The single-particle density matrix of a quantum bright soliton

Alex Ayet[1][2] and Joachim Brand[1]

[1] Dodd-Walls Centre for Photonics and Quantum Technologies, New Zealand Institute for Advanced Study, Massey University, New Zealand

[2] Département de physique, École Normale Supérieure, Paris, France

Introduction

We have developed a novel approach for computing reduced density matrices for superpositions of eigenstates of a Bethe-ansatz solvable model, using a diagrammatic approach [1]. We present its application to the computation of the single-particle density matrix and its eigenvalues (including the condensate fraction) for a quantum bright soliton with up to $N = 10$ bosons. The underlying approach is suitable for studying time-dependent problems and generalises to higher-order correlation functions.

Model

We consider the Lieb-Liniger model, describing a one dimensional system of N bosons with coordinates x_i interacting with a contact potential of strength c. Its hamiltonian is

$$H = -\sum_{i=1}^{N} \frac{\partial^2}{\partial x_i^2} + 2c \sum_{\langle i,j \rangle} \delta(x_i - x_j), \qquad (1)$$

whose eigenstates are obtained with the coordinate Bethe-ansatz as string states $|p\rangle$.
We construct a bright soliton of mean momentum n_0 as

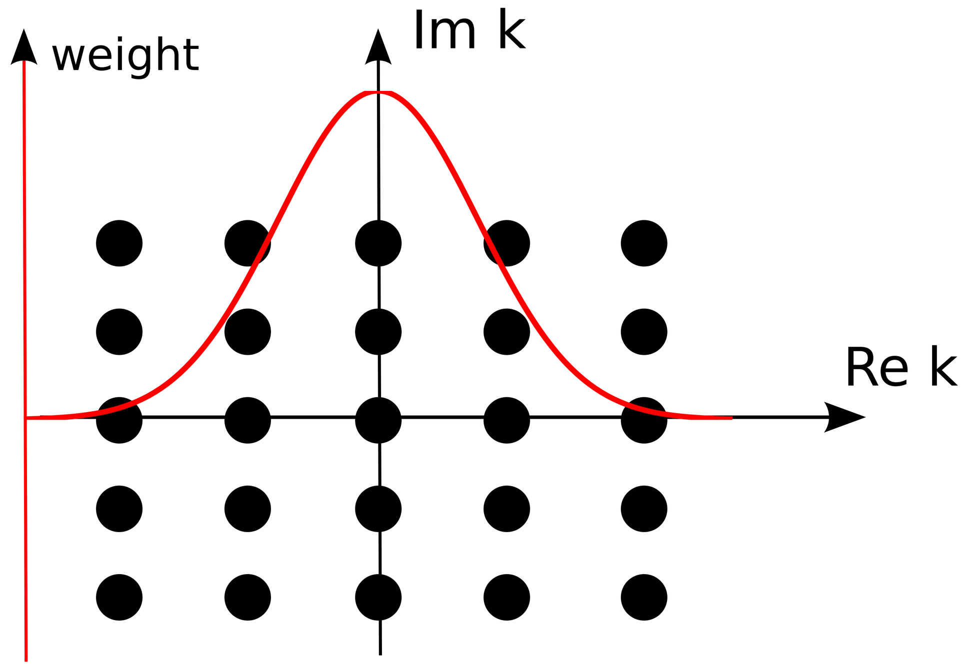
$$|S\rangle = \mathcal{G} \sum_{n=n_0-\frac{s}{2}}^{n_0+\frac{s}{2}} \exp\left[-\frac{\pi^2}{L^2\Delta}(n-n_0)^2\right] \left|\frac{\pi}{L}n\right\rangle . \qquad (2)$$

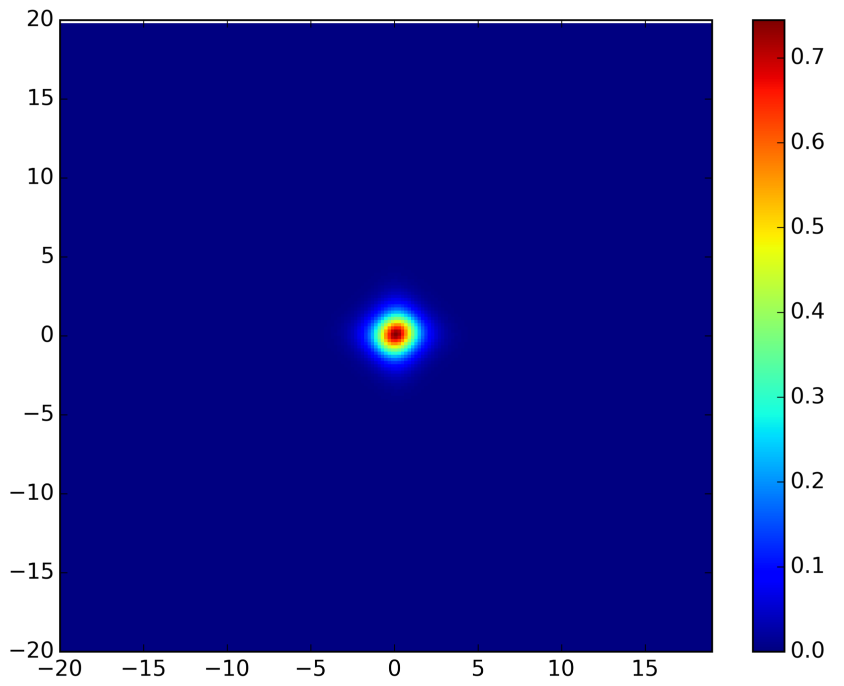
In order the get the single-particle density matrix, we have to compute the form factors (e.g. [2])

