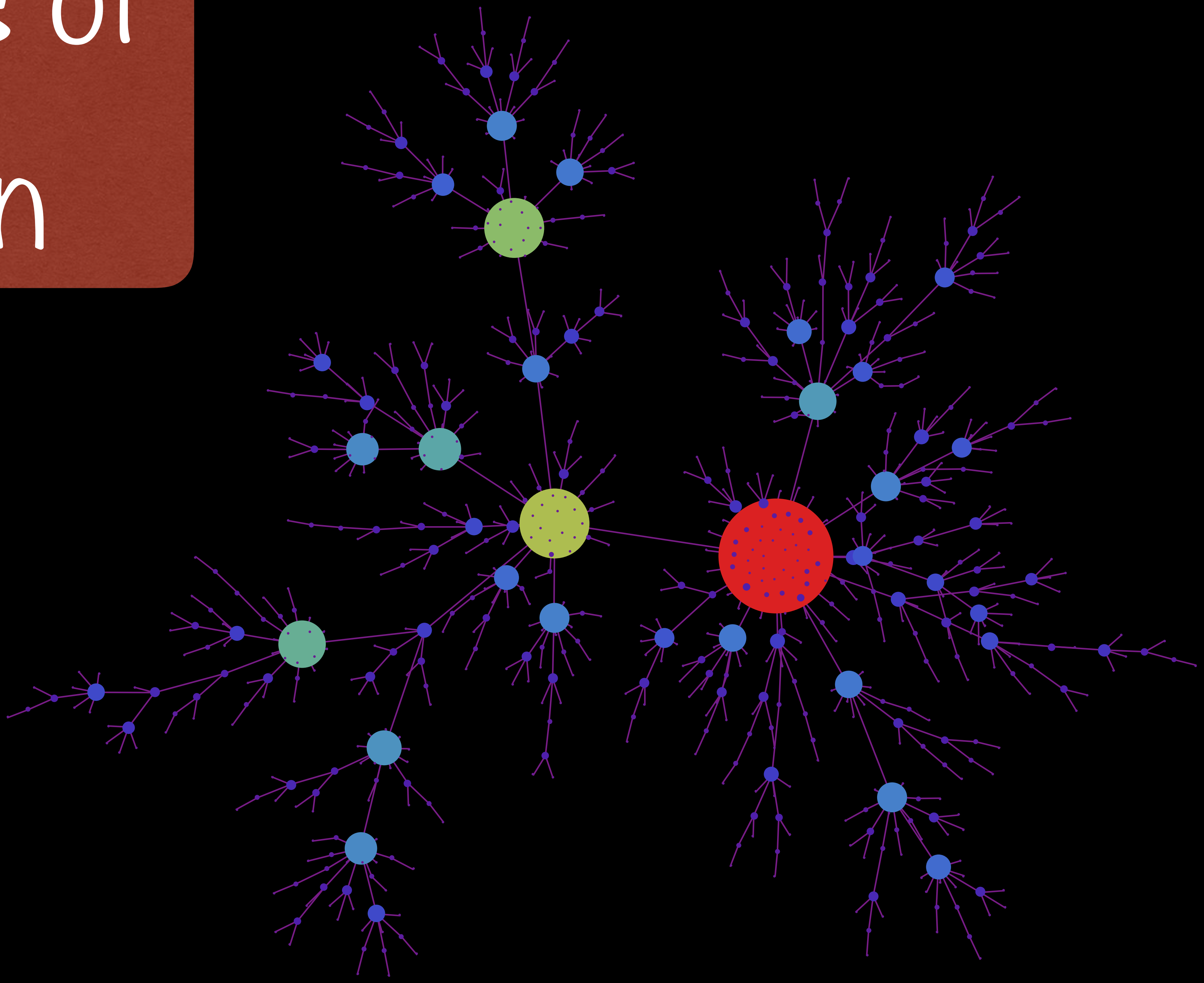
Discrete mathematics

MAA 103

What is mathematics?

