Philosophy of Mathematics and Natural Science

Hermann WEYL
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34 ORIGINE DE LA RÉCURRENCE

The first explicit mention of the principle of complete induction seems to be with B. Pascal (1654) and Jacob Bernoulli (1686).

34-35 PRIMAUTÉ DES ORDINAUX SUR LES CARDINAUX : COMPTER SE FAIT PAR L'*ACTION*

The question has been argued extensively whether the concept of cardinal, rather than ordinal, number is not the primary one. The former, if it is to be introduced independently of an ordinal arrangement, has to be defined by abstraction […]. This definition is not even restricted to finite sets; a theory of infinite cardinal numbers based thereon was developed by G. Cantor within the framework of his general set theory. But the criterion of numerical equivalence makes use of the possibility of pairing, which can only be ascertained is the acts of correlation are carried out one after another in temporal succession and the elements of the sets themselves are thereby arranged in order. Even if one follows the road of abstraction and splits up the act of numerical comparison of two sets by first ascribing a number to each set and then comparing these numbers, it remains indispensable to order each individual set itself by exhibiting its elements one by one in temporal succession. (Such a one-by-one exhibition is necessary anyhow if an aggregate is to be considered as concretely given; and the numbers employed by us in everyday life concern only such aggregates.) For this reason it seems to me unquestionable that the concept of ordinal number is the primary one. Modern research in the foundations of mathematics, which has destroyed dogmatic set theory, confirms this view

35-36 ENTIERS PRIMITIFS

the representation of the data by strokes has the function of putting these data into a 'normal' form of such a kind that a difference in shape at once indicates a difference in number. […] The immediate recognition of equality or disparity of two symbols consisting of successions of strokes is possible, however, for the lowest numbers only. In general one has to proceed by using the strokes recorded during the first sequence over again, say, by crossing them out one by one; for this purpose it is required that the first sequence stays put (and does not disappear […]

If one wants to speak, all the same, of numbers as concepts or ideal objects, one must at any rate refrain from giving them independent existence; their being exhausts itself in the functional role which they play and their relations of more or less

65-66 FONDEMENTS MATHÉMATIQUES, DU DONNÉ AU TRANSCENDANT

The stages through which research in the foundations of mathematics has passed in recent times correspond to the three basic possibilities of epistemological attitude. The set-theoretical approach is the stage of *naive realism* which is unaware of the transition from the given to the transcendent. Brouwer represents *idealism*, by demanding the reduction of all truth to the intuitively given. In axiomatic formalism, finally, consciousness makes the attempt to 'jump over its own shadow,' to leave behind the stuff of the given, to represent the transcendent – but how could it be otherwise?, only through the *symbol*. […] It cannot be denied that a theoretical desire, incomprehensible from the merely phenomenal point of view, is alive in us which urges toward totality. Mathematics shows that with particular clarity; but it also teaches us that that desire can be fulfilled on one condition only, namely, **that we are satisfied with the symbol and renounce the mystical error of expecting the transcendent ever to fall within the lighted circle of our intuition**.

68 LES NOMBRES COMPTENT MAIS AUSSI *MESURENT*

Analytic geometry reduces every geometric problem to an algebraic one. This presupposes that the number concept, by the inclusion of fractions and irrational numbers, has acquired that width which makes it suitable, not only for counting, but also for measuring.

86 SIMILARITÉ EN GÉOMÉTRIE

We start with a group Γ of transformations. It describes, as it were, to what degree our point field is homogeneous. Once the group is given we know what likeness or similarity means – namely two figures are similar (or alike, or equivalent) that arise from each other by a transformation of Γ –, and also under what condition a relation is objective, namely if it is invariant with respect to all transformations of Γ. It is in this sense that Felix Klein in his famous Erlanger Program (1872) promulgated the conception that a geometry is determined by a group of transformations.

116 LE MONDE *EST*

The objective world simply *is*, is does not *happen*. Only to the gaze of my consciousness, crawling upward along the life line of my body, does a section of this world come to life as a fleeting image in space which continuously changes in time.

125 LE MONDE, LA CONSCIENCE, DIEU : ENTRE SUJET ET OBJET

Postulation of the external world does not guarantee that much a world will rise from the phenomena through the cognitive work of reason which attempts to create concordance. For this to take place it is necessary that the world be governed throughout by simple elementary laws. Thus the mere positing of the external world does not really explain what it was meant to explain, the question of the reality of the world mingles inseparably with the question of the reason for its lawful mathematical harmony. The latter clearly points in another direction of transcendency than that of a transcendental world; towards the origin rather than the product. Thus the ultimate answer lies beyond all knowledge, in God alone; emanating from him, the light of consciousness, its own origin hidden from it, grasps itself in self-penetration, divided and suspended between subject and object, between meaning and being.

157 HYPOTHÈSE : GUIDE & MOTEUR

"The essential function of a hypothesis," according to Mach (*Erkenntnis und Irrtum*, p. 237), "consists in the guidance it affords to new observations and experiments, by which our conjecture is either confirmed, refuted, or modified, by which – in short – our experience is broadened." "The seafarer, in whose imagination the objects thrown up the ocean upon the beach create a vivid picture of the distant land, sets out to find that land. Whether his search will succeed or not, whether in place of the expected Indian or Chinese coast he discovers a new one, at any rate his experience has been widened" (*op. cit.*, p. 213)

274 RESPECT DU MEDIUM SYMBOLIQUE

A picture of reality drawn in a few sharp lines can not be expected to be adequate to the variety of its shades. […] if the transcendental is accessible to us only through the medium of images and symbols, let the symbols at least be as distinct and unambiguous as mathematics will permit.