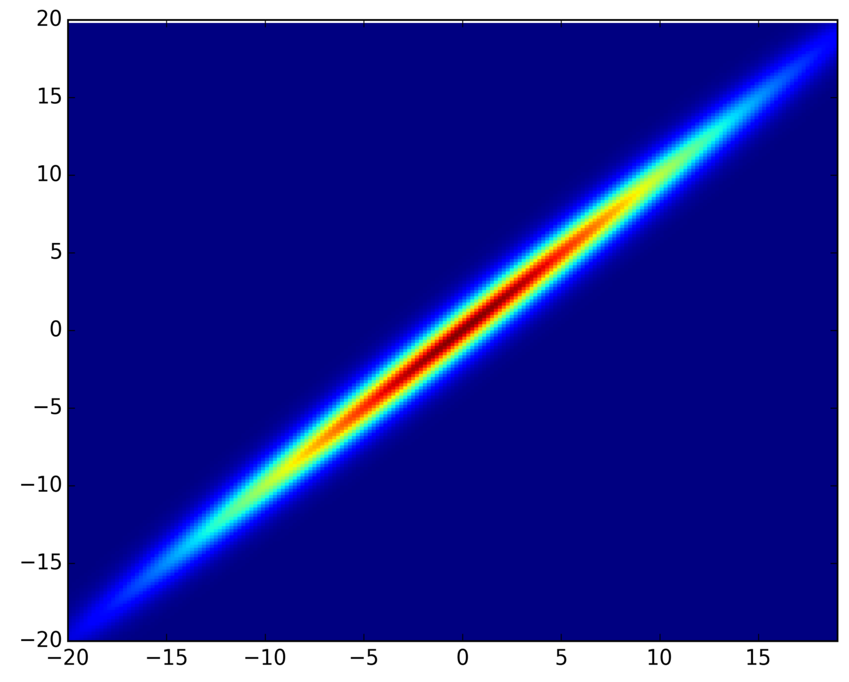
$$\mathcal{F}_{p',p}(x',x) = \int_{[-L,L]^{N-1}} dx_1 ... dx_{N-1} \times \langle p' | \, x_1, ..., x_{N-1}, x' \rangle \langle x_1, ..., x_{N-1}, x | \, p \rangle , \qquad (3)$$

which is done diagramatically in [1].