Six degrees of separation



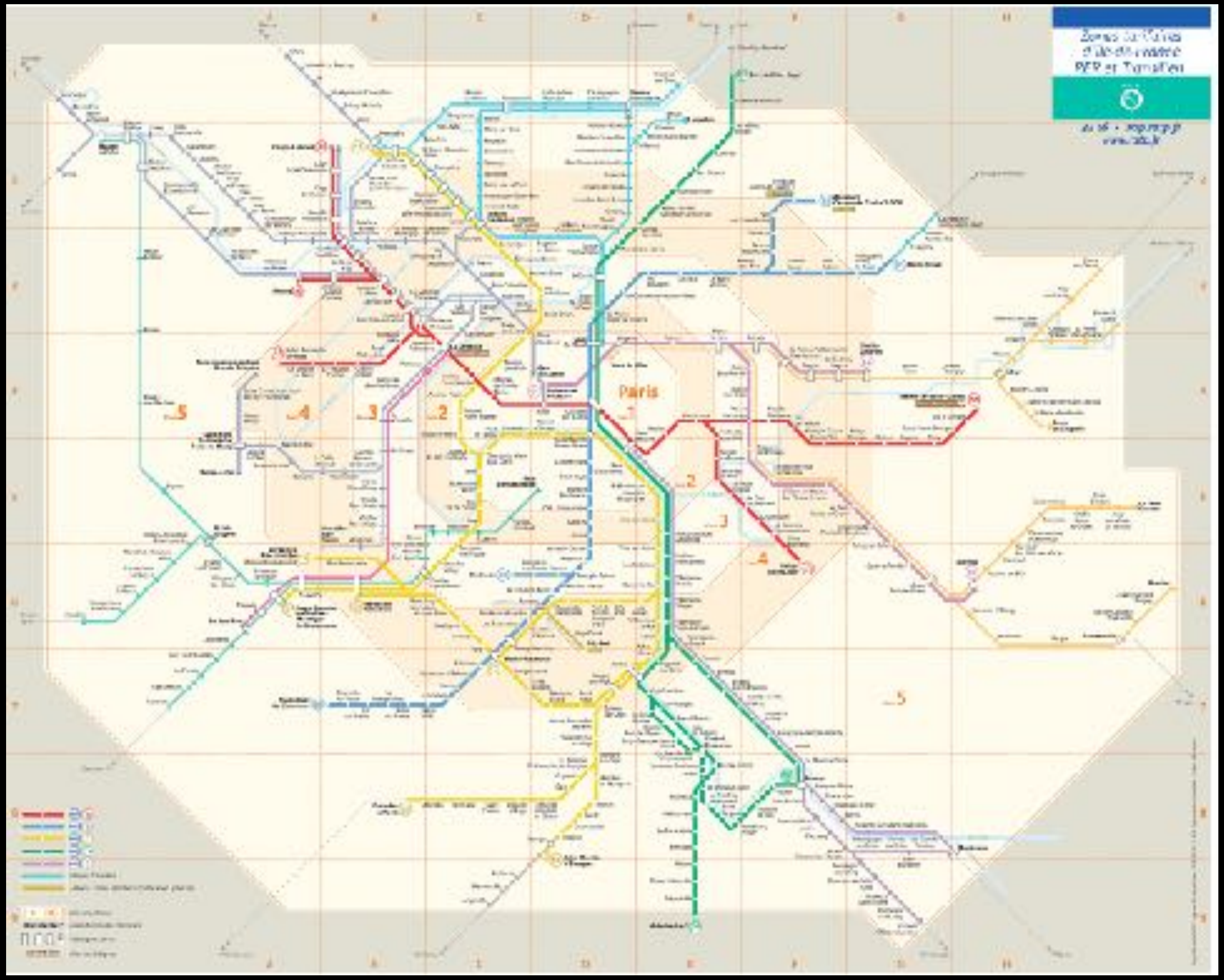
What is the common point between…

What is the common point between…

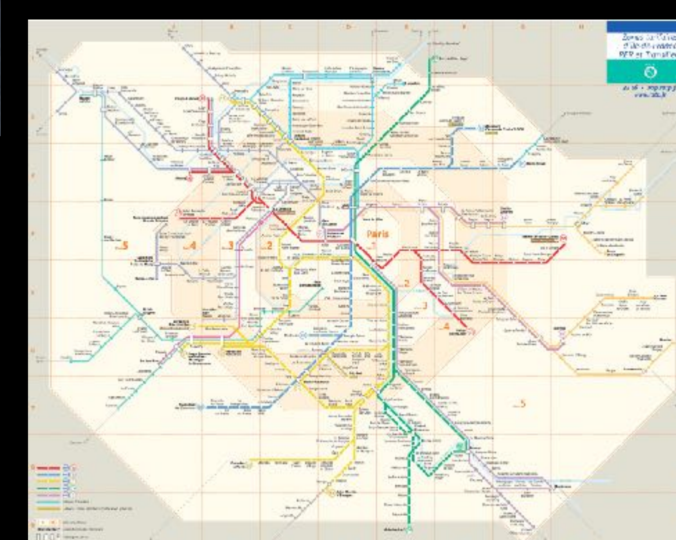
facebook 2560

What is the common point between…



What is the common point between…

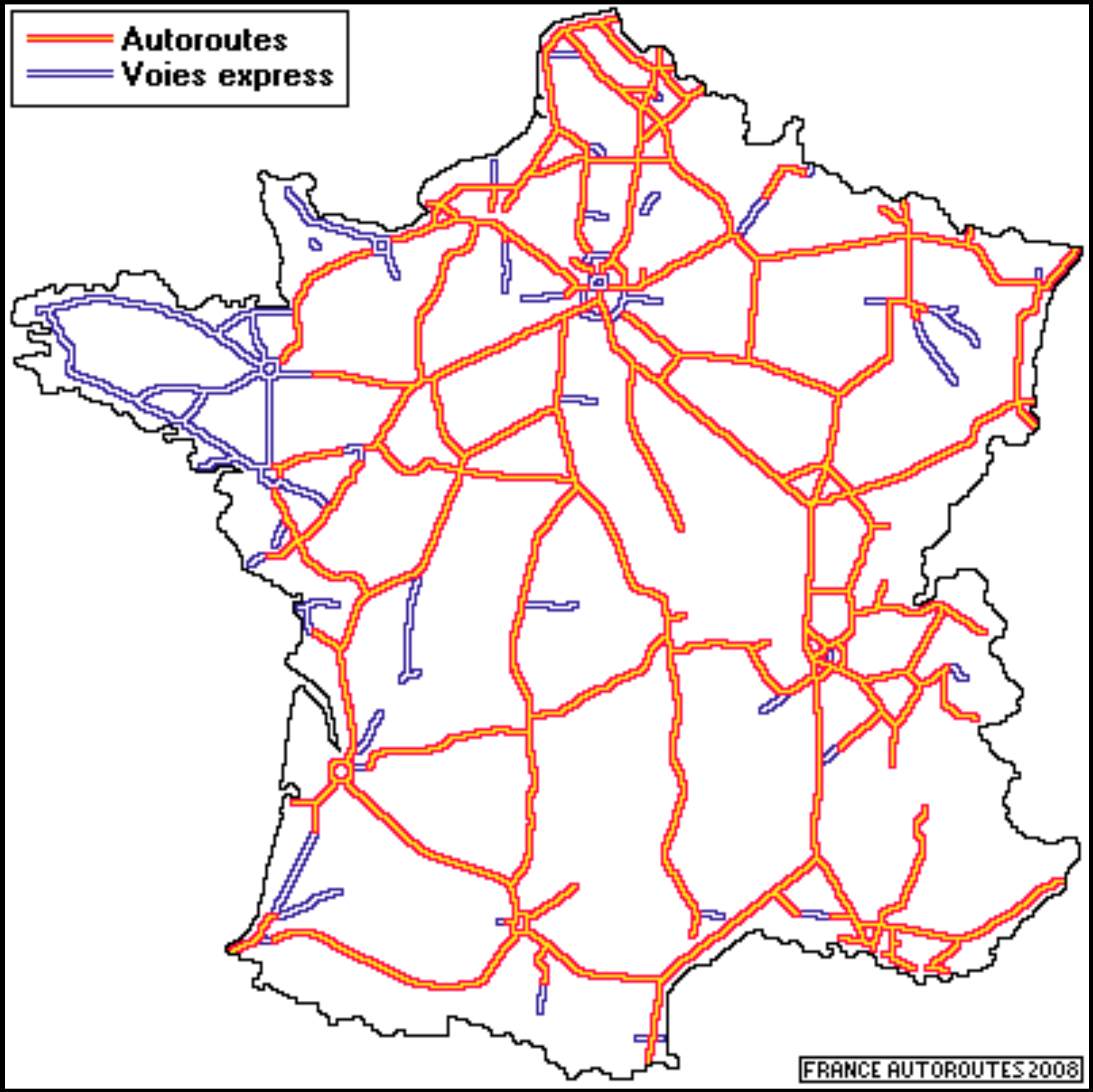
facebook 2560



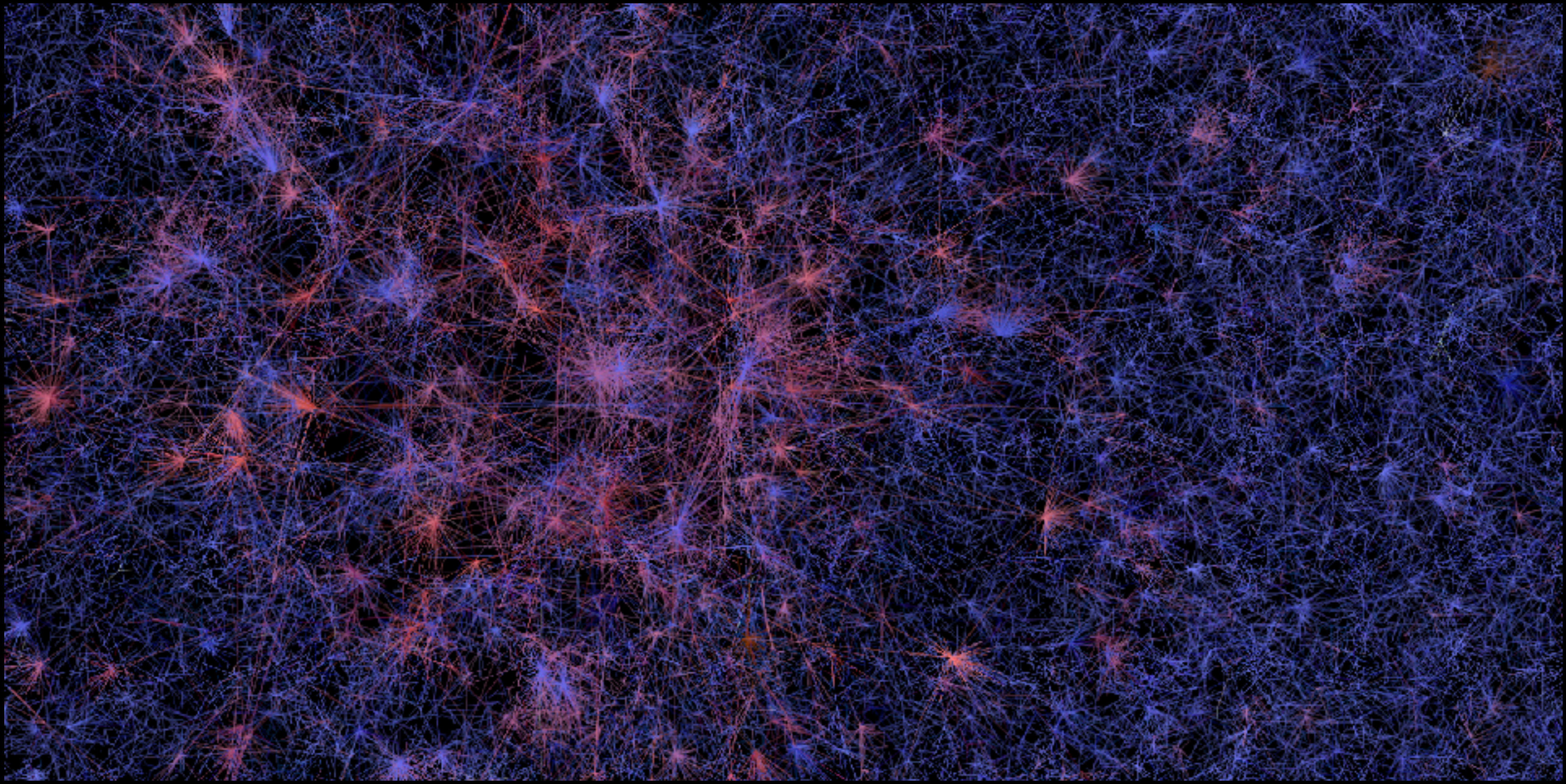
Networks of interacting objects!

Networks of interacting objects

Networks of interacting objects



Networks of interacting objects



Six degrees of separation

Six degrees of separation

Frigyes Karinthy, *Chain-links*, 1929:

Six degrees of separation

