An important NP-hard problem
Pecha Kucha talk

Miguel Acosta

University of Luxembourg

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$P = NP$?

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- You can win \$1.000.000 if you solve it

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- You are not allowed to write "$P = NP$, so $1 = N$"

P = NP ?

- You can win \$1.000.000 if you solve it
- You are not allowed to write "$\not P = N\not P$, so $1 = N$"
- How easy/difficult it is to solve problems that depend on an input

P = NP ?

P = **P**olynomial time

P = NP ?

P = **P**olynomial time
  → Find a solution in polynomial time on the size of the input

P = NP ?

P = **P**olynomial time

$\rightarrow$ Find a solution in polynomial time on the size of the input

NP = **N**on deterministic **P**olynomial time

P = NP ?

P = **P**olynomial time
   $\rightarrow$ Find a solution in polynomial time on the size of the input

NP = **N**on deterministic **P**olynomial time
   $\rightarrow$ Check that a solution is correct in polynomial time on the size of the input

Problems in P : Linear Equality

Linear Equality

Input : $A \in \mathrm{Mat}_n(\mathbb{Q})$, $B \in \mathbb{Q}^n$.

Question : Is there $X \in \mathbb{Q}^n$ such that $AX = B$?

Output : Such an X.

Problems in P : Packing is correct

Packing is correct

Input : Sizes of boxes, size of truck, places for the boxes in the truck

Question : Is the input a correct way to put the boxes in the truck ?

Output : Yes/No

Problems in NP : Packing

Packing

Input : Sizes of boxes, size of truck

Question : Is it possible to put all the boxes in the truck ?

Output : Instructions to put the boxes in the truck

Problems in NP : SAT

SAT

Input : $(x_1 \vee x_2 \vee \neg x_3) \wedge (x_1 \vee \neg x_4 \vee x_5) \wedge (x_2 \vee x_4 \vee x_6) \wedge$
$(\neg x_1 \vee \neg x_5 \vee \neg x_7) \wedge (x_1 \vee x_4 \vee x_7) \wedge (\neg x_5 \vee \neg x_6 \vee x_7)$

Question : Is the formula satisfiable ?

Output : True/False values for all variables

P=NP
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Problems in P
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Problems in NP
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NP-hardness
00

Best NP-hard problem solvers
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Problems in NP : SuperMario

SuperMario

Input : A level in SuperMario

Question : Is it possible to win the level ?

Output : A winning path

NP-hardness

Reduction

$\rightsquigarrow$ $(x_1 \lor x_2 \lor \neg x_3) \land (x_1 \lor \neg x_4 \lor x_5) \land$
$(x_2 \lor x_4 \lor x_6) \land (\neg x_1 \lor \neg x_5 \lor \neg x_7) \land$
$(x_1 \lor x_4 \lor x_7) \land (\neg x_5 \lor \neg x_6 \lor x_7)$

NP-hardness

Reduction

$\rightsquigarrow$ $(x_1 \lor x_2 \lor \neg x_3) \land (x_1 \lor \neg x_4 \lor x_5) \land$
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$(x_1 \lor x_4 \lor x_7) \land (\neg x_5 \lor \neg x_6 \lor x_7)$

NP-hard problem : Every problem in **NP** can be reduced to it

NP-hard problems

- SAT
- Packing
- SuperMario

NP-hard problems

- SAT
- Packing
- SuperMario

SCHEDULE

Input : Availability of students, availability of lecturers, availability of rooms

Question : Is there a schedule satisfying everyone ?

Output : A schedule

Best NP-hard problem solvers

Camille Peignois

Florence Magi

Marie Leblanc

Katharina Heil