Bright soliton state



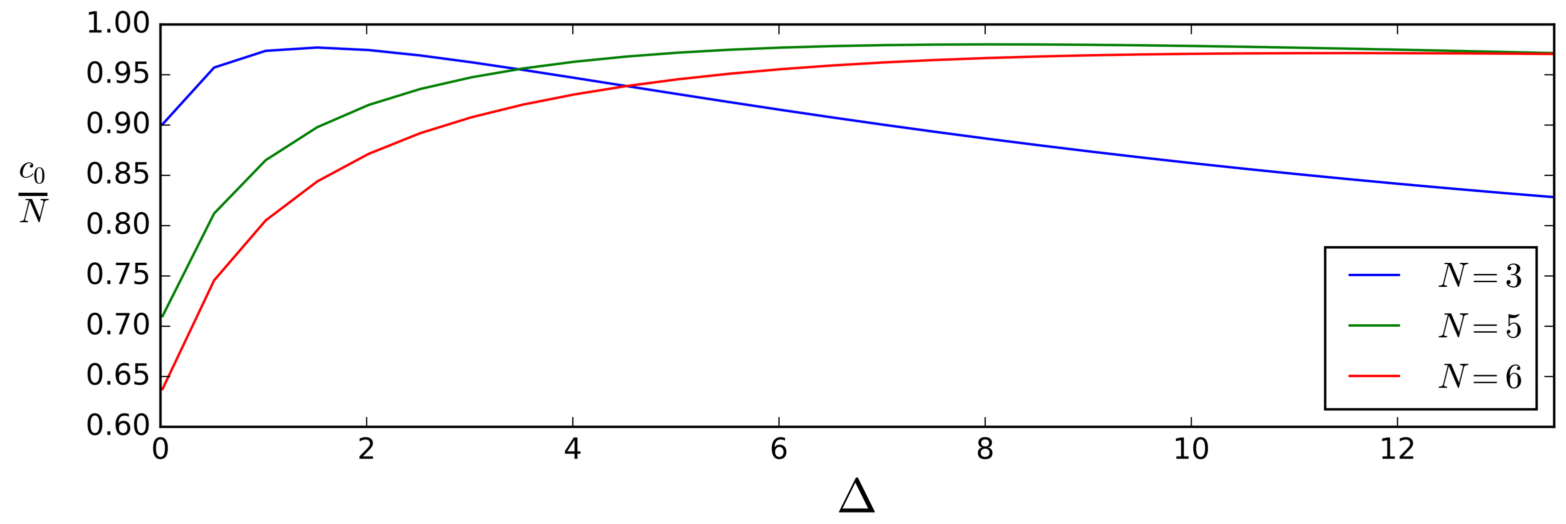


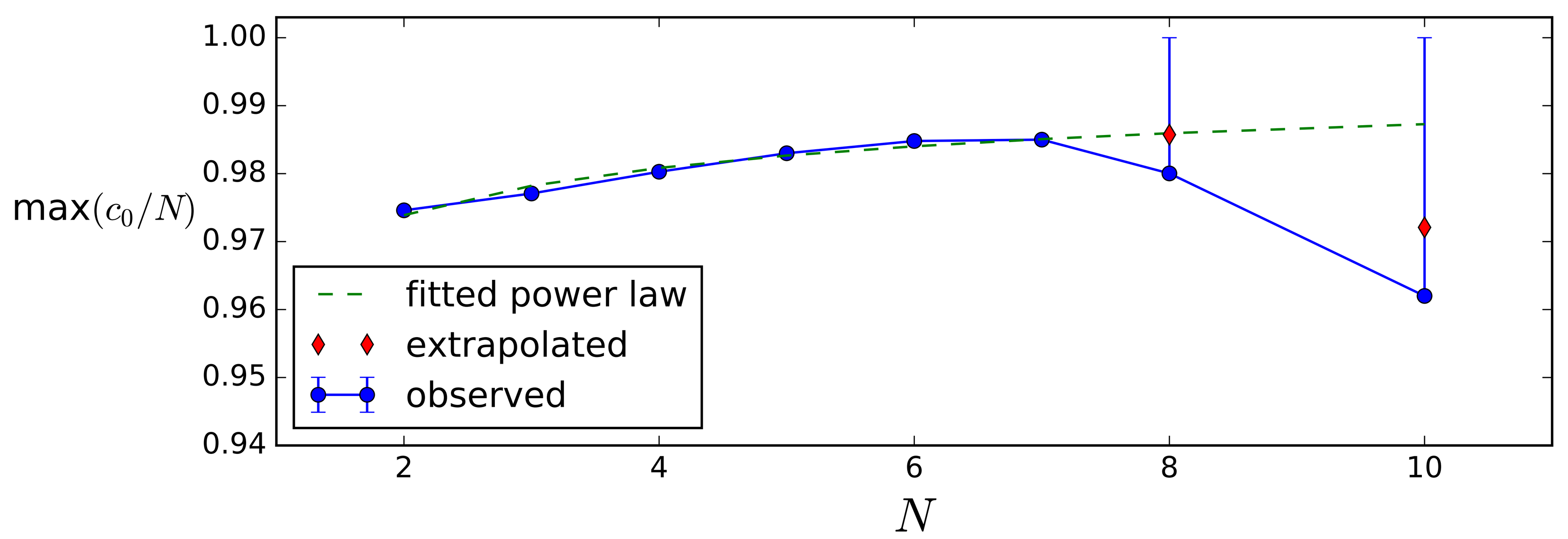


Results

We have computed three important quantities:

- the largest eigenvalue c_0 of the density matrix as function of the spread Δ.
- for a given N, the maximum of this value, w.r.t. Δ.
- for a given N, the maximum of the spatial variance σ^2 of the density matrix, w.r.t. Δ. It is compared to the mean field Hartree-Fock solution.