Frigyes Karinthy, *Chain-links*, 1929:

"There has to be something of crucial importance," I said in the middle of debate [...]: Planet Earth has never been as **tiny** as it is now - relatively speaking, of course - due to the quickening pulse of both physical and verbal communication.

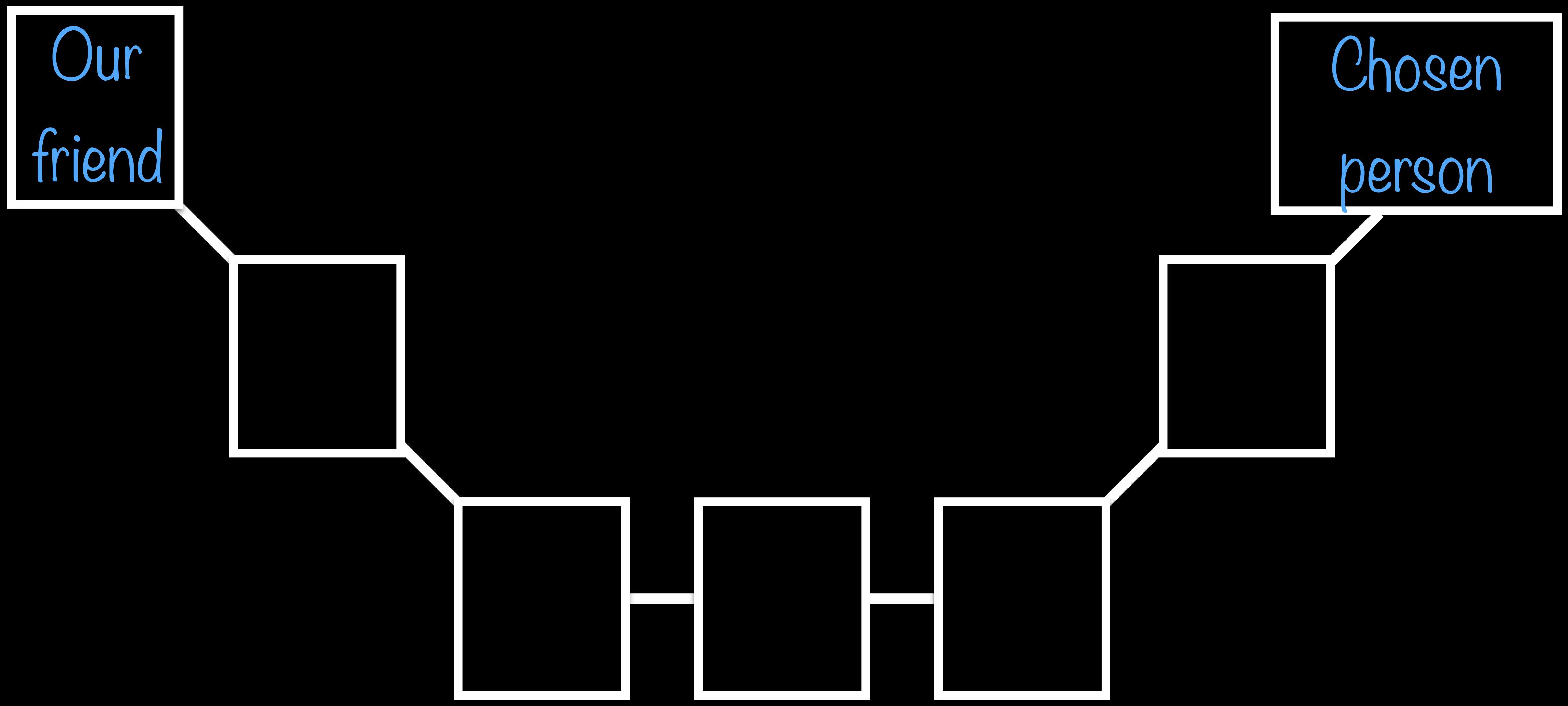
Six degrees of separation

Frigyes Karinthy, *Chain-links*, 1929:

One of us suggested to select any person from the 1.5 billion inhabitants of the Earth.

He **bet** us that, using no more than **five** other individuals, he could contact the selected individual using nothing except the network of personal acquaintances.

