

Week 11: Combinatorics: additional exercises

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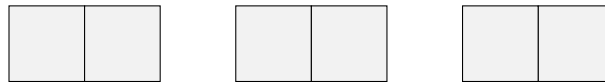
Exercise 1.

1. In how many ways can we write the 6 integers between 1 and 6 in the following squares



so that the first number is less than the second number?

2. In how many ways can we write the 6 integers between 1 and 6 in the following squares



so that the three numbers are in increasing order?

Exercise 2. Let $1 \leq n \leq p$ be integers. How many (strictly) increasing functions from $\{1, 2, \dots, n\} \rightarrow \{1, 2, \dots, p\}$ are there?

Exercise 3. Let $n \geq 2$ be an integer and let us consider a deck of n cards numbered from 1 to n .

1. In how many ways is it possible to shuffle the deck so that the card with number 1 is further in the deck than the card 2?
2. In how many ways is it possible to shuffle the deck so that the cards with numbers 1 and 2 are neighbours?

Exercise 4. Let $1 \leq p \leq n$ be integers. Let E be a set with n elements and A a subset of E with p elements.

- 1) How many subsets X of E such that $A \subset X$ are there?
- 2) If $p \leq m \leq n$, how many subsets X of E such that $A \subset X$ are there?
- 3) How many couples (X, Y) of subsets of E such that $X \cap Y = A$ are there?

Exercise 5. Let $n \geq 2$ be an integer. Find the number of permutations $\sigma \in S_n$ such that 1 and n belong to the same orbit of σ (that is, such that there exists an integer $k \geq 1$ with $\sigma^k(1) = n$).