An important NP-hard problem Pecha Kucha talk

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P=NP				
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P = NP?

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P=NP ●○			
P = N	IP?		

■ You can win \$1.000.000 if you solve it

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P=NP ●0	Problems in P 00	Problems in NP 000	NP-hardness 00	Best NP-hard problem solvers 0
P = N	NP?			

- You can win \$1.000.000 if you solve it
- You are not allowed to write " $\not\!\!P = N \not\!\!P$, so 1 = N"

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P=NP ●0		
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P = NP ?

- You can win \$1.000.000 if you solve it
- You are not allowed to write " $\not P = N \not P$, so 1 = N"
- How easy/difficult it is to solve problems that depend on an input

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P=NP	Problems in NP	

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P = NP?

P = Polynomial time

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P = N	IP ?		

P = Polynomial time

 $\rightarrow\,$ Find a solution in polynomial time on the size of the input

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- P = Polynomial time
 - $\rightarrow\,$ Find a solution in polynomial time on the size of the input
- NP = Non deterministic **P**olynomial time

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P = NP?

- P = Polynomial time
 - $\rightarrow\,$ Find a solution in polynomial time on the size of the input
- NP = Non deterministic **P**olynomial time
 - $\rightarrow\,$ Check that a solution is correct in polynomial time on the size of the input

Problems in P : Linear Equality

Linear Equality

Input : $A \in \operatorname{Mat}_n(\mathbb{Q})$, $B \in \mathbb{Q}^n$.

Question : Is there $X \in \mathbb{Q}^n$ such that AX = B?

Output : Such an X.

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Problems in P : Packing is correct

Packing is correct





- Input : Sizes of boxes, size of truck, places for the boxes in the truck
- Question : Is the input a correct way to put the boxes in the truck ?
 - Output : Yes/No

	Problems in NP	
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Problems in NP : Packing

Packing





Input : Sizes of boxes, size of truck Question : Is it possible to put all the boxes in the truck ? Output : Instructions to put the boxes in the truck

	Problems in NP	
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Problems in NP : SAT

SAT

$$\begin{array}{l} \mathsf{Input}: \ (x_1 \lor x_2 \lor \neg x_3) \land (x_1 \lor \neg x_4 \lor x_5) \land (x_2 \lor x_4 \lor x_6) \land \\ (\neg x_1 \lor \neg x_5 \lor \neg x_7) \land (x_1 \lor x_4 \lor x_7) \land (\neg x_5 \lor \neg x_6 \lor x_7) \end{array}$$

Question : Is the formula satisfiable?

Output : True/False values for all variables

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	Problems in NP	
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Problems in NP : SuperMario

SuperMario



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Input : A level in SuperMario Question : Is it possible to win the level? Output : A winning path

	NP-hardness	
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NP-hardness



Reduction



$$\begin{array}{c} \begin{array}{c} \begin{array}{c} (x_1 \lor x_2 \lor \neg x_3) \land (x_1 \lor \neg x_4 \lor x_5) \land \\ (x_2 \lor x_4 \lor x_6) \land (\neg x_1 \lor \neg x_5 \lor \neg x_7) \land \\ (x_1 \lor x_4 \lor x_7) \land (\neg x_5 \lor \neg x_6 \lor x_7) \end{array} \end{array}$$

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	NP-hardness	
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NP-hardness



$$\begin{array}{c} \hookrightarrow \quad (x_1 \lor x_2 \lor \neg x_3) \land (x_1 \lor \neg x_4 \lor x_5) \land \\ (x_2 \lor x_4 \lor x_6) \land (\neg x_1 \lor \neg x_5 \lor \neg x_7) \land \\ (x_1 \lor x_4 \lor x_7) \land (\neg x_5 \lor \neg x_6 \lor x_7) \end{array}$$

NP-hard problem : Every problem in NP can be reduced to it

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	NP-hardness	Best NP-hard problem solvers
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NP-hard problems

SAT

Packing

SuperMario

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	NP-hardness	
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NP-hard problems

- SAT
- Packing
- SuperMario

SCHEDULE

Input : Availability of students, availability of lecturers, availability of rooms

- Question : Is there a schedule satisfying everyone?
 - Output : A schedule

				Best NP-hard problem solvers
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Best NP-hard problem solvers



Camille Peignois



Florence Magi



Marie Leblanc



Katharina Heil

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