Six degrees of separation



Six degrees of separation

Frigyes Karinthy, *Chain-links*, 1929:

- "An interesting idea!" – said a friend – "Let's give it a try. How would you contact Selma Lagerlöf?"

Selma Lagerlöf

Gustav of Sweden

Gustav of Sweden

Kehrling

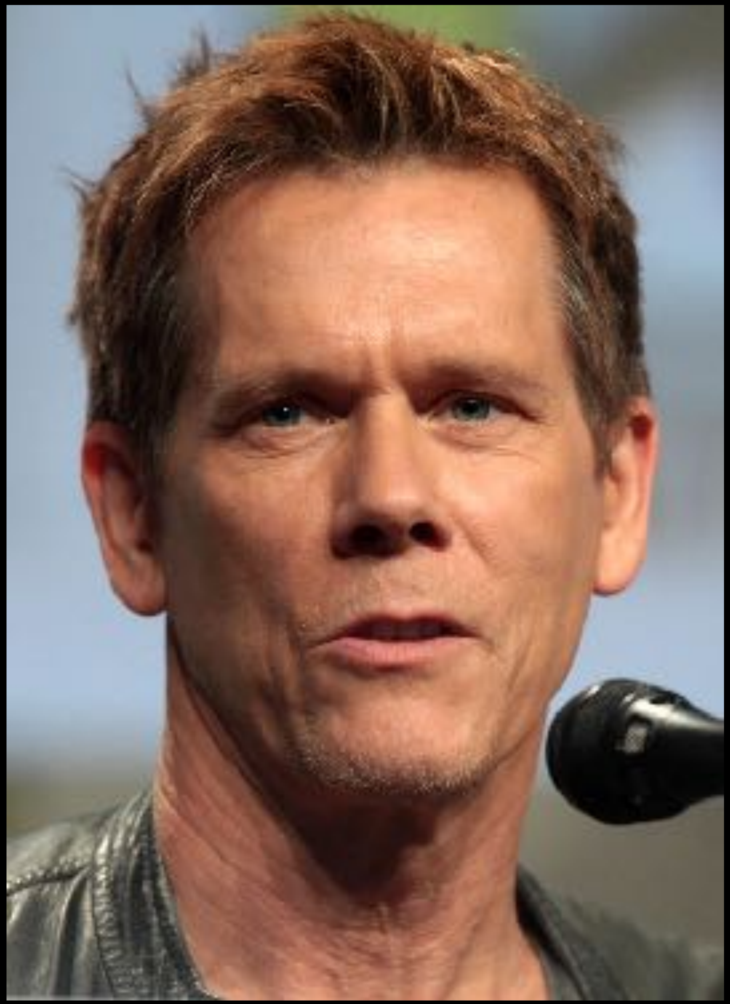
Kehrling

Our Friend

Six degrees of separation… in movies

Six degrees of separation… in movies

In 1994, Kevin Bacon commented that he had worked with everybody in Hollywood or someone who's worked with them.



Six degrees of separation…
in movies

The Bacon number of an actress or an actor is the degree of separation she or he has with Kevin Bacon.

For example: what is the Bacon number of Ryan Gosling?

Ryan Gosling

Kevin Bacon

Crazy Stupid Love

The Bacon number of Ryan Gosling is 1.

Six degrees of separation… in movies

What is the Bacon number of Natalie Portman ?

Natalie Portman

Robert de Niro

Heat

Robert de Niro

Sleapers

Kevin Bacon

The Bacon number of Natalie Portman is 2 (because she didn't shoot with Bacon).

Cf oracleofbacon.org !

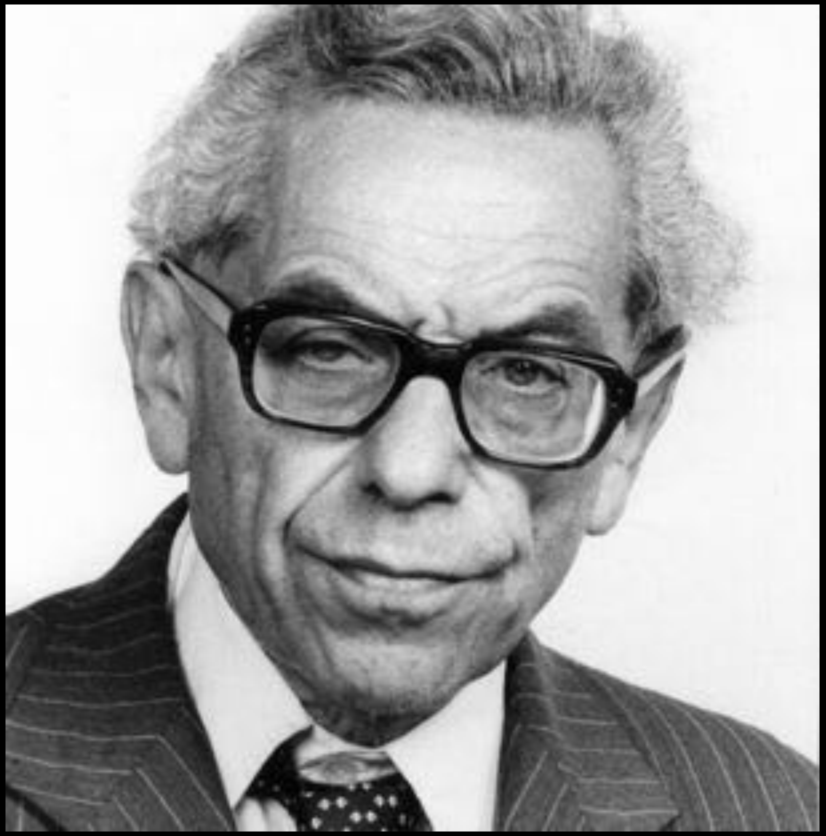
Six degrees of separation… in mathematics

Six degrees of separation… in science

The Erdős number of a mathematicain is the degree of separation, in terms of mathematical collaboration, she or he has with Paul Erdős.

Six degrees of separation… in science

The Erdős number of a mathematicain is the degree of separation, in terms of mathematical collaboration, she or he has with Paul Erdős.



Paul Erdős (1913-1996) wrote more than 1500 articles with more than 500 collaborators!

Six degrees of separation…
… in science

The Erdős number of a mathematicain is the degree of separation, in terms of mathematical collaboration, she or he has with Paul Erdős.

What is the Erdős number of Natalie Portman ?

Six degrees of separation…
in science

The Erdős number of a mathematicain is the degree of separation, in terms of mathematical collaboration, she or he has with Paul Erdős.

The Erdős number of Natalie Portman is 5:

Six degrees of separation… in science

The Erdős number of a mathematicain is the degree of separation, in terms of mathematical collaboration, she or he has with Paul Erdős.

The Erdős number of Natalie Portman is 5:

- Natalie Portman wrote an article with Abigail A. Baird ;

Six degrees of separation… in science

The Erdős number of a mathematicain is the degree of separation, in terms of mathematical collaboration, she or he has with Paul Erdős.

