. We have nine 1$ bills to distribute among five persons, how many different repartitions are there? What if we have the constraint that everyone gets at least one bill?
3. We have the players of three soccer teams (11 players each) sit in line for a picture. If we consider only the alternation of shirt colours, how many different pictures can we take?

B Arrangements and combinations

1. Show that $\binom{n}{k} = \binom{n}{n-k}$.
   Can you give a combinatorial (*i.e.* without any computation) argument for this fact?
2. Show that $\dfrac{(3n)!}{6^n}$ is an integer for any n. *(hint: a combinatorial proof would be nice)*
3. Show that for any n, $\binom{2n}{n} - \binom{2n}{n+1} = \frac{1}{n+1}\binom{2n}{n}$.

C Binomial theorem

1. What is the value of the coefficient of x^4y^3 in $(x+y)^7$?
2. What is the value of the coefficient of $xy^2z^2t^3$ in $(x+y+z+t)^8$?
3. What is the value of $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \cdots + (-1)^n\binom{n}{n}$?
   (hint: try to write it with the $\sum_{k=0}^{n} \ldots$ notation)
4. And that of $\binom{n}{0} + 2\binom{n}{1} + 4\binom{n}{2} + 8\binom{n}{3} + \cdots + 2^n\binom{n}{n}$?

Grimaldi's exercises 1.3: 4, 13, 15, 23, 28.
Grimaldi's exercises 1.4: 1, 3, 7, 16, 18, 23.