ondes acoustique bilan énergétique

Carnet de n...	Premier carnet de notes
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Auteur :	sophie.kervazo@gmail.com

$$e_k = \frac{1}{2}\rho\left(\frac{\partial \psi}{\partial t}\right)^2$$

$$\frac{de_k}{\partial t} = \rho\left(\frac{\partial \psi}{\partial t}\right)\left(\frac{\partial^2 \psi}{\partial t^2}\right)$$

On définit $\delta \mathcal{P}$ la puissance reçut par un élément dx du milieu

$$\delta \mathcal{P} = \delta p(x,t) S v(x,t) - \delta p(x+dx,t) S v(x+dx,t)\, dx$$

$$= \frac{-\partial}{\partial x}\underbrace{\left(\delta p \cdot \frac{\partial \psi}{\partial t}\right)}\, S\, dx \quad \text{analogue au vecteur de Poynting}$$

$$= \frac{-\partial}{\partial x}\left(\frac{-1}{K_S}\frac{\partial \psi}{\partial x}\frac{\partial \psi}{\partial t}\right)\underbrace{S\, dx}_{d\tau} \quad \text{car } \delta p = \frac{-1}{K_S}\frac{\partial \psi}{\partial x}$$

$$= \frac{1}{K_S}\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x} \cdot \frac{\partial \psi}{\partial t}\right) d\tau$$

$$= \frac{1}{K_S}\left(\frac{1}{v_\phi^2}\right)\frac{\partial}{\partial t}\left[\left(\frac{\partial \psi}{\partial t}\right)^2\right] d\tau \quad \text{car } \frac{\partial}{\partial x} = \frac{-1}{v_\phi}\frac{\partial}{\partial t}$$

$$= \rho\frac{\partial}{\partial t}\left[\left(\frac{\partial \psi}{\partial t}\right)^2\right] d\tau$$

$$= 2\rho\frac{\partial \psi}{\partial t}\frac{\partial^2 \psi}{\partial t^2}\, d\tau$$

$$\delta p_V = \frac{\delta \mathcal{P}}{d\tau} = 2\rho\frac{\partial \psi}{\partial t}\frac{\partial^2 \psi}{\partial t^2} = 2\frac{de_k}{dt}$$

énergie potentielle volumique

$$\delta p_V = \frac{\partial}{\partial t}\left(e_k + e_p\right)$$

$$\frac{\partial e_p}{\partial t} = \frac{\partial e_k}{\partial t}$$

$$\rho \frac{\partial \psi}{\partial t} \frac{\partial^2 \psi}{\partial t} = \frac{\partial e_r}{\partial t}$$

$$e_p = \frac{1}{2} \rho \left(\frac{\partial \psi}{\partial t}\right)^2$$

$$= \frac{1}{2} \rho v_\phi^2 \left(\frac{\partial \psi}{\partial x}\right)^2$$

$$= \frac{1}{2 K_s} \left(\frac{\partial \psi}{\partial x}\right)^2 = \frac{1}{2} K_s \delta p^2$$

énergie mécanique volumique

$$e_m = \frac{1}{2} K_s \delta p^2 + \frac{1}{2} \rho \left(\frac{\partial \psi}{\partial t}\right)^2$$