The Erdős number of Natalie Portman is 5:

- Natalie Portman wrote an article with Abigail A. Baird ;
- Abigail A. Baird wrote an article with Michael S. Gazzaniga ;

Six degrees of separation… in science

The Erdős number of a mathematicain is the degree of separation, in terms of mathematical collaboration, she or he has with Paul Erdős.

The Erdős number of Natalie Portman is 5:

- Natalie Portman wrote an article with Abigail A. Baird ;
- Abigail A. Baird wrote an article with Michael S. Gazzaniga ;
- Michael S. Gazzaniga wrote an article with Jonathan D. Victor ;

Six degrees of separation… in science

The Erdős number of a mathematicain is the degree of separation, in terms of mathematical collaboration, she or he has with Paul Erdős.

The Erdős number of Natalie Portman is 5:

- Natalie Portman wrote an article with Abigail A. Baird ;
- Abigail A. Baird wrote an article with Michael S. Gazzaniga ;
- Michael S. Gazzaniga wrote an article with Jonathan D. Victor ;
- Jonathan D. Victor wrote an article with Joseph Gillis.

Six degrees of separation… in science

The Erdős number of a mathematicain is the degree of separation, in terms of mathematical collaboration, she or he has with Paul Erdős.

The Erdős number of Natalie Portman is 5:

- Natalie Portman wrote an article with Abigail A. Baird ;
- Abigail A. Baird wrote an article with Michael S. Gazzaniga ;
- Michael S. Gazzaniga wrote an article with Jonathan D. Victor ;
- Jonathan D. Victor wrote an article with Joseph Gillis.
- Joseph Gillis wrote an article with Paul Erdős.

Six degrees of separation… in science

The Erdős number of a mathematicain is the degree of separation, in terms of mathematical collaboration, she or he has with Paul Erdős.

My Erdős number is 4

- I have written an article with Céline Abraham;

ESAIM: PROCEEDINGS AND SURVEYS, October 2015, Vol. 51, p. 133-149
A. Garivier et al, Editors

RANDOM MAPS

Céline Abraham[1], Jérémie Bettinelli[2], Gwendal Collet[3] and Igor Kortchemski[4]

Abstract. This is a quick survey on some recent works done in the field of random maps, which, very roughly speaking, are graphs embedded without edge crossings in a surface. We present the main results and tools in this area then summarize the original contributions presented during the conference *JournÃľes MAS 2014*.

Six degrees of separation…
in mathematics

The Erdős number of a mathematicain is the degree of separation, in terms of mathematical collaboration, she or he has with Paul Erdős.

My Erdős number is 4

- I have written an article with Nicolas Curien;
- Céline Abraham has written an article with Jean-François Le Gall;

J. Eur. Math. Soc. 20, 2951–3016

DOI 10.4171/JEMS/827

Céline Abraham · Jean-François Le Gall

Excursion theory for Brownian motion indexed by the Brownian tree

Received September 24, 2015

Abstract. We develop an excursion theory for Brownian motion indexed by the Brownian tree, which in many respects is analogous to the classical Itô theory for linear Brownian motion. Each excursion is associated with a connected component of the complement of the zero set of the tree-indexed Brownian motion. Each such connected component is itself a continuous tree, and we introduce a quantity measuring the length of its boundary. The collection of boundary lengths coincides with the collection of jumps of a continuous-state branching process with branching mechanism $\psi(u) = \sqrt{8/3}\, u^{3/2}$. Furthermore, conditionally on the boundary lengths, the different excursions are independent, and we determine their conditional distribution in terms of an excursion measure $\mathbb{M}_0$ which is the analog of the Itô measure of Brownian excursions. We provide various descriptions of $\mathbb{M}_0$, and we also determine several explicit distributions, such as the joint distribution of the boundary length and the mass of an excursion under $\mathbb{M}_0$. We use the Brownian snake as a convenient tool for defining and analysing the excursions of our tree-indexed Brownian motion.

Keywords. Excursion theory, tree-indexed Brownian motion, continuum random tree, Brownian snake, exit measure, continuous-state branching process

Six degrees of separation… in mathematics

The Erdős number of a mathematicain is the degree of separation, in terms of mathematical collaboration, she or he has with Paul Erdős.

My Erdős number is 4

- I have written an article with Nicolas Curien;
- Nicolas Curien has written an article with Jean-François Le Gall;
- Jean-François Le Gall has written an article with Samuel J. Taylor;

ELSEVIER

Stochastic Processes and their Applications 59 (1995) 1–20

stochastic processes and their applications

The packing measure of the support of super-Brownian motion

J.-F. Le Gall[a], E.A. Perkins[b], S.J. Taylor[c,*]

[a] *Laboratoire de Probabilités, Université Pierre et Marie Curie, 4, Place Jussieu, 75252 Paris 05, France*

[b] *Department of Mathematics, University of British Columbia, Vancouver, B.C., Canada V$\leqslant$T 1Z2*

[c] *Department of Mathematics, University of Virginia, Charlottesville, Virginia 22903, USA*

Received 21 June 1994; revised 9 February 1995

Abstract

Our object is to obtain more information about the fractal properties of super-Brownian motion. For $d \geqslant 2$ the closed support $S(Y_t)$ of super-Brownian motion has zero Lebesgue measure and fractal dimension 2. The exact Hausdorff measure properties of $S(Y_t)$ are also known. In this paper we show that, for $d \geqslant 3$ there is no measure function ϕ such that the packing measure $\phi - p(S(Y_t))$ is finite and positive, and give an integral test which distinguishes those ϕ which make the packing measure 0 or $+\infty$. Incomplete results are also obtained for $d = 2$.

Six degrees of separation…
… in mathematics

The Erdős number of a mathematicain is the degree of separation, in terms of mathematical collaboration, she or he has with Paul Erdős.

My Erdős number is 4

- I have written an article with Nicolas Curien;
- Nicolas Curien has written an article with Jean-François Le Gall;
- Jean-François Le Gall has written an article with Samuel J. Taylor;
- Samuel J. Taylor has written an article with Paul Erdős.

SOME PROBLEMS CONCERNING THE STRUCTURE OF RANDOM WALK PATHS

By

P. ERDŐS (Budapest), corresponding member of the Academy,
and S. J. TAYLOR (Birmingham)

1. Introduction. We restrict our consideration to symmetric random walk, defined in the following way. Consider the lattice formed by the points of d-dimensional Euclidean space whose coordinates are integers, and let a point $S_d(n)$ perform a move randomly on this lattice according to the rules: at time zero it is at the origin and if at any time $n-1$ $(n=1,2,\ldots)$ it is at some point S of the lattice, then at time n it will be at one of the $2d$ lattice points nearest S, the probability of it being at any specified one of these being $\frac{1}{2d}$.

In the present note we examine in some detail the structure of the *path* formed by the points $S_d(0), S_d(1), \ldots, S_d(n), \ldots$. We will sometimes be interested in the first n points of the path, and at others in some property of the infinite path obtained as $n\to\infty$. Our results will depend to a large extent on those obtained in [2]; for convenience we shall use a notation which is consistent with that paper. In Section 2 we summarise the notations used and obtain some preliminary results which will be needed in the sequel.

Six degrees of separation… in mathematics

The Erdős number of a mathematicain is the degree of separation, in terms of mathematical collaboration, she or he has with Paul Erdős.

