MAT 2348 — exercise sheet #1

A Basic exercises

A.1 Sets

1. Let $A = \{1,2,4\}$ and $B = \{1,4,5\}$. Compute $A \cup B, A \cap B, A \setminus B, A \times B$.
2. Show that if $A \subseteq B$ and $B \subseteq A$, then $A = B$.
3. Show that if $A \subseteq B$ then $\mathtt{card}(B) = \mathtt{card}(A) + \mathtt{card}(B \setminus A)$. *(hint: use the sum principle)*

A.2 Sum and product principles

1. A store sells 3 different types of fruit, 4 types of vegetable and 6 different types of cookies. How many different choices can be made if we want to buy:
   (a) One item?
   (b) One of each type of items? (i.e. *one fruit, one vegetable, one cookie)*
   (c) Two items of different type?
2. We consider sequences of length 5 such that the first character is either a letter of a digit, and the remaining ones are digits. How many such sequences are there? Same question with sequences of length n.

A.3 Arrangements and combinations

1. We draw a sequence of 5 cards from a deck of 32 cards. How many such sequences are there?
2. We draw a hand of 5 cards (*i.e.* the order in which we draw them do not count) from a deck of 32 cards. How many such hands are there?
3. How many different shuffles of the word HELLO are there? Of the word ELLEN?
4. Same question as above, but considering only shuffles where the two L are side by side.

B Advanced exercises

B.1 Sets

1. Can you guess (and even prove?) a formula for $\mathtt{card}(A \cup B)$? That is, the sum principle when $A \cap B$ is not empty. *(hint: draw potatoids)*
2. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. This is called the distributivity of $\cap$ over $\cup$.

   What about the distributivity of $\cup$ over $\cap$?
3. Show that A.1.3 is false when $A \not\subseteq B$.

B.2 Counting subsets

If A is a set, then a *subset* of A is any B such that $B \subseteq A$. Suppose A has n elements.

1. How many subsets of A of size k (for $0 \leq k \leq n$) are there?
2. Show that there are 2^n different subsets of A.
3. Can you use this to compute $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n-1} + \binom{n}{n}$?

B.3 The round table

A group of 6 people sit at a round table. We consider that they are in the same *configuration* if each one of them has the same neighbour (*i.e.* configurations are considered up to rotation of the table). How many configurations are there?

If you want to push further, you can do Grimaldi's 1.1-1.2 exercises: 3, 5, 9, 11, 21, 31, 37.