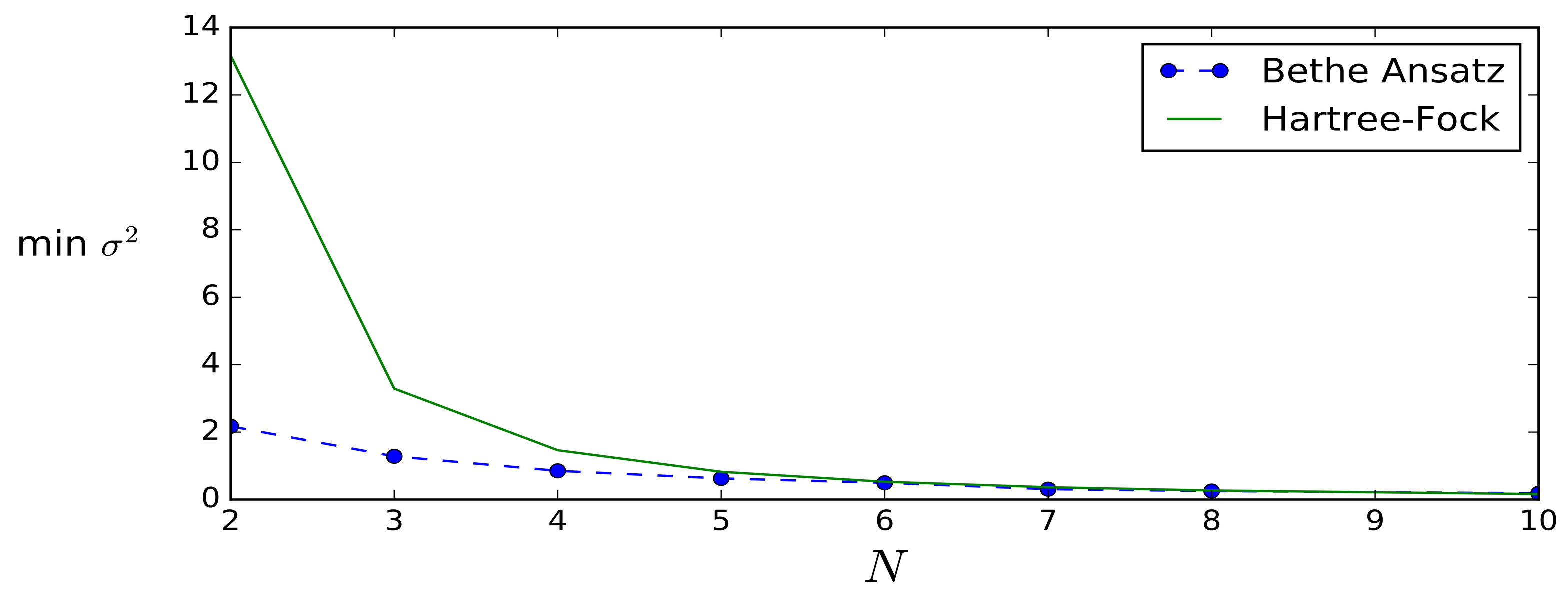
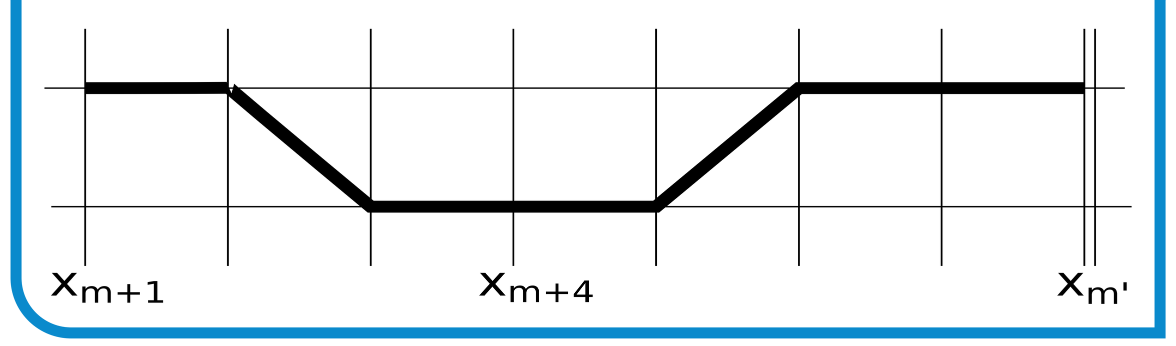


Diagram example



References

[1] Alex Ayet and Joachim Brand. The single-particle density matrix of a quantum bright soliton from the coordinate bethe ansatz. *J. Stat. Mech. Theor. Exp.*, 2017(2):023103, 2017.

[2] Pasquale Calabrese and Jean-Sebastien Caux. Dynamics of the attractive 1D Bose gas: analytical treatment from integrability. *J. Stat. Mech. Theory Exp.*, pages 08032–08032, 2007.