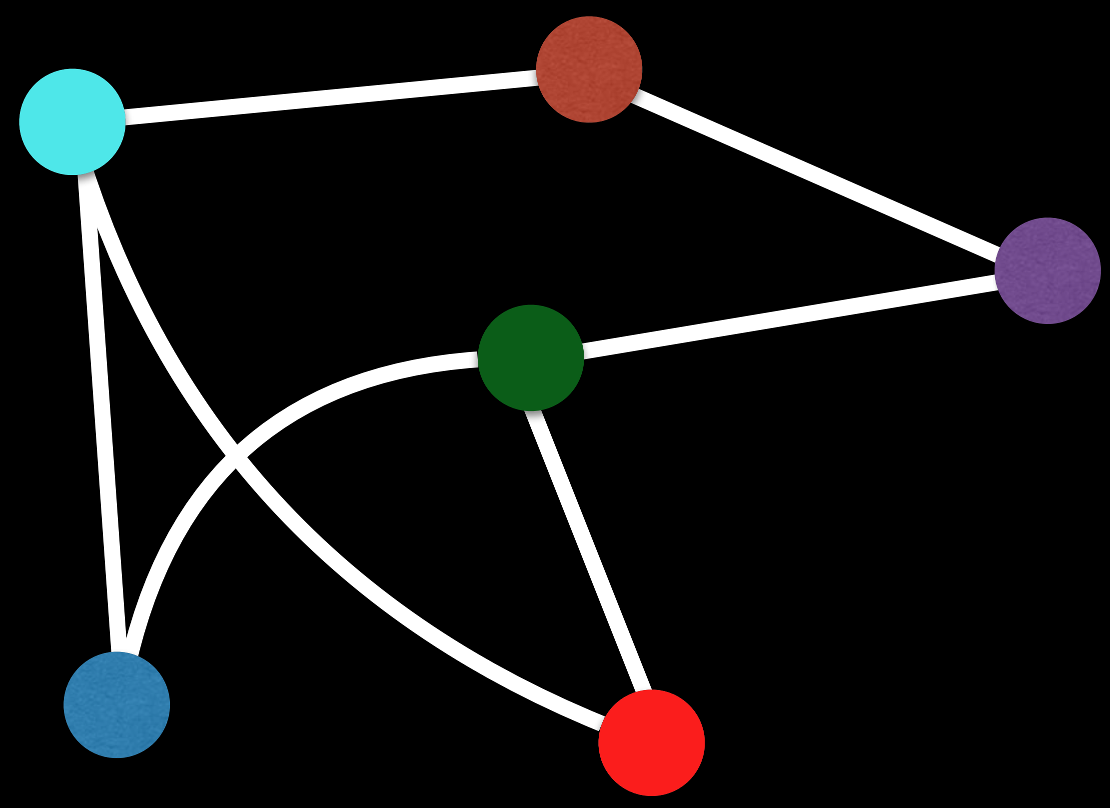
My Erdős number is 4

- I have written an article with Nicolas Curien;
- Nicolas Curien has written an article with Jean-François Le Gall;
- Jean-François Le Gall has written an article with Samuel J. Taylor;
- Samuel J. Taylor has written an article with Paul Erdős.

How can we explain the phenomenon of six degrees of separation?

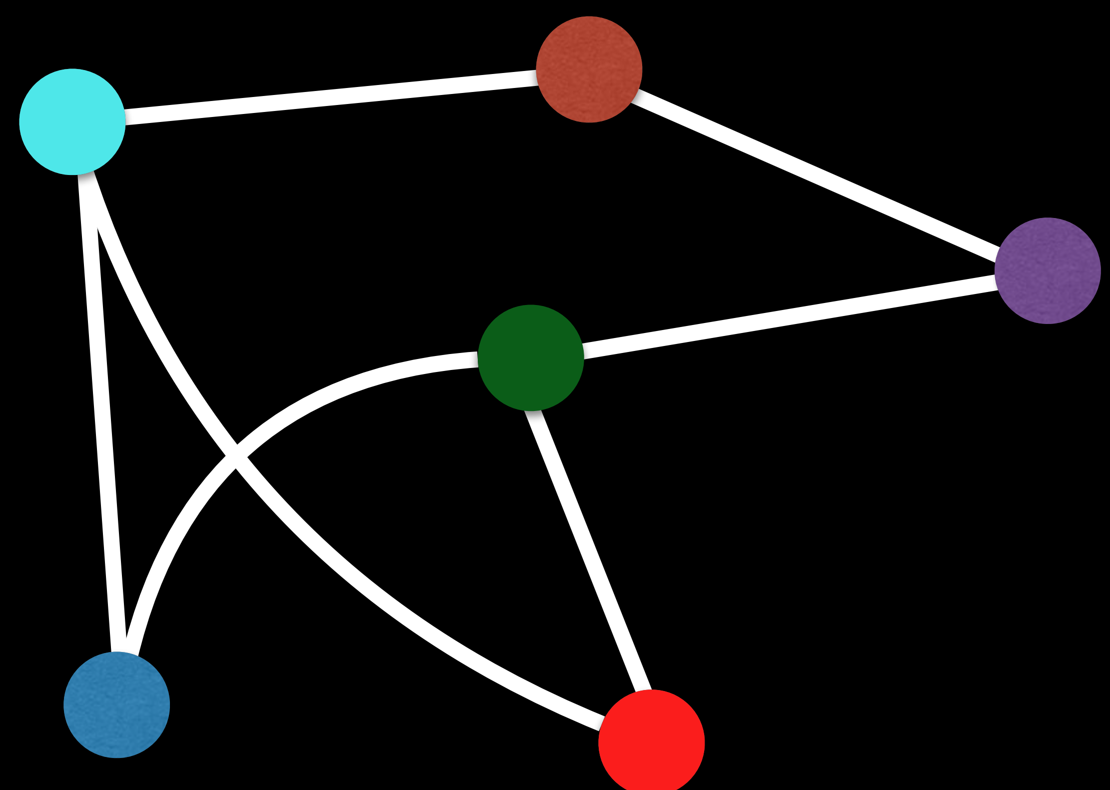
Modelling: graphs

Notion of graph



Notion of graph

A **graph** is a collection of vertices and of edges connecting some vertices between them.



Notion of graph

A **graph** is a collection of vertices and of edges connecting some vertices between them.

