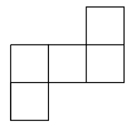
MAT 2348 — assignment #3

Due: March 26th. Numeric answers need to be justified. Don't Panic.

A. Compute the rook polynomial for the board  .

B. If you have not already done it in #7.B.1, find the generating function for the sequence of cubes: $f(X) = 1^3 + 2^3X + 3^3X^2 + \cdots$

Then use it to compute the formula for the sum of cubes $F(n) = 1^3 + 2^3 + 3^3 + \cdots + n^3$.

C. Give the sequence generated by the following functions:

$$\frac{2X-3}{1+X} \qquad \frac{1}{4-X} \qquad \frac{1}{(1-X)(2+X)} \qquad \frac{1+X}{(1-X^2)}$$

D. Consider the equation $x_1 + x_2 + x_3 = n$ with the x_i positive integers.

Give the generating functions for the problem with the following constraints:

1. $x_1 \geq 2$ and $x_2 \geq 4$
2. $x_3 \leq 9$
3. $x_1 \geq 3$ and x_1 is odd

E. We consider a generating function $f(X) = a_0 + a_1X + a_2X^2 + \cdots$

1. We set $f(X)f(X) = b_0 + b_1X + b_2X^2 + \cdots$, compute the value of b_i in terms of the a_i.
2. Considering the special case where $f(X) = (1+X)^n$, express the coefficient of X^n in

$$(1+X)^n(1+X)^n$$

3. Give the coefficient of X^n in $(1+X)^{2n}$ directly.
4. Use this to show that $\binom{2n}{n} = \sum_{i=0}^{n} \binom{n}{i}^2$.