The Philosophy of Mathematical Practice

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36 Proclus

In short, Proclus' philosophy of mathematics, to the extent that it might be of interest to our discussion, involces the following elements: *(a)* Platonic ideas (still conceived as inaccessible objects), among which are, for instance, the idea of triangle and unit; *(b) logoi*, that could be seen as the concepts associated to those ideas, like the concepts of triangle and unit; *(c) nous*, that lets us contemplate ideas, recalls them to the soul when this becomes embodied, and projects *logoi* on the extended (though immaterial) space of imagination, or on some other suitable receptacle, so as to generate appropriate and purely intelligible quantities, such as the different triangles or the different numbers; *(d) dianoia*, which is exerted on the *logoi* and lets us formulate definitions, postulates and theorems that concern projections of the same *logoi* and are also justified by the *logoi*, although they are made true by their agreement with the ideas. As regards both geometry and arithmetic, then, *dianoia*, *logoi*, their imaginative projections, and *nous* too – due to its activity of projecting *logoi* – are used for obtaining the theorems of mathematics and the earthly knowledge they express. While ideas, and *nous*, too, again – due to its contemplative ideas – are used for assigning truth to theses theorems.

41 Kant

in establishing whether a judgement is analtytic or synthetic, what is most important to Kant is whether its articulation requires appeal to intuition, and not that whether the predicat is or is not contained in the subject.

55 Principe de Hume

If there is a one-to-one correspondence between the objects falling under the concepts *F* and *G*, we say that *F* and *G* are equinumerous. We then have the folowing principle, which is known today as 'Hume's principle' (henceforth, HP), since Frege introduces it by quoting (in § 63) a passage from Hume's *Treatise of Human nature* (Book I, pArt III, Section I: "when two numbers are so combin'd, as that the one has always an unite asnwering to every unite of the other, we pronounce them equal"):

$$Num.(F) = Num(G) \text{ iif } F \text{ and } G \text{ are equinumerous,}$$

where *F* and *G* are any two concepts, and for every concept *X*, '*Num.*(*X*)' denotes the number belonging to *X*.

57 Frege platoniste *via* les extensions de concepts

The solution to the Caesar problem [is Caesar a number?], […] has a deep significance for Frege's philosophy of arithmetic: according to him, it is just what makes it a platonist philosophy.

The solution is offered in § 68, and relies on a notion Frege borrows from the traditional logical vocabulary: the notion of extension (*Umfang*) of a concept. Kant abudantly employs this notion in the *Critique of Pure Reason*, and in the *Vienna Logic* he defines it as a "*spharea*", constituted by a "multitude of things that are contained under the concept" […]. Quite surprinsingly, Frege elaborated no further this notion, and in a footnote to § 68, he even acknowledges that he is assuming "that it is known what the extension of a concept is".

62 Les entiers comptent, les réels mesurent

Frege's main point on this respect is that, just as natural numbers provide the cardinality of concepts (namely are things belonging to sortal concepts in relation to the size of the domain of individuals falling under them), so real numbers provide the measures of magnitudes (namely are things belonging to magnitudes, in relation to their respective sizes).

84 Arithmétique hilbertienne

The crucial point is just here: proofs like that of Hilbert, which do not follow from axioms but rather appeal to properties of natural numbers that immediately stem from their being given as signs, can only secure theorems of finitary arithmetic.

The difference between a (necessarily finitary) theory including only such proofs, like Hilbert's own finitary version of artihmetic, and an axiomatic (possibliy infinitary) theory is crucial in his view. His basic point is that, since the former deals with objets that are, as said, “intuitively present as immediate experience prior to all thought”, it is free from any risk of contradiction.

109 Dilemme de Benacerraf (paradoxe de Poincaré)

Benacerraf's point, in short, is that a good semantics for mathematics goes together with a bad epistemology, and a good epistemology goes together with a bad semantics. This is what has becom known as Benacerraf's dilemma.

113 Réponse de Field

According to Field, all these [indispensability] arguments rely on an implicit, often unquestioned, assumption: that the truth of the theorems of mathematics is a necessay condition for the successful applicability of mathematics to science. This is what he denies. More generally, he denies that the virtues of mathematics depend on the truth of its theorems, that theses theorems must be true in order for mathematics to be “good”

137-138 Structuralisme éliminatif

A structure can be defined without defining any particular system of objetcs that instantiates it: it is enough that the conditions that the relevant functions and relations must satistfy are laid down. The definition of a group show this very neatly: it does not establish the existence of the structure (at least if one accepts that a strcture exists only if there is a system of objets instantiating it), but it suffices to establish that the objects that are supposed to form its domain must have some relational properties. The basic idea is thus that a mathematical statement cannot say more: such a statement should thus be reformulated as a statement universally quantifying over the systems of objects instantiating a given structure. For example, a theorem such as '3 is a prime number' should thus be reformulated as “For any system of objects σ, if σ instantiates the structure of progression, then the object occupying the 3-place in σ is σ-prime', where the term '3-place' is defined for all progressions in general. Parsons takes this option to lead to a form of **eliminative structuralism**: a view that “begins with the basic idea of the structuralist view of mathematical objects and develops it into an analysis in which reference or commitment to such objects, or to mathematical objects of a specific kind such as natural numbers, is claimed to be eliminated”

148 Empirisme mathématique ?

the real issue is that the shift from our empirically grounded beliefs to the principles underpinning our mathematical theories, despite its crucial and obvious importance in the edification of mathematics, does not seem to be empirically justifiable.