Another example of a graph:

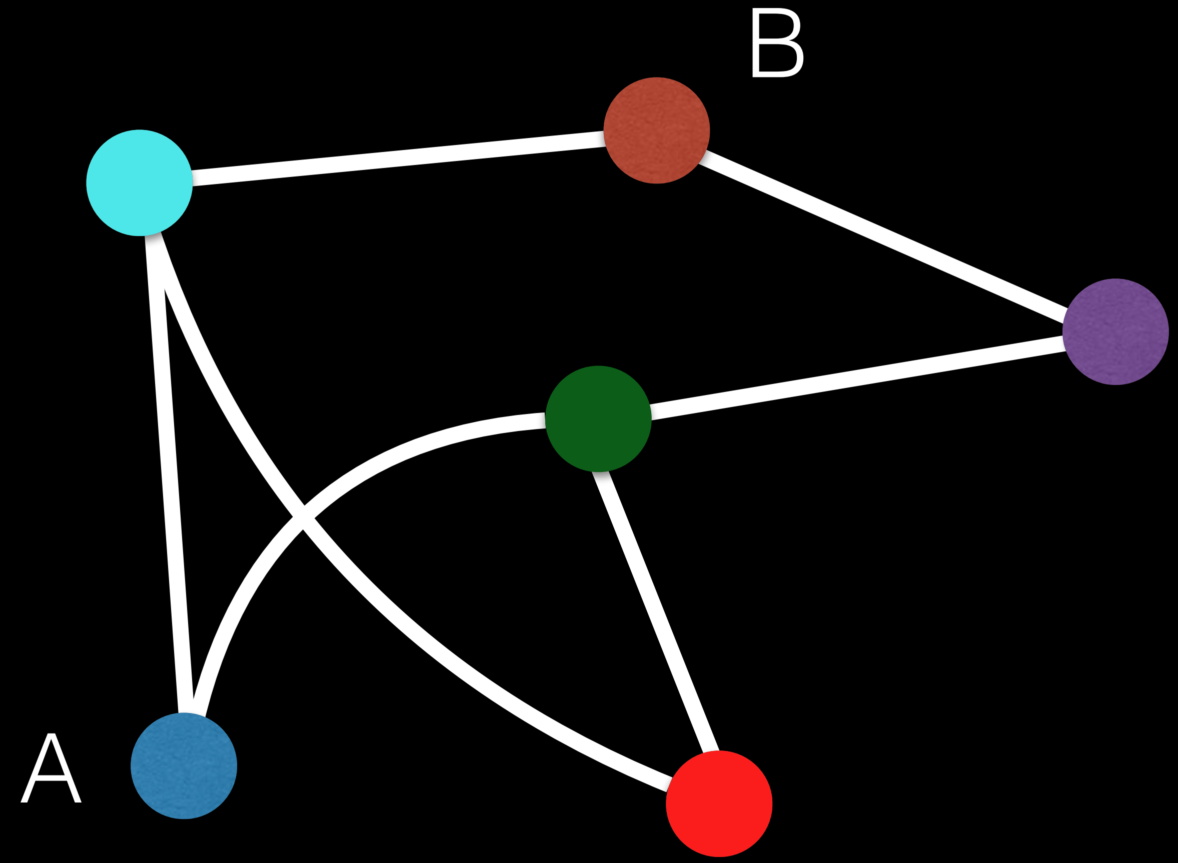
- the vertices are the people in this room
- two persons are connected by an edge if they know each other

Distance between 2 points

The **distance** between two vertices of a graph is the minimal number of edges separating them.

Distance between 2 points

The **distance** between two vertices of a graph is the minimal number of edges separating them.



The distance between A and B is 2.

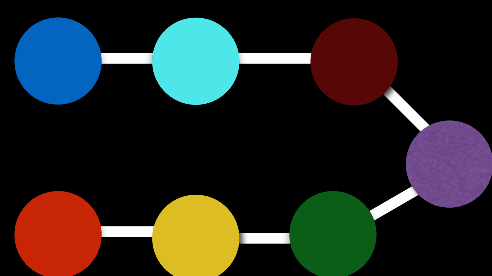
Diameter of a graph

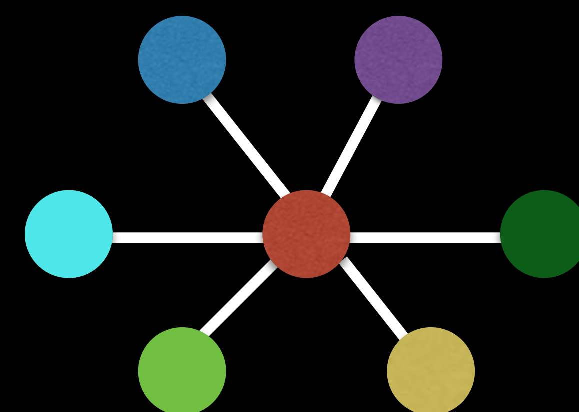
The **diameter** of a graph is the distance between two vertices who are the farthest apart.

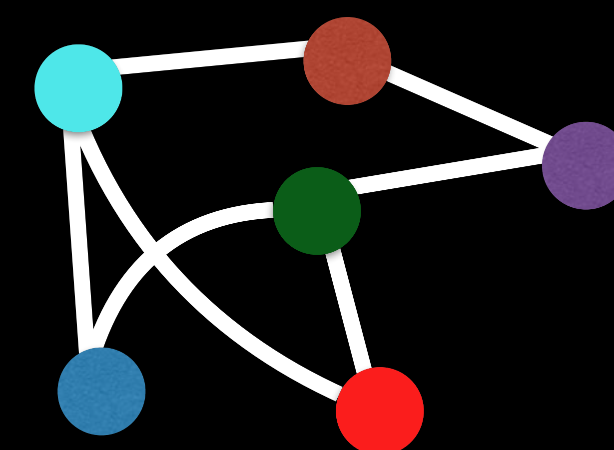
Diameter of a graph

The **diameter** of a graph is the distance between two vertices who are the farthest apart.

Graphs:





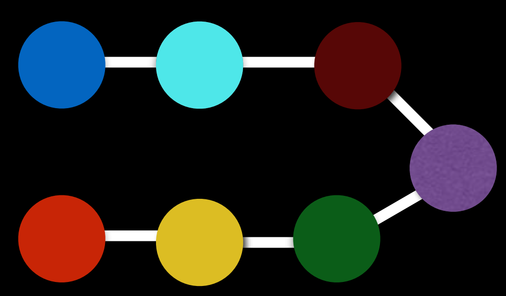


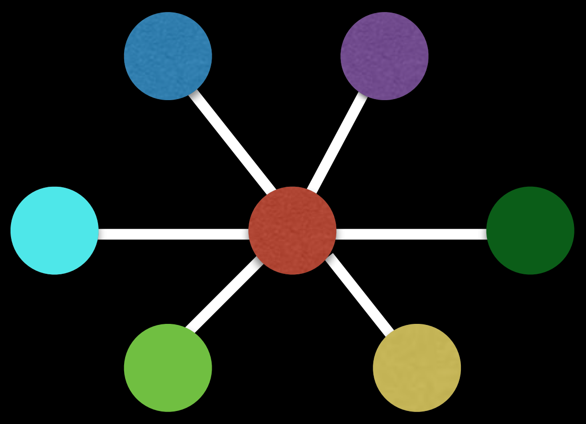
Diameter: 6 2 2

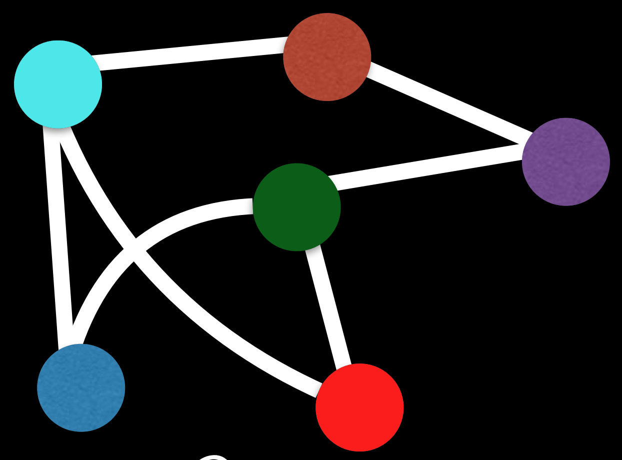
Diameter of a graph

The **diameter** of a graph is the distance between two vertices who are the farthest apart.

Graphs:







Diameter: 6 2 2

Saying that a graph satisfies the six degrees of separation phenomenon amounts to saying that its diameter is at most 6!

How can we explain the phenomenon of six degrees of separation?

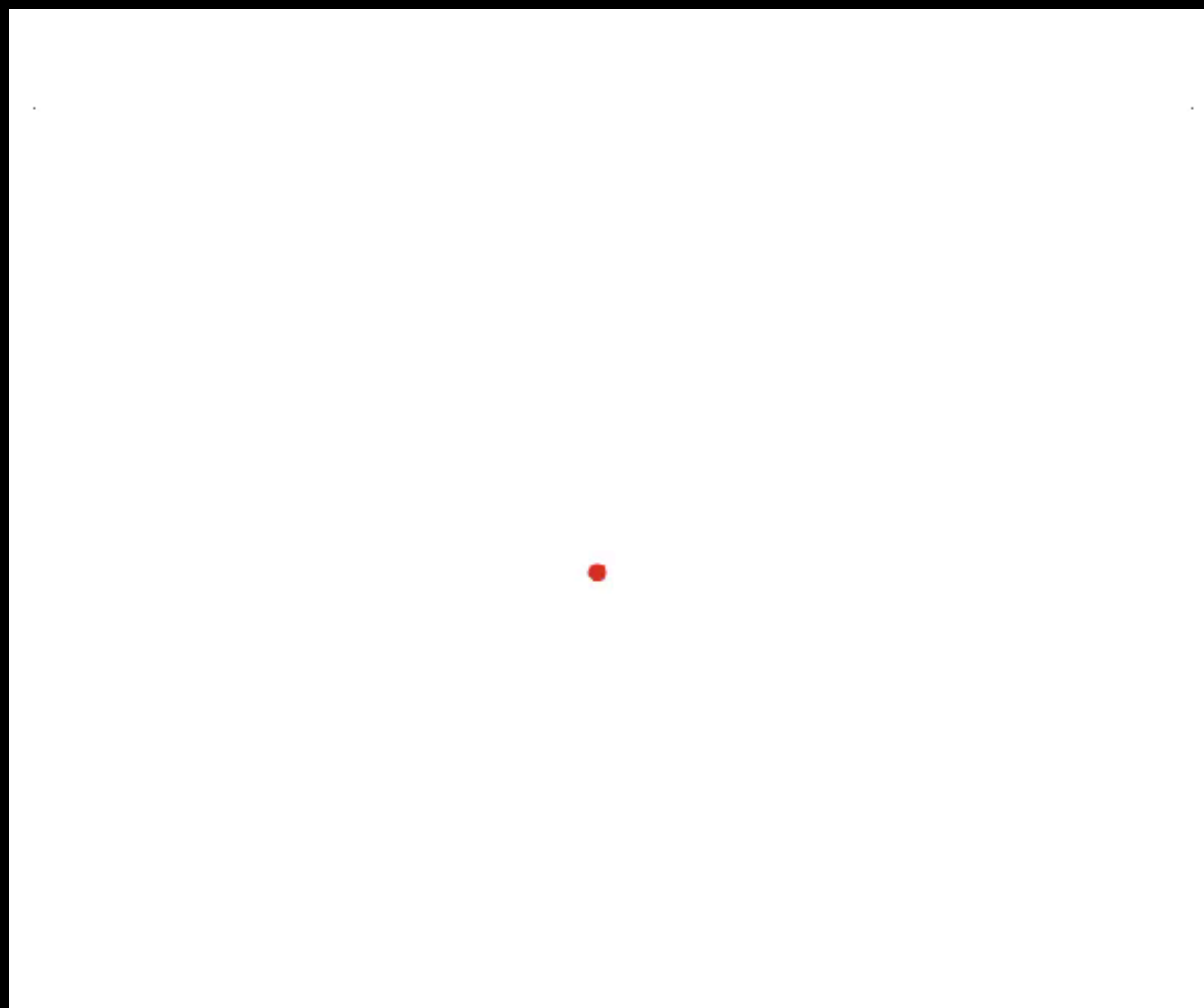
A « simulator »

Graphs and randomness

In 1999, Albert and Barabási suggested to use a simulator based on randomness and on a mechanism of popularity reinforcment.

Graphs and randomness

In 1999, Albert and Barabási suggested to use a simulator based on randomness and on a mechanism of popularity reinforcment.



Conclusion

The simulator of Albert and Barabási produces graphs that look like those of the real world.

In this simulator, the diameter grows (roughly) by 1 when one multiplies by 10 the number of vertices.

Karinthy was partly right, partly wrong.

What is **discrete** mathematics?

Discrete mathematics

Mathematics can by roughly divided in two realms:

- the continuous (real numbers)
- the discrete (integer numbers)

Comparison with watches:

- analog watches: continuous
- digital watches: discrete

Discrete mathematics:

- provides excellent tools and models for analyzing real-world phenomena.
- is the entrance to mathematics.

What is

mathematics?

Mathematics

What is success in mathematics?

In mathematics, the main success is a **proof**.

Actually, mathematics is **proofs**.

> A **proof** is an essay in which, starting from axioms, an assertion (such as « there are infinitely many prime numbers ») is incontrovertibly and universally shown to be correct.

A main goal of this course is to learn how to write **proofs**.

Proofs train us to think and communicate clearly, and present our case logically.

Mathematics in the real world

In mathematics, the axioms are taken for granted.

What are the axioms of the real world?

Mathematics in the real world

In order to apply mathematics in the real world, one has to choose a model (=«the axioms ») to make predictions.

If the model has nothing to do with reality, one can still study the model, but beware the consequences!

Questions concern models in the real world/our society:

- how to define happiness?
- how to define the quality of Education?
- ...

Mathematics

Proofs train us to think and communicate clearly, present our case logically and keep a critical mind.

Discrete mathematics

MAA 103

Discrete mathematics MAA 103

- Course on the blackboard, following Discrete Mathematics by Schneierman.
- Course webpage (there is a link on the moodle):

  http://www.cmap.polytechnique.fr/~kortchemski/dmaths/

Grading:

- 50% homework (each exercise sheet will contains a homework assignement, which has to be handed the next week to your TA).
- 25% midterm exam (probably November 12)
- 35% final exam (end of January)

You ARE are allowed and encouraged to discuss the homework problems with other students. However, all written solutions must be individually submitted and must not be copied from somewhere else.

Discrete mathematics MAA 103

Work regularly and keep up the pace!

- Tutoring sessions (only if needed)