Finding a supersingular isogeny path with only one isogeny computation

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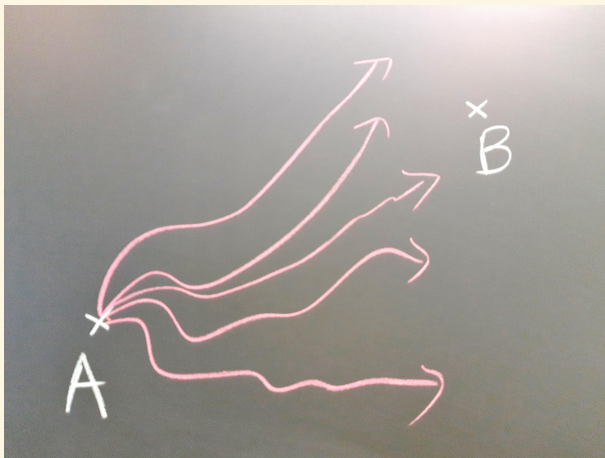Équipe LFANT, Inria Bordeaux Sud-Ouest
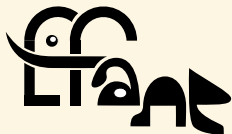
Goal

- Luca[1] wants to compute a bicycle path between Zürich and Lyon
- He is only allowed to do one isogeny request!

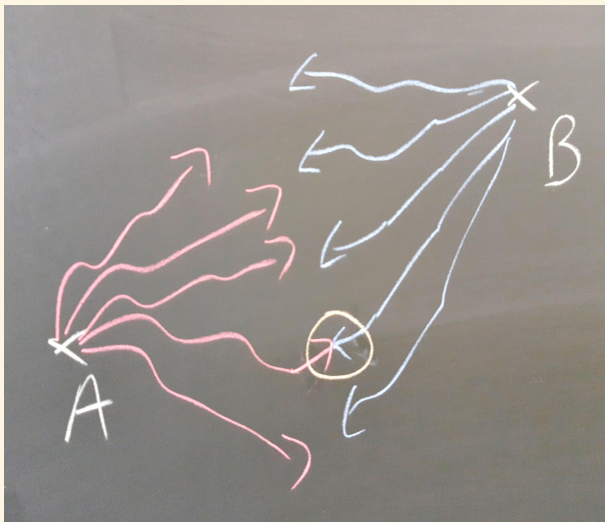[1]any similarity to an actual person is purely coincidental…

Depth search

Meet in the middle

Special points

A solution

- $E, E'/\mathbb{F}_{p^2}$ two supersingular curves ($p \approx 256$ bits)
- Goal: find a 2^e-isogeny $\phi : E_1 \to E_2$

- $E_\lambda : y^2 = x(x-1)(x-\lambda)$ (Legendre)
- $A = \prod_{\lambda \in \mathbb{F}_{p^2} - \{0,1\}} E_\lambda$
- $K \subset A[2]$ generated by the 2-torsion points $(0,0)$ on each E_λ
- $\Phi : A \to B = A/K$ encodes the full supersingular 2-isogeny graph!

Complexity

- Computing a 2-isogeny in dimension $\approx 2^{512}$ may seem expensive
- Good news! Restricting to $A = \prod E_\lambda$ with E_λ supersingular we are only in dimension $\approx 2^{256}$
- The point $(0,0)$ on E_λ and on $E_{1/\lambda}$ encode the same isogeny
- ⟹ Gains a factor two!

To infinity and beyond

- Abelian scheme of unbounded dimension:

$$A = \coprod_p \prod_{\lambda \in \mathbb{F}_{p^2}-\{0,1\}} E_{\mathbb{F}_{p^2},\lambda}$$

- A single 2-isogeny encode all supersingular 2-isogeny graphs over all primes

Slogan: higher dimensional isogeny = the ability to put your bike in a train!