Corrigendum for the paper
The geometry of random minimal factorizations of a long cycle via biconditioned bitype random trees
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The last display in the proof of Lemma 4.12 is incorrect; the authors are grateful to Victor Dubach for pointing out this mistake. As a consequence, Propositions 4.8, Lemmas 4.12, Lemma 4.13, Corollary 4.14 and Proposition 4.15 are incorrect as stated in the article due to a missing constant. We explain here how to correct them.

Most importantly, we emphasize that the corrections do not affect the proof of Theorem 1.3 (iii).

The correct statements are the following.

PROPOSITION 4.8. *Assume that* $\frac{K_n}{\sqrt{n}} \to \infty$ *and* $\frac{K_n}{n} \to \gamma \in [0,1)$ *as* $n \to \infty$*. Set* $D_n^\circ = \sqrt{(\sigma_\circ^n)^2 K_n}$*. Then*

$$\left(\frac{\overline{B}^n_{\lfloor u(n-K_n)\rfloor}(\tilde{\mathscr{T}}_n)}{\sqrt{1+\alpha_\gamma^2} \cdot D_n^\circ} : 0 \leq u \leq 1 \right) \quad \xrightarrow[n\to\infty]{(d)} \quad \mathbbm{e}$$

for a certain constant $\alpha_\gamma \geq 0$*. In addition, if* $\gamma > 0$*, then*

$$\sup_{0\leq u\leq 1} \left| \frac{\overline{H}^n_{\lfloor u(n-K_n)\rfloor}(\tilde{\mathscr{T}}_n)}{K_n} - u \right| \quad \xrightarrow[n\to\infty]{(\mathbb{P})} \quad 0 \qquad \textit{and} \qquad \frac{1}{n} \max_{1\leq i\leq n} \ell_i^{\bullet,n} \quad \xrightarrow[n\to\infty]{(\mathbb{P})} \quad 0.$$

The last display in the proof of Lemma 4.12 should read

$$\left(\widehat{H}_1^{(n)}, \frac{\overline{B}^n_{n-K_n} - \frac{n-K_n}{K_n+1} \widehat{H}_1^{(n)} D_n^\bullet}{D_n^\circ} \right) \xrightarrow[n\to\infty]{} (W_1, X_1),$$

where W_1, X_1 are independent standard Gaussian random variables. Accordingly, one needs to modify the definition of $\widehat{B}^{(n)}$ in the following way (the factor $D_n^\bullet$ was forgotten in the article):

$$\widehat{B}_u^{(n)} = \frac{\overline{B}^n_{\lfloor u(n-K_n)\rfloor} - \frac{n-K_n}{K_n+1} D_n^\bullet \widehat{H}_u^{(n)}}{D_n^\circ}.$$

The statements of Lemma 4.12 and Lemma 4.13 are then correct with this modified definition of $\widehat{B}^{(n)}$. However, the statement of Corollary 4.14 should be modified as follows.

COROLLARY 4.14. *Assume that, as* $n \to \infty$*, we have* $\frac{K_n}{\sqrt{n}} \to \infty$ *and* $\frac{K_n}{n} \to \gamma$ *with* γ *in* $[0,1)$*. There is a constant* $\alpha_\gamma \geq 0$ *such that conditionally given the event* $\{\overline{H}^n_{n-K_n} = K_n + 1, \overline{B}^n_{n-K_n} = -1\}$*, the following convergence in distribution holds jointly*

$$\left(\frac{\overline{B}^n_{\lfloor u(n-K_n)\rfloor}}{D_n^\circ} \right)_{0\leq u\leq 1} \xrightarrow[n\to\infty]{(d)} X^{\mathrm{br}} + \alpha_\gamma W^{\mathrm{br}}, \qquad \sup_{0\leq u\leq 1} \left| \frac{\overline{H}^n_{\lfloor u(n-K_n)\rfloor}}{K_n} - u \right| \xrightarrow[n\to\infty]{} 0.$$

Proof. We have

$$\frac{\overline{B}^n_{\lfloor u(n-K_n)\rfloor}}{D_n^\circ} = \widehat{B}_u^{(n)} + \frac{(n-K_n)D_n^\bullet}{(K_n+1)D_n^\circ}\,\widehat{H}_u^{(n)}$$

When $\gamma > 0$, we have

$$\frac{D_n^\bullet(n-K_n)}{D_n^\circ(K_n+1)} \sim \frac{1-\gamma}{\gamma^{3/2}}\frac{\sigma_\bullet^n}{\sigma_\circ^n}.$$

In the regime $\gamma > 0$, the quantities $\sigma_\bullet^n$ and $\sigma_\circ^n$ have positive limits as n tends to $+\infty$, so that $\frac{D_n^\bullet(n-K_n)}{D_n^\circ(K_n+1)}$ tends to a positive limit denoted by α_γ. When $\gamma = 0$, $\frac{D_n^\bullet(n-K_n)}{D_n^\circ(K_n+1)} \to \alpha_0 := 0$. The first part of Corollary 4.14 (convergence of $\frac{\overline{B}^n_{\lfloor u(n-K_n)\rfloor}}{D_n^\circ}$) then follows from Lemma 4.13. The proof of the convergence of $\sup_{0\le u\le 1}\left|\frac{\overline{H}^n_{\lfloor u(n-K_n)\rfloor}}{K_n} - u\right|$ is unchanged. $\square$

Using the fact that $X^{\mathrm{br}} + \alpha_\gamma \cdot W^{\mathrm{br}}$ has the same distribution as $\sqrt{1+\alpha_\gamma^2}\cdot X^{\mathrm{br}}$, Proposition 4.15 is modified as follows.

PROPOSITION 4.15. *Assume that* $\frac{K_n}{\sqrt{n}} \to \infty$ *and* $\frac{K_n}{n} \to \gamma \in [0,1)$ *as* $n \to \infty$. *There is a constant* $\alpha_\gamma \ge 0$ *such that the following convergences hold jointly in distribution*

$$\left(\frac{\overline{B}_{\lfloor u(n-K_n)\rfloor}(\mathscr{T}_n)}{\sqrt{1+\alpha_\gamma^2}\cdot D_n^\circ} : 0 \le u \le 1\right) \xrightarrow[n\to\infty]{(d)} \mathbb{e}, \qquad \sup_{0\le u\le 1}\left|\frac{\overline{H}_{\lfloor u(n-K_n)\rfloor}(\mathscr{T}_n)}{K_n} - u\right| \xrightarrow[n\to\infty]{} 0.$$

The multiplicative constant $\sqrt{1+\alpha_\gamma^2}$ does not affect the proof of Theorem 1